



Everyone's Smart
University

Uncertainty Modeling

Lecture 11

PMG510

Project Risk Management

Modeling Uncertainty: From Scenarios to Continuous Distributions



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Scenario	Probability
-0.00024	0.05
0.01760	0.05
-0.02114	0.05
-0.01178	0.05
-0.01515	0.05
-0.00353	0.05
-0.01772	0.05
-0.02345	0.05
0.03562	0.05
0.03108	0.05
0.01557	0.05
0.00073	0.05
-0.02188	0.05
0.02063	0.05
0.03044	0.05
0.01276	0.05
0.01214	0.05
0.00138	0.05
-0.00507	0.05
0.01134	0.05

Scenario 1: Return $R_1 = -0.00024$, occurring with probability $p_1 = 0.05$

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Reward



Expected Value of R:

$$E(R) = p_1 * R_1 + p_2 * R_2 + \dots + p_{19} * R_{19} + p_{20} * R_{20}$$

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Risk

Standard Deviation of R:

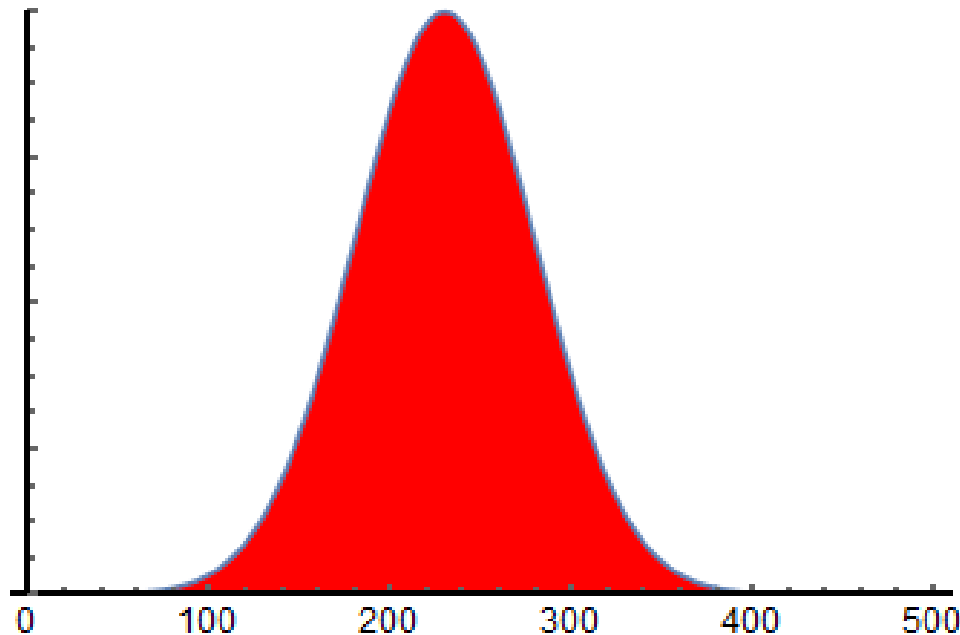
$SD(R) =$

$$p_1*(R_1 - E(R))^2 + \dots + p_{20}*(R_{20} - E(R))^2$$

Modeling Uncertainty: From Scenarios to Continuous Distributions

- ◆ What if the number of possible values of the random variable (scenarios) is *infinite*, and the future is described by a continuous distribution?

Probability
Density



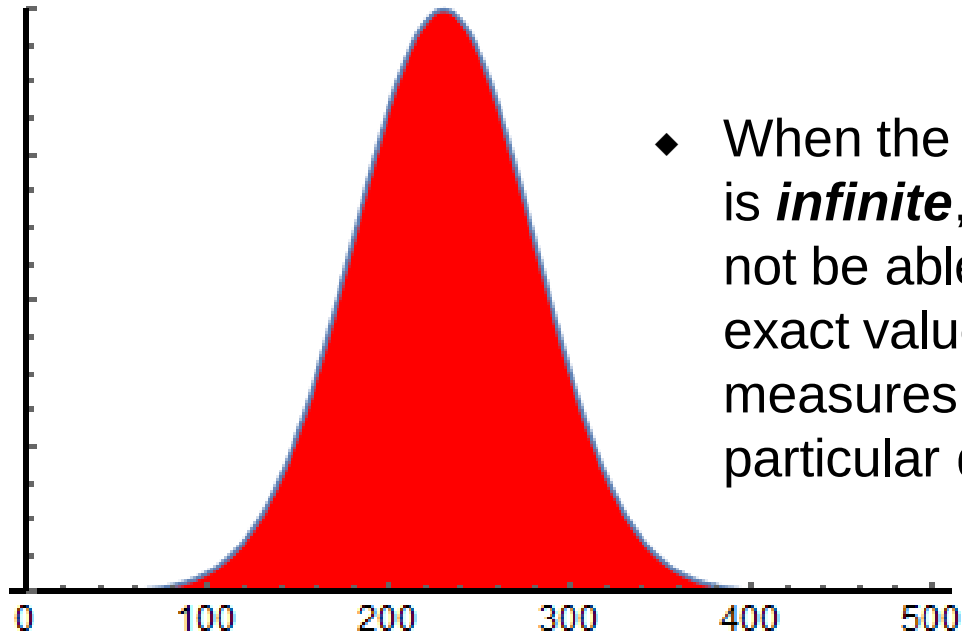
Modeling Uncertainty: From Scenarios to Continuous Distributions



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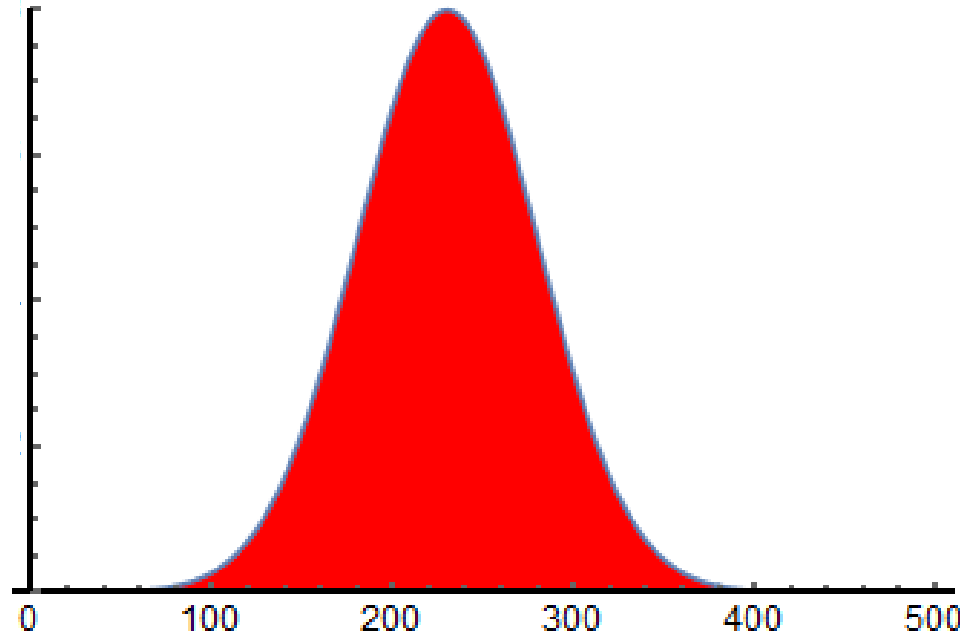


- ◆ When the number of scenarios is *infinite*, Excel, in general, will not be able to calculate the exact values of reward and risk measures corresponding to a particular decision

Simulation: Making Decisions when Randomness is Described by a Continuous Distribution

- ◆ For a number of probability distributions (for example, normal distribution) Excel can generate instances of a random variable coming from that distribution

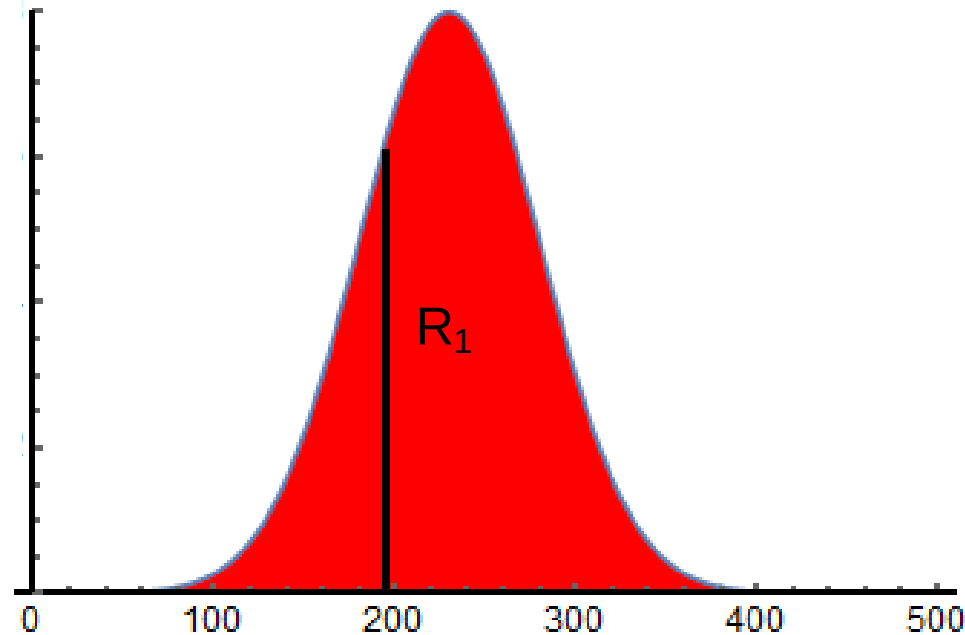
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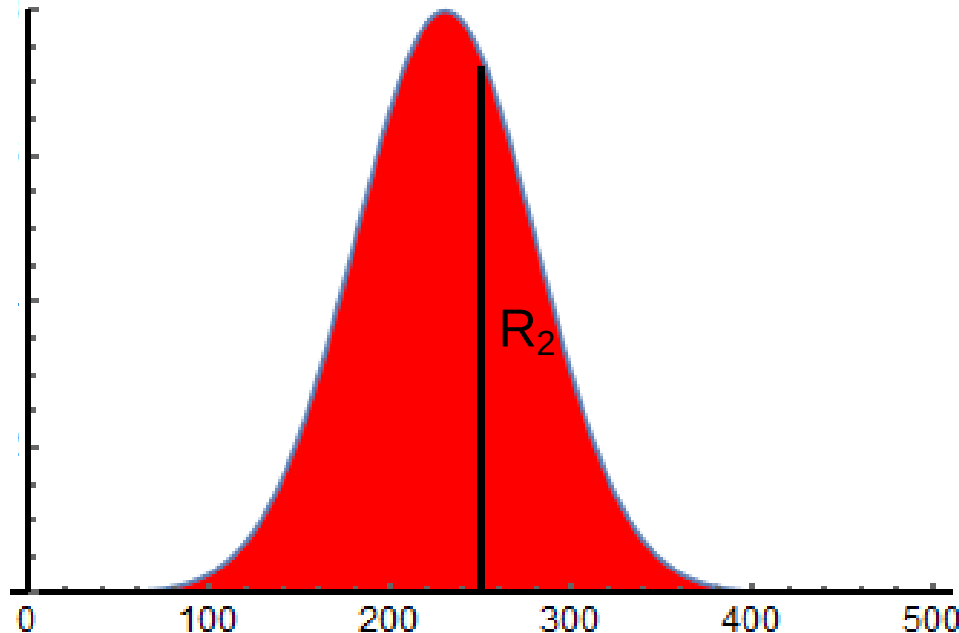
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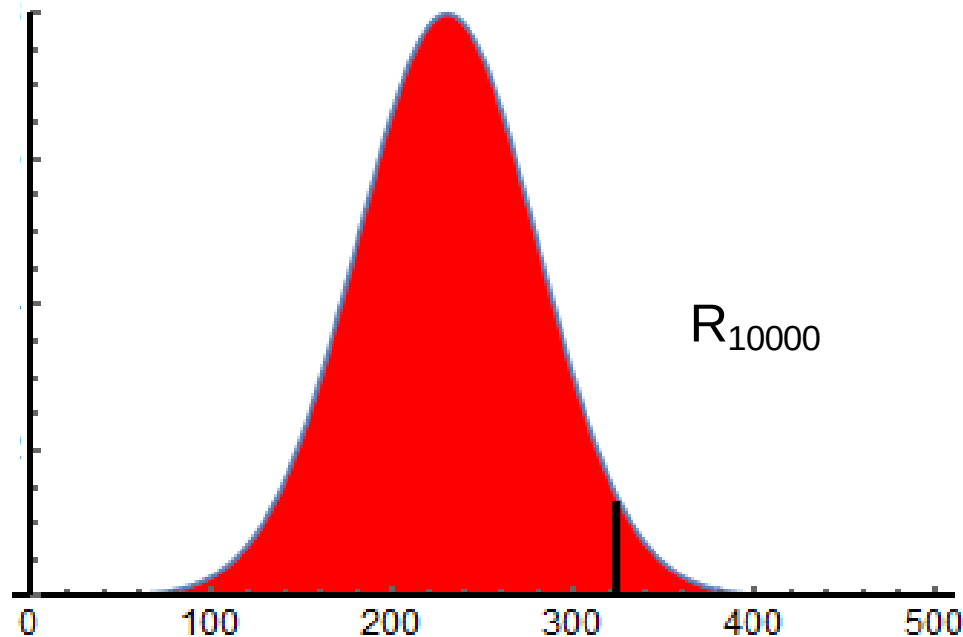
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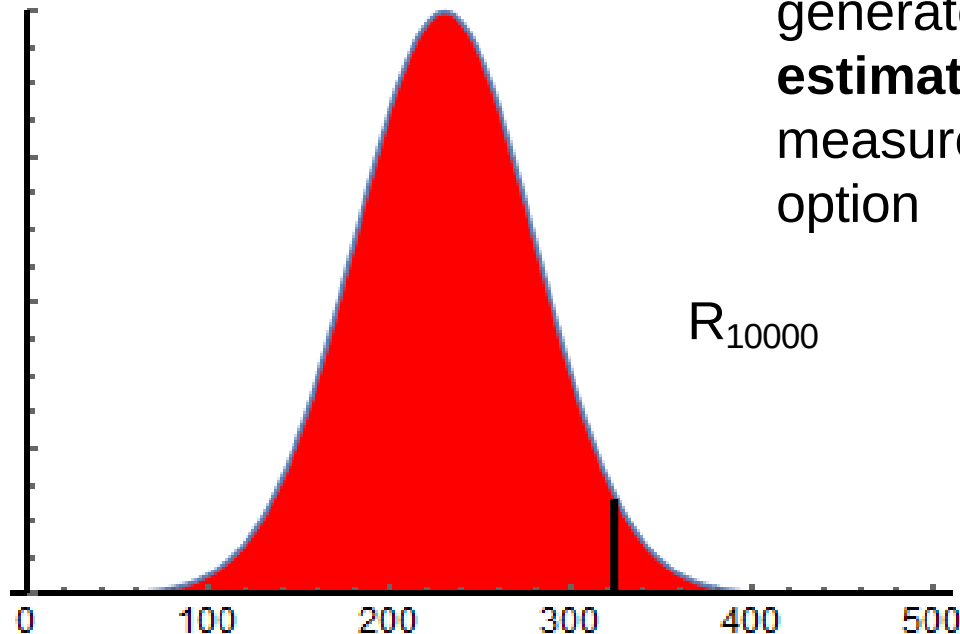
**Probability
Density**



Simulation: Making Decisions when Randomness is Described by a Continuous Distribution

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Probability
Density



One can use scenarios generated in such a way to **estimate** the reward and risk measures for any decision option

Example: Designing a New Apartment Building



- ◆ The Stargrove Development Corporation is planning a new apartment building in Philadelphia

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- ◆ The building will have two kinds of apartments, regular and luxury
- ◆ The building will have 15 floors, with lower floors housing regular apartments and higher floors – luxury apartments
- ◆ Each floor will contain only one kind of apartments – either regular or luxury
- ◆ Each floor can house either 8 regular apartments or 4 luxury apartments

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- ◆ The building will have 15 floors, with lower floors housing regular apartments and higher floors – luxury apartments
- ◆ Each floor will contain only one kind of apartments – either regular or luxury
- ◆ Each floor can house either 8 regular apartments or 4 luxury apartments
- ◆ Stargrove needs to decide how many floors to allocate to regular apartments and how many floors to allocate to luxury apartments

Example: Designing a New Apartment Building



- ◆ Stargrove expects to complete construction within the next year, and during that period, it plans to sell apartments to prospective buyers
- ◆ Stargrove expects to obtain a profit of $P_R = \$500,000$ for each regular apartment it sells during the next year, and a profit of $P_L = \$900,000$ for each luxury apartment it sells during the next year
- ◆ If, at the end of the next year, there are unsold apartments, Stargrove will sell all of them to a real estate investment company at a “salvage” profit of $S_R = \$100,000$ for each remaining regular apartment and $S_L = \$150,000$ for each remaining luxury apartment

Example: Designing a New Apartment Building



- ◆ Stargrove analysts project that at the price levels that the company will charge for the apartments, the total demand for **regular apartments** over the next year will be **normally distributed with a mean of 90 and a standard deviation of 25**, and the total demand for **luxury apartments** over the next year will be **normally distributed with a mean of 10 and a standard deviation of 3**. The demands for two apartment types will be modeled as independent (non-correlated) random variables

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- ◆ Normal random variables can take fractional and negative values, while the demand values must be positive and integer. The values used for estimating the reward and risk measures will take this into account

Example: Designing a New Apartment Building



- ◆ Stargrove knows that if the demand for either type of apartment exceeds the availability of the apartments, all those extra prospective buyers will be lost
- ◆ None of the prospective buyers looking for a regular apartment will pay for a luxury one
- ◆ None of the prospective buyers looking for a luxury apartment will want a regular one

Number of Sold Apartments

- ◆ So, if the number of regular apartments built is R , and the demand for regular apartments during the next year is D_R , then the number of regular apartments sold during the next year at the profit $P_R = \$500,000$ each will be $\min(R, D_R)$, i.e., the smallest of two numbers, R and D_R

Number of Sold Apartments



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- ◆ For example, if $R = 96$, and $D_R = 90$, the number of regular apartments sold during the year is $\min(R, D_R) = \min(96, 90) = 90$

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- ◆ For example, if $R = 96$, and $D_R = 90$, the number of regular apartments sold during the year is $\min(R, D_R) = \min(96, 90) = 90$
- ◆ On the other hand, if $R = 96$, and $D_R = 100$, the number of regular apartments sold during the year is $\min(R, D_R) = \min(96, 100) = 96$

Number of Sold Apartments



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- ◆ For example, if $R = 96$, and $D_R = 90$, the number of regular apartments sold during the year is $\min(R, D_R) = \min(96, 90) = 90$
- ◆ On the other hand, if $R = 96$, and $D_R = 100$, the number of regular apartments sold during the year is $\min(R, D_R) = \min(96, 100) = 96$
- ◆ The rest of the regular apartments, $R - \min(R, D_R)$ will be “salvaged” at the profit $S_R = \$100,000$ each

Number of Sold Apartments



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- ◆ Similarly, if the number of luxury apartments built is L , and the demand for luxury apartments during the next year is D_L , then the number of luxury apartments sold during the next year at the profit $P_L = \$900,000$ each will be $\min(L, D_L)$
- ◆ The number of luxury apartments “salvaged” at the profit $S_L = \$150,000$ each will be $L - \min(L, D_L)$

Evaluating an Alternative



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- ◆ Stargrove wants to evaluate the plan of having 12 regular floors (with $R = 12 \times 8 = 96$ regular apartments) and 3 luxury floors (with $L = 3 \times 4 = 12$ luxury apartments)

Evaluating an Alternative

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- ◆ The company needs to evaluate a distribution of profit Π it will earn under this plan

Evaluating an Alternative

- ◆ Stargrove wants to evaluate the plan of having 12 regular floors (with $R = 12 \times 8 = 96$ regular apartments) and 3 luxury floors (with $L = 3 \times 4 = 12$ luxury apartments)
- ◆ The company needs to evaluate a distribution of profit Π it will earn under this plan
- ◆ In particular, Stargrove is interested in estimating the expected profit $E(\Pi)$ and the likelihood that the profit value will be below \$45,000,000

Number of Sold Apartments

- ◆ In Stargrove's case, the profit Π is a function of the demand values for regular and luxury apartments, D_L and D_R



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Number of Sold Apartments

- ◆ In Stargrove's case, the profit Π is a function of the demand values for regular and luxury apartments, D_L and D_R
- ◆ Since the demand values are random, the profit may also be random



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Number of Sold Apartments



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- ◆ In Stargrove's case, the profit Π is a function of the demand values for regular and luxury apartments, D_L and D_R
- ◆ Since the demand values are random, the profit may also be random
- ◆ In algebraic terms, the profit value (in \$) can be expressed as

$$\begin{aligned}\Pi = & 500,000 * \min(D_R, R) + 900,000 * \min(D_L, L) \\ & + 100,000 * (R - \min(D_R, R)) + 150,000 * (L - \min(D_L, L))\end{aligned}$$

Number of Sold Apartments



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- ◆ In a simulation, the demand values for regular and luxury apartments are **random inputs** and the profit value is a **random output**
- ◆ Simulation generates instances of random inputs and calculates the corresponding instances of random outputs

Number of Sold Apartments

- ◆ In algebraic terms, the profit is

$$\begin{aligned}\Pi = & 500,000 * \min(D_R, R) + 900,000 * \min(D_L, L) \\ & + 100,000 * (R - \min(D_R, R)) + 150,000 * (L - \min(D_L, L))\end{aligned}$$

- ◆ What is the distribution of Π ?
- ◆ What is the expected value of Π ?
- ◆ What is the probability that Π is less than 45,000,000?

Expected Value of Π ?

- ◆ In algebraic terms, the profit is

$$\begin{aligned}\Pi = & 500,000 * \min(D_R, R) + 900,000 * \min(D_L, L) \\ & + 100,000 * (R - \min(D_R, R)) + 150,000 * (L - \min(D_L, L))\end{aligned}$$

- ◆ Expected value of D_R is 90, and the expected value of D_L is 10
- ◆ For $R=96$, and $L=12$, can't we just plug in the expected values of D_R and D_L into the above formula and get the expected value of Π ?
- ◆ In general, we **do not** get the correct value for the expected random output that way

Expected Value of Π ?

- ◆ In algebraic terms, the profit is

$$\begin{aligned}\Pi = & 500,000 * \min(D_R, R) + 900,000 * \min(D_L, L) \\ & + 100,000 * (R - \min(D_R, R)) + 150,000 * (L - \min(D_L, L))\end{aligned}$$

- ◆ Example: D_R takes values of 65 and 115, each with probability 0.5, and D_L takes the values of 7 and 13, each with probability 0.5. D_R and D_L take these values in independent manner

Expected Value of Π ?

- ◆ In algebraic terms, the profit is

$$\begin{aligned}\Pi = & 500,000 \cdot \min(D_R, R) + 900,000 \cdot \min(D_L, L) \\ & + 100,000 \cdot (R - \min(D_R, R)) + 150,000 \cdot (L - \min(D_L, L))\end{aligned}$$

- ◆ Example: D_R takes values of 65 and 115, each with probability 0.5, and D_L takes the values of 7 and 13, each with probability 0.5. D_R and D_L take these values in independent manner
- ◆ The expected value of D_R is 90, and the standard deviation of D_R is 25, the expected value of D_L is 10, and the standard deviation of D_L is 3

Expected Value of Π ?

- ◆ In algebraic terms, the profit is

$$\begin{aligned}\Pi = & 500,000 * \min(D_R, R) + 900,000 * \min(D_L, L) \\ & + 100,000 * (R - \min(D_R, R)) + 150,000 * (L - \min(D_L, L))\end{aligned}$$

- ◆ Example: D_R takes values of 65 and 115, each with probability 0.5, and D_L takes the values of 7 and 13, each with probability 0.5. D_R and D_L take these values in independent manner
- ◆ The expected value of D_R is 90, and the standard deviation of D_R is 25, the expected value of D_L is 10, and the standard deviation of D_L is 3
- ◆ If we “plug” the expected values of D_R (90) and D_L (10) into the formula above, we get, for $R=96$ and $L=12$, the value of $500,000*90 + 900,000*10 + 100,000*6 + 150,000*2 = 54,900,000$

Expected Value of Π ?



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- ◆ The correct expected value of Π in this case can be calculated as follows
- ◆ If $D_R = 65$ and $D_L = 7$ (this happens with probability $0.5 \times 0.5 = 0.25$), then the value of profit Π is
$$\begin{aligned} & 500,000 \cdot \min(65, 96) + 900,000 \cdot \min(7, 12) \\ & + 100,000 \cdot (96 - \min(65, 96)) + 150,000 \cdot (12 - \min(7, 12)) \\ & = 500,000 \cdot 65 + 900,000 \cdot 7 + 100,000 \cdot 31 + 150,000 \cdot 5 = 42,650,000 \end{aligned}$$

Expected Value of Π ?



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- ◆ The correct expected value of Π in this case can be calculated as follows
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- ◆ If $D_R = 115$ and $D_L = 7$ (this also happens with probability $0.5 \times 0.5 = 0.25$), then the value of profit Π is
$$\begin{aligned} & 500,000 \cdot \min(115, 96) + 900,000 \cdot \min(7, 12) \\ & + 100,000 \cdot (96 - \min(115, 96)) + 150,000 \cdot (12 - \min(7, 12)) \\ & = 500,000 \cdot 96 + 900,000 \cdot 7 + 100,000 \cdot 0 + 150,000 \cdot 5 = 55,050,000 \end{aligned}$$

Expected Value of Π ?



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- ◆ If $D_R = 65$ and $D_L = 13$ (this happens with probability $0.5 \times 0.5 = 0.25$), then the value of profit Π is

$$\begin{aligned} & 500,000 \cdot \min(65, 96) + 900,000 \cdot \min(13, 12) \\ & + 100,000 \cdot (96 - \min(65, 96)) + 150,000 \cdot (12 - \min(13, 12)) \\ & = 500,000 \cdot 65 + 900,000 \cdot 12 + 100,000 \cdot 31 + 150,000 \cdot 0 = 46,400,000 \end{aligned}$$



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Expected Value of Π ?

- ◆ If $D_R = 65$ and $D_L = 13$ (this happens with probability $0.5 \times 0.5 = 0.25$), then the value of profit Π is

$$\begin{aligned} & 500,000 \cdot \min(65, 96) + 900,000 \cdot \min(13, 12) \\ & + 100,000 \cdot (96 - \min(65, 96)) + 150,000 \cdot (12 - \min(13, 12)) \\ & = 500,000 \cdot 65 + 900,000 \cdot 12 + 100,000 \cdot 31 + 150,000 \cdot 0 = 46,400,000 \end{aligned}$$

- ◆ Finally, if $D_R = 115$ and $D_L = 13$ (this also happens with probability $0.5 \times 0.5 = 0.25$), then the value of profit Π is

$$\begin{aligned} & 500,000 \cdot \min(115, 96) + 900,000 \cdot \min(13, 12) \\ & + 100,000 \cdot (96 - \min(115, 96)) + 150,000 \cdot (12 - \min(13, 12)) \\ & = 500,000 \cdot 96 + 900,000 \cdot 12 + 100,000 \cdot 0 + 150,000 \cdot 0 = 58,800,000 \end{aligned}$$

- ◆ The expected value Π is

$$\begin{aligned} & 0.25 \cdot (42,650,000) + 0.25 \cdot (55,050,000) + 0.25 \cdot (46,400,000) \\ & + 0.25 \cdot (58,800,000) = 50,725,000 - \text{much lower value than } 54,900,000 \end{aligned}$$