



مجموعة جامعة
الجميع الذكية
للتعليم القابضة

MIDOCEAN
UNIVERSITY
BY EVERYONE'S SMART UNIVERSITY

Mathematical Foundations For Data Science and AI

Lecture 08

AI102

Probability; Basic Concepts

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www.midocean.ae

Lecture

08

LEARNING OUTCOMES

- Understanding the meaning of Probability.
- Understanding The Complement Rule.
- Understanding Sum of Probabilities(Disjoint-Events).
- Understanding Sum of Probabilities(Joint-Events).
- Understanding Independence.
- Understanding Conditional Probability.
- Understanding Bayes Theorem.
- Understanding the use of probability in machine learning.

Probability is a measure of how likely an event is to occur.



$$P_{(\text{Football player})} = \frac{\text{No. of Football players}}{\text{Total No. of Students}} = \frac{3}{10} = 0.3$$



50 %

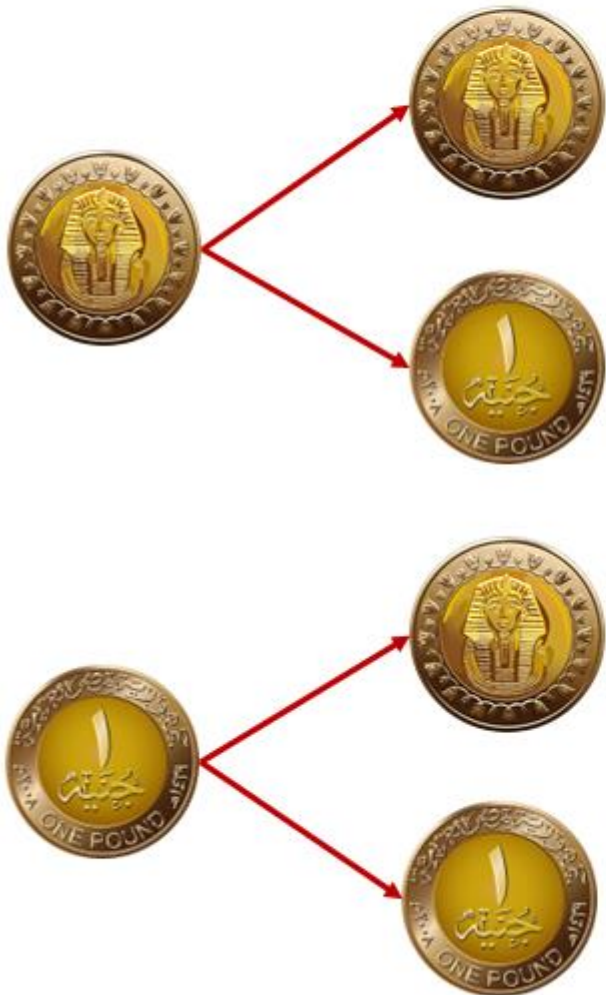


50 %

$$P_{(Head)} =$$



$$P_{(Head)} = \frac{1}{2} = 0.5$$

*1st Round**2nd Round*

$$P_{(Head, Head)} = \frac{1}{4} = 0.25$$



$$P_{(6)} =$$



$$P_{(6)} = \frac{1}{6} = 0.167$$



	1,1	1,2	1,3	1,4	1,5	1,6
	2,1	2,2	2,3	2,4	2,5	2,6
	3,1	3,2	3,3	3,4	3,5	3,6
	4,1	4,2	4,3	4,4	4,5	4,6
	5,1	5,2	5,3	5,4	5,5	5,6
	6,1	6,2	6,3	6,4	6,5	6,6



$$P_{(6,6)} =$$

	1,1	1,2	1,3	1,4	1,5	1,6
	2,1	2,2	2,3	2,4	2,5	2,6
	3,1	3,2	3,3	3,4	3,5	3,6
	4,1	4,2	4,3	4,4	4,5	4,6
	5,1	5,2	5,3	5,4	5,5	5,6
	6,1	6,2	6,3	6,4	6,5	6,6

$$P_{(6,6)} = \frac{1}{36} = 0.0278$$

Complement Rule



$$P_{(\text{Football player})} = \frac{\text{No. of Football players}}{\text{Total No. of Students}} = \frac{3}{10} = \boxed{0.3}$$

$$P_{(\text{Non-Football player})} = \frac{\text{No. of Non - Football players}}{\text{Total No. of Students}} = \frac{7}{10} = \boxed{0.7}$$

$$P_{(\text{Non-Football player})} = 1 - P_{(\text{Football player})} = 0.7$$

The complement rule works off of the idea that two parts make a whole. In probability, the "whole" refers to all possible outcomes.

$$P(A') = 1 - P(A)$$

Complement Rule

Near to 1 the probability of occurrence increases

$$0 < P < 1$$

Probability

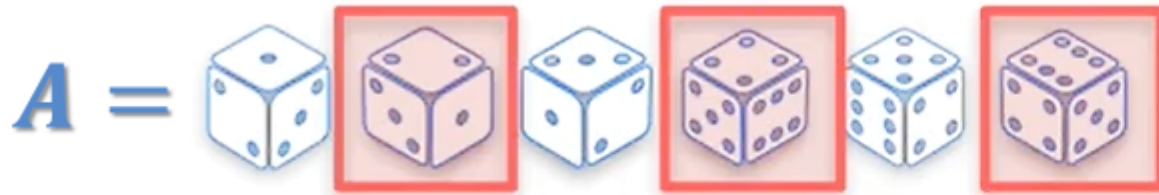
Near to 0 the probability of occurrence decreases

Sum of Probabilities (Disjoint-Events)

*The **Sum of probability (Disjoint-Events)** of a union of events can be calculated by summing probabilities together.*



What is the probability of obtaining an even number or number (5)?



$$\frac{3}{6} + \frac{1}{6} = \frac{4}{6} = \frac{2}{3} = 0.67$$

Sum of Probabilities (Disjoint-Events)



What is the probability of obtaining the sum of (7) or the sum of (10)?

A = Sum of (7)

	1,1	1,2	1,3	1,4	1,5	1,6
	2,1	2,2	2,3	2,4	2,5	2,6
	3,1	3,2	3,3	3,4	3,5	3,6
	4,1	4,2	4,3	4,4	4,5	4,6
	5,1	5,2	5,3	5,4	5,5	5,6
	6,1	6,2	6,3	6,4	6,5	6,6

B = Sum of (10)

	1,1	1,2	1,3	1,4	1,5	1,6
	2,1	2,2	2,3	2,4	2,5	2,6
	3,1	3,2	3,3	3,4	3,5	3,6
	4,1	4,2	4,3	4,4	4,5	4,6
	5,1	5,2	5,3	5,4	5,5	5,6
	6,1	6,2	6,3	6,4	6,5	6,6

Sum of Probabilities (Disjoint-Events)

	1,1	1,2	1,3	1,4	1,5	1,6
	2,1	2,2	2,3	2,4	2,5	2,6
	3,1	3,2	3,3	3,4	3,5	3,6
	4,1	4,2	4,3	4,4	4,5	4,6
	5,1	5,2	5,3	5,4	5,5	5,6
	6,1	6,2	6,3	6,4	6,5	6,6

Sum of (7)

$$\frac{6}{36} + \frac{3}{36} = \frac{9}{36} = \frac{1}{4} = 0.25$$

Sum of (10)

Sum of Probabilities (Disjoint-Events)



What is the probability of obtaining a difference of (2) or a difference of (1)?

A = Difference of (2)

	1,1	1,2	1,3	1,4	1,5	1,6
	2,1	2,2	2,3	2,4	2,5	2,6
	3,1	3,2	3,3	3,4	3,5	3,6
	4,1	4,2	4,3	4,4	4,5	4,6
	5,1	5,2	5,3	5,4	5,5	5,6
	6,1	6,2	6,3	6,4	6,5	6,6

B = Difference of (1)

	1,1	1,2	1,3	1,4	1,5	1,6
	2,1	2,2	2,3	2,4	2,5	2,6
	3,1	3,2	3,3	3,4	3,5	3,6
	4,1	4,2	4,3	4,4	4,5	4,6
	5,1	5,2	5,3	5,4	5,5	5,6
	6,1	6,2	6,3	6,4	6,5	6,6

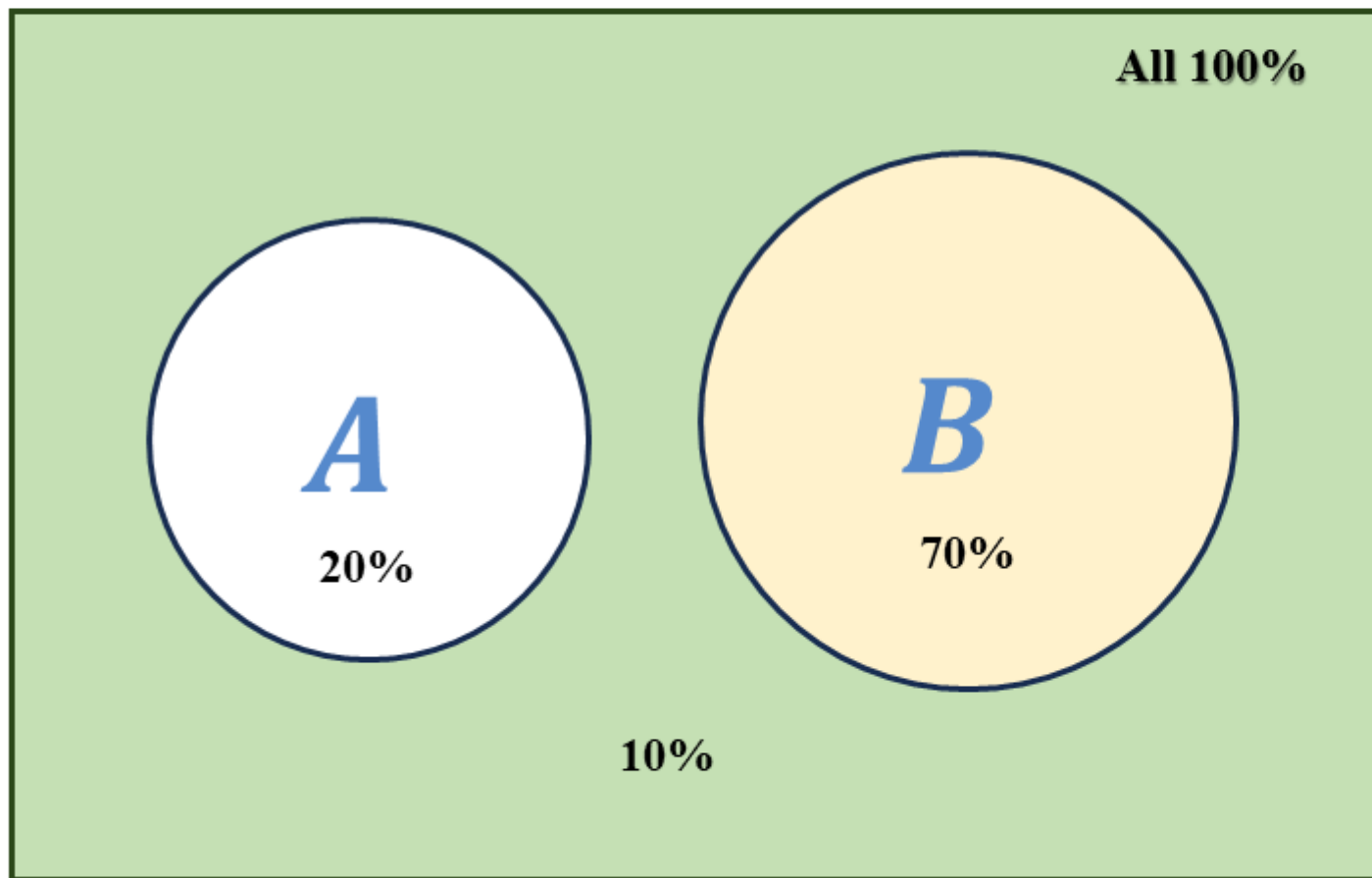
Sum of Probabilities (Disjoint-Events)

	1,1	1,2	1,3	1,4	1,5	1,6
	2,1	2,2	2,3	2,4	2,5	2,6
	3,1	3,2	3,3	3,4	3,5	3,6
	4,1	4,2	4,3	4,4	4,5	4,6
	5,1	5,2	5,3	5,4	5,5	5,6
	6,1	6,2	6,3	6,4	6,5	6,6

$$\frac{8}{36} + \frac{10}{36} = \frac{18}{36} = \frac{1}{2} = 0.5$$

Sum of Probabilities (Disjoint-Events)

Using Venn diagram



Union (Or)

$$P(A \cup B) = P(A) + P(B)$$

In this diagram = 90%

$$P = 0.9$$

Sum of Probabilities (Joint-Events)

The Sum of Probabilities (Joint-Events) is the likelihood of two events occurring together and at the same point in time.



$$P(F) = 0.6$$

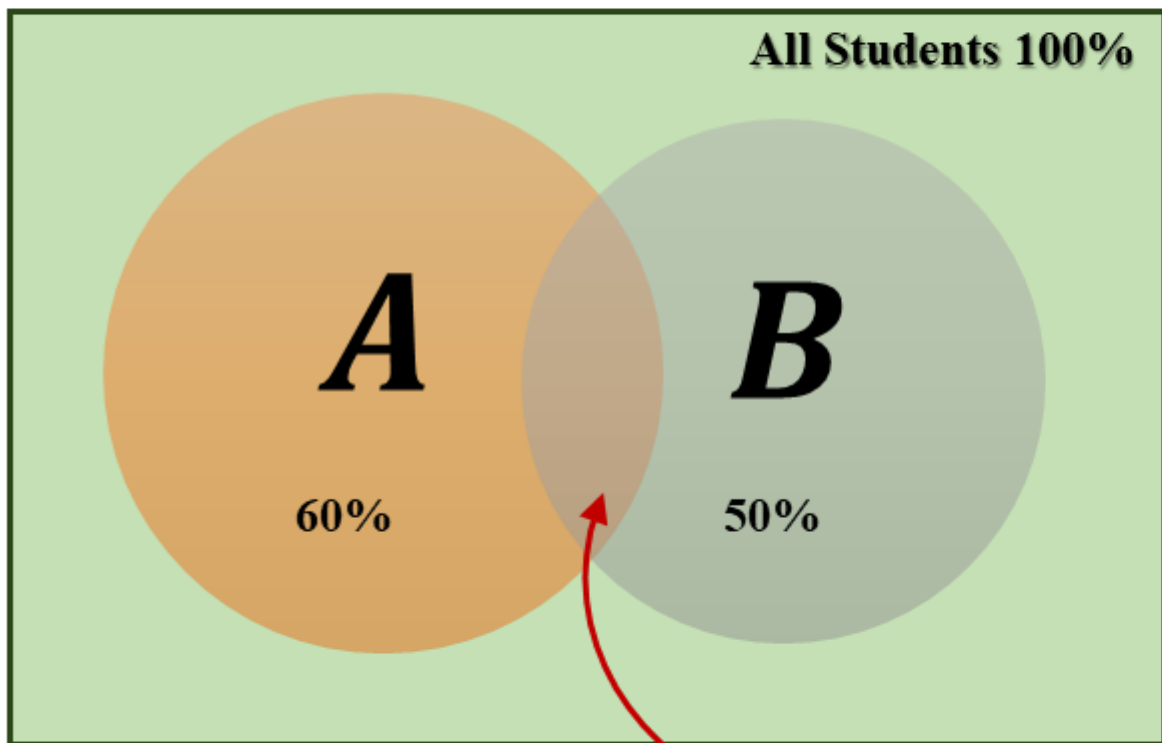


$$P(B) = 0.5$$

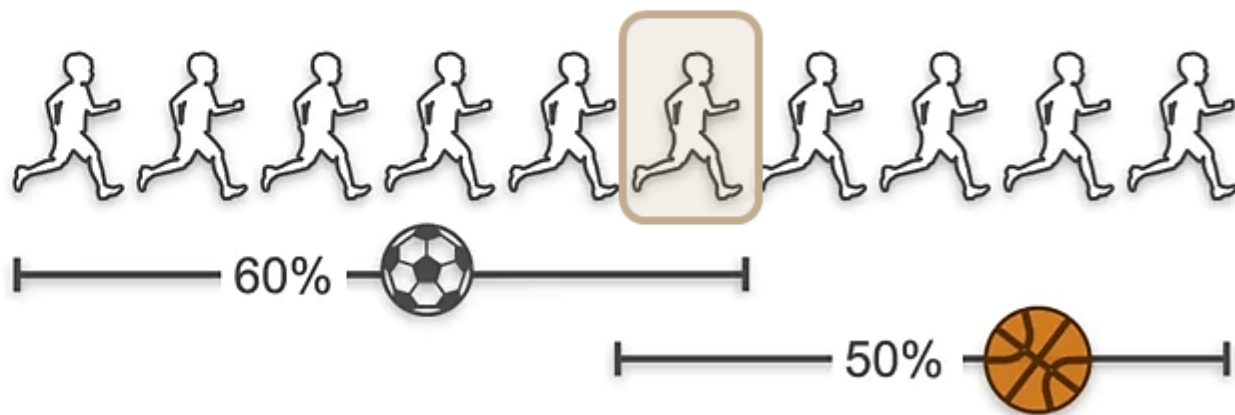
There is only (10) students in the School, What is the probability of the student plays football or basketball?

Sum of Probabilities (Joint-Events)

Using Venn diagram



$P(A \cap B)$

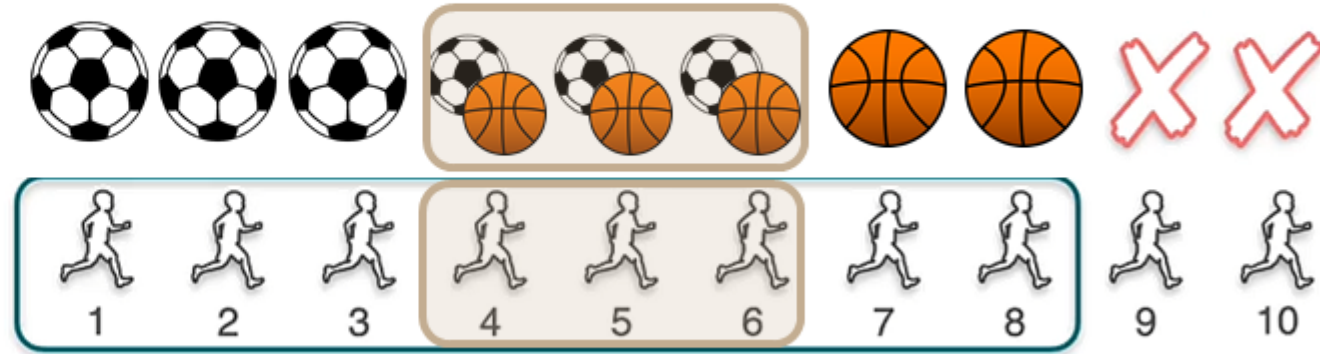
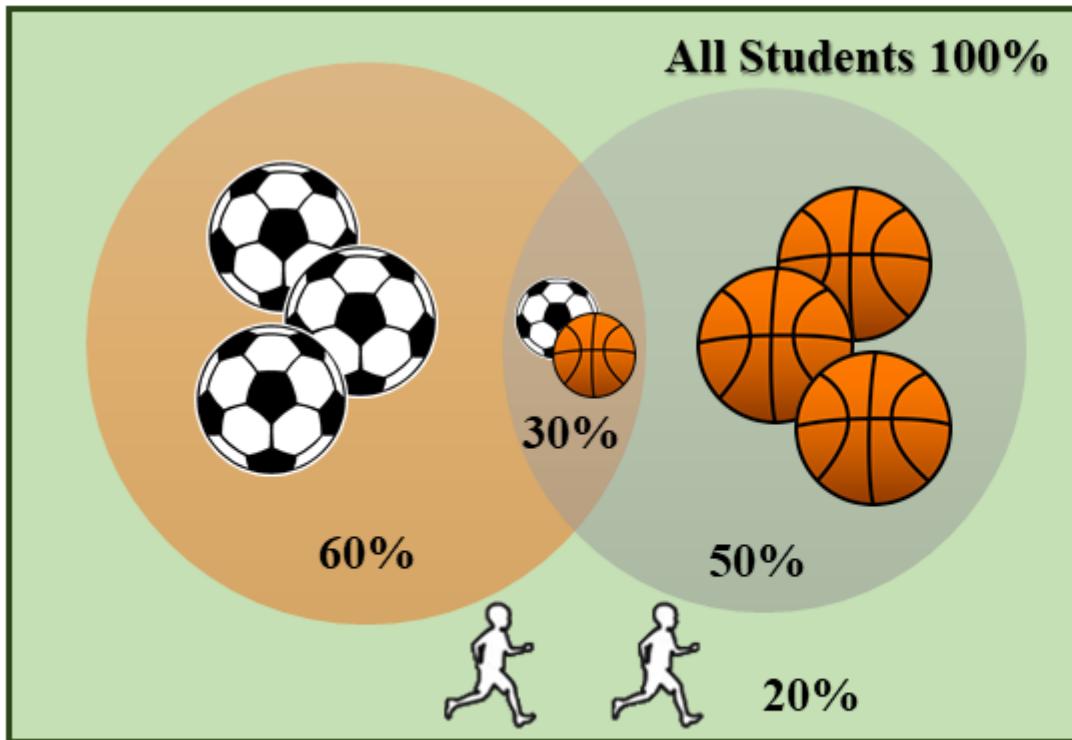


$$P(A \cup B) = P(A) + P(B)$$

Not Logic, the summation exceeds 100%

Sum of Probabilities (Joint-Events)

Using Venn diagram



$$\begin{aligned}
 P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\
 &= 0.6 + 0.5 - 0.3 \\
 &= 0.8
 \end{aligned}$$

Sum of Probabilities (Joint-Events)



What is the probability of obtaining sum of (7) and a difference of (1)?

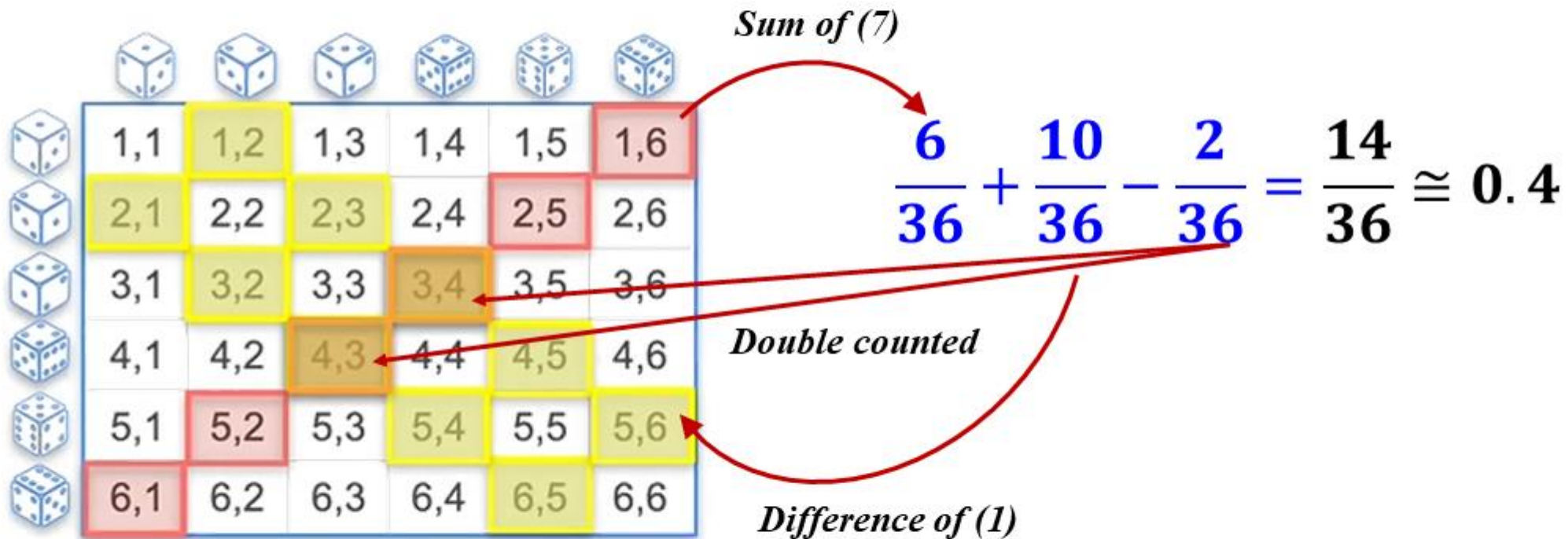
A = Sum of (7)

	1,1	1,2	1,3	1,4	1,5	1,6
	2,1	2,2	2,3	2,4	2,5	2,6
	3,1	3,2	3,3	3,4	3,5	3,6
	4,1	4,2	4,3	4,4	4,5	4,6
	5,1	5,2	5,3	5,4	5,5	5,6
	6,1	6,2	6,3	6,4	6,5	6,6

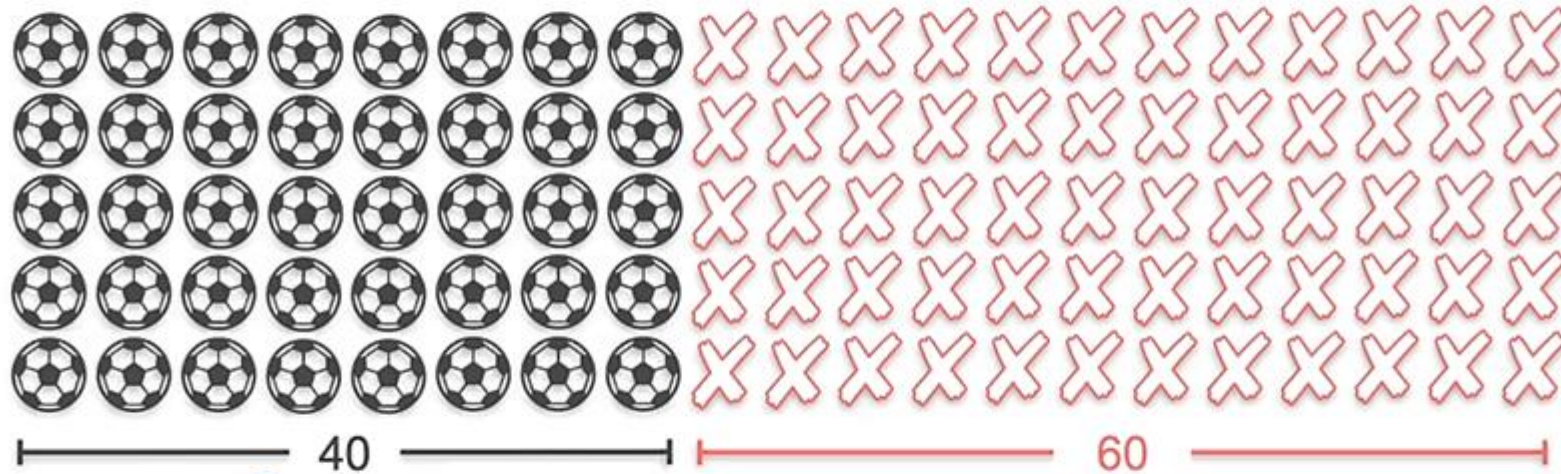
B = Difference of (1)

	1,1	1,2	1,3	1,4	1,5	1,6
	2,1	2,2	2,3	2,4	2,5	2,6
	3,1	3,2	3,3	3,4	3,5	3,6
	4,1	4,2	4,3	4,4	4,5	4,6
	5,1	5,2	5,3	5,4	5,5	5,6
	6,1	6,2	6,3	6,4	6,5	6,6

Sum of Probabilities (Joint-Events)

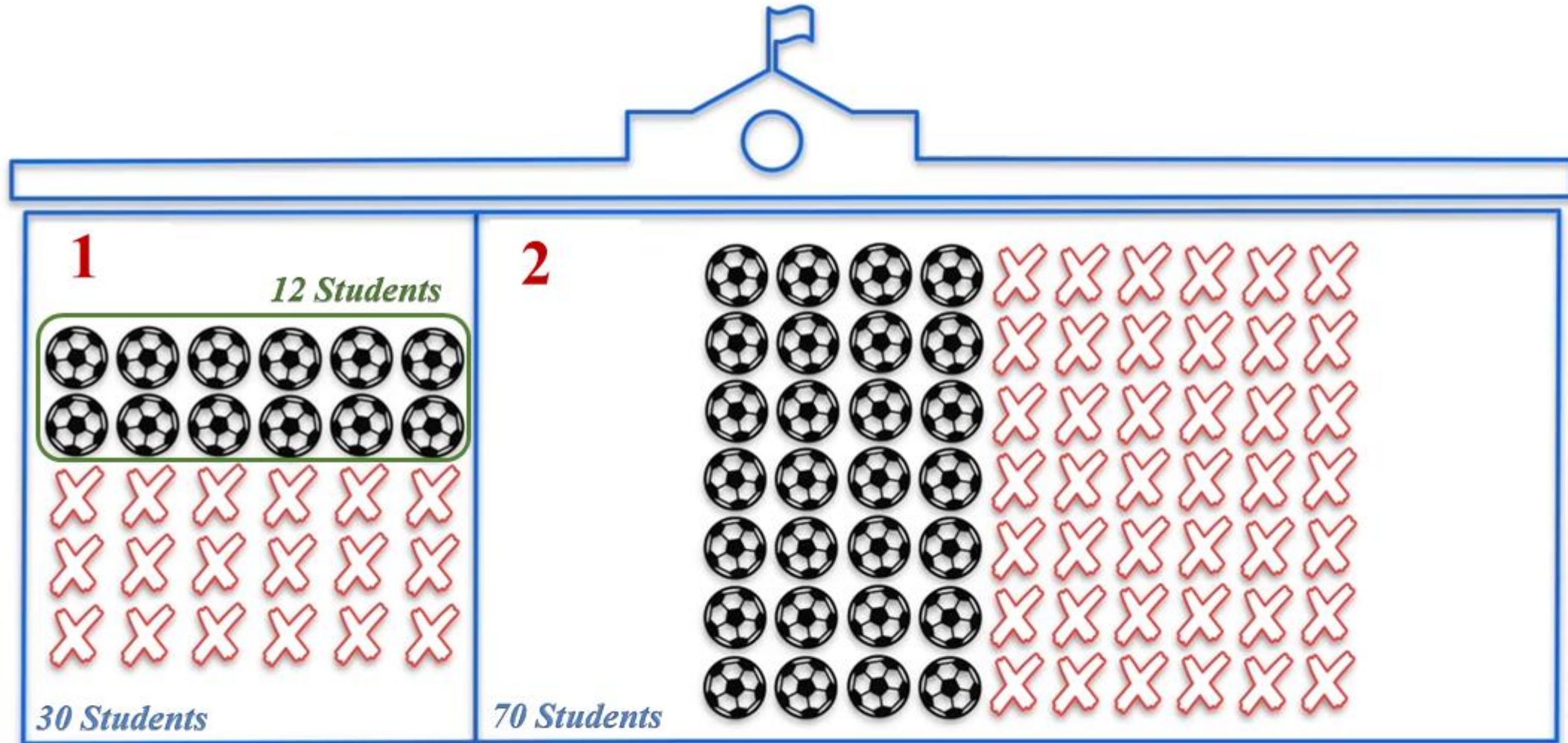


*In probability, any two events are **independent** if knowing one event occurred doesn't change the probability of the other event.*



	
1	2
30	70
<i>Students</i>	<i>Students</i>

How many students in room (1) like to play football?



$P(\text{Football and Room 1})$

$$P(F \cap R_1) = P(F) \times P(R_1)$$

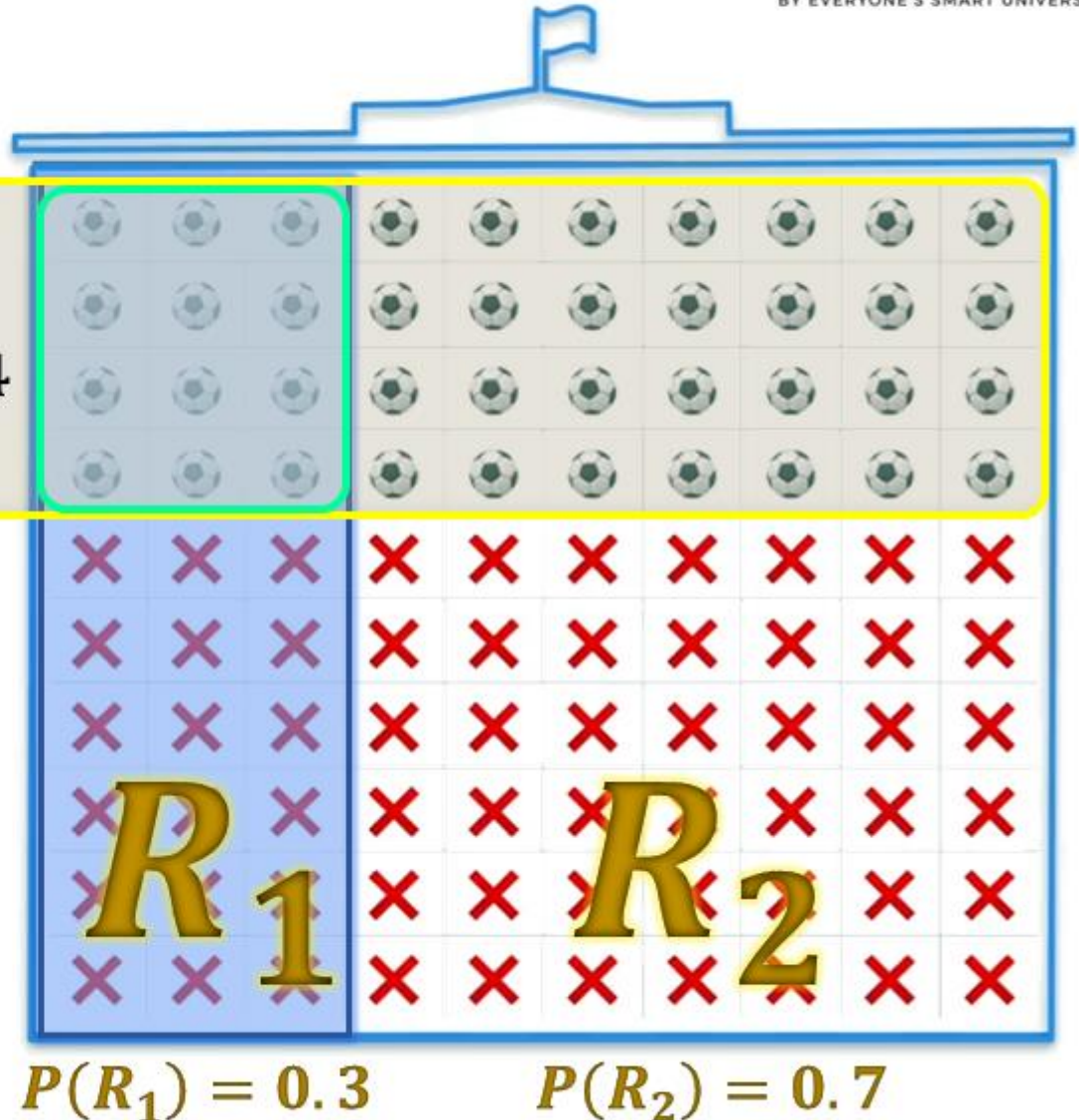
$$= 0.4 \times 0.3 = 0.12$$



$$P(\text{Not} - F) = 0.6$$



$$P(F) = 0.4$$

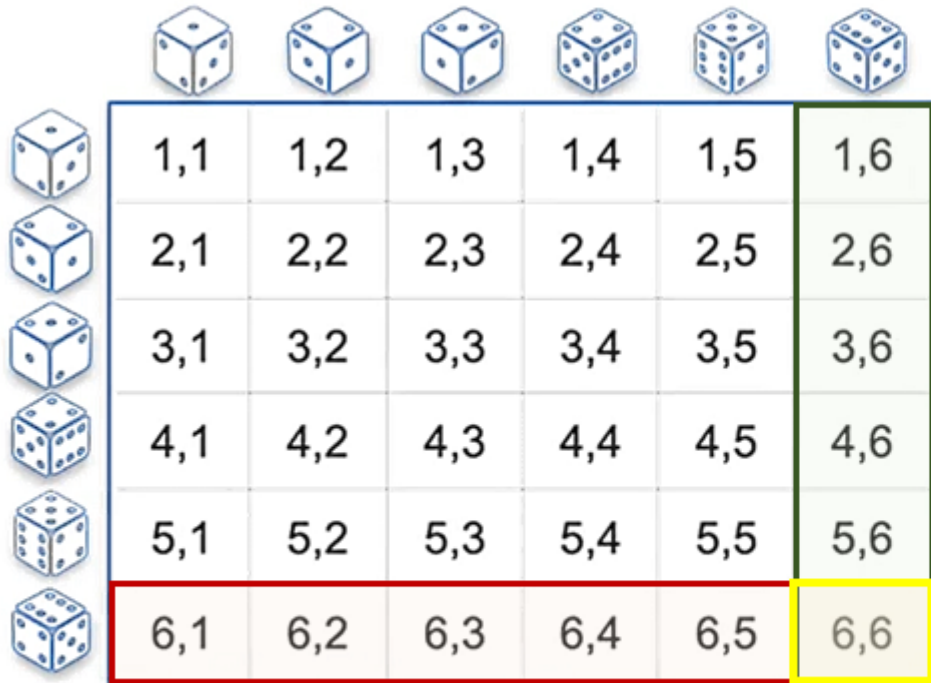




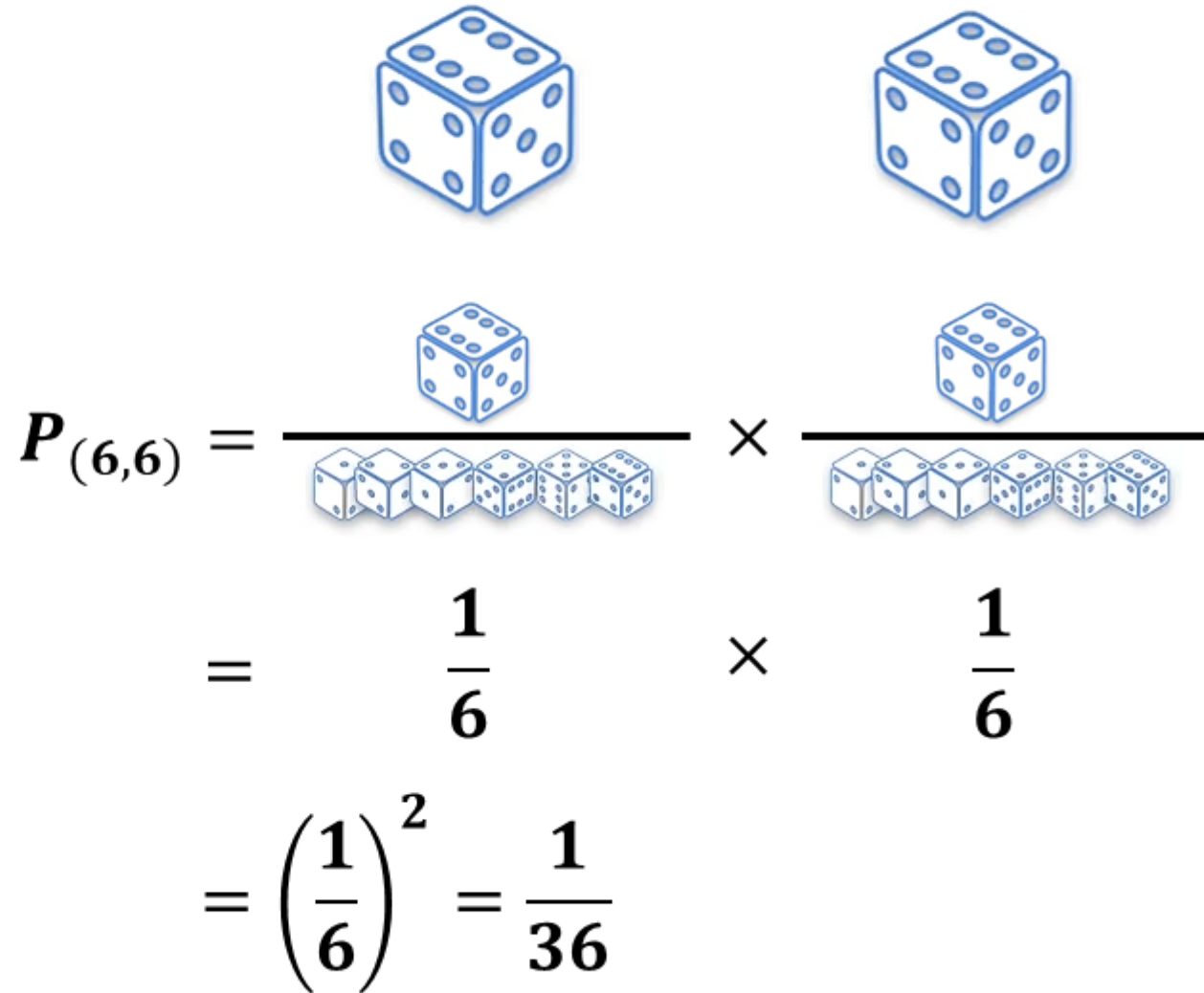
What is the probability of obtaining (6) for two dice?

	1,1	1,2	1,3	1,4	1,5	1,6
	2,1	2,2	2,3	2,4	2,5	2,6
	3,1	3,2	3,3	3,4	3,5	3,6
	4,1	4,2	4,3	4,4	4,5	4,6
	5,1	5,2	5,3	5,4	5,5	5,6
	6,1	6,2	6,3	6,4	6,5	6,6

$$P_{(6,6)} = \frac{1}{36}$$



	1,1	1,2	1,3	1,4	1,5	1,6
	2,1	2,2	2,3	2,4	2,5	2,6
	3,1	3,2	3,3	3,4	3,5	3,6
	4,1	4,2	4,3	4,4	4,5	4,6
	5,1	5,2	5,3	5,4	5,5	5,6
	6,1	6,2	6,3	6,4	6,5	6,6



$$\begin{aligned}
 P_{(6,6)} &= \frac{\text{1 die showing 6}}{\text{6 possible outcomes}} \times \frac{\text{1 die showing 6}}{\text{6 possible outcomes}} \\
 &= \frac{1}{6} \times \frac{1}{6} \\
 &= \left(\frac{1}{6}\right)^2 = \frac{1}{36}
 \end{aligned}$$



What is the probability of landing on Head (5) times?

$$\begin{aligned} P_{(Head)} &= \frac{\text{Head}}{\text{Head} + \text{Tail}} \times \frac{\text{Head}}{\text{Head} + \text{Tail}} \times \frac{\text{Head}}{\text{Head} + \text{Tail}} \times \frac{\text{Head}}{\text{Head} + \text{Tail}} \times \frac{\text{Head}}{\text{Head} + \text{Tail}} \\ &= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \\ &= \left(\frac{1}{2}\right)^5 = \frac{1}{32} \end{aligned}$$

Conditional probability refers to the chances that some outcome occurs given that another event has also occurred.



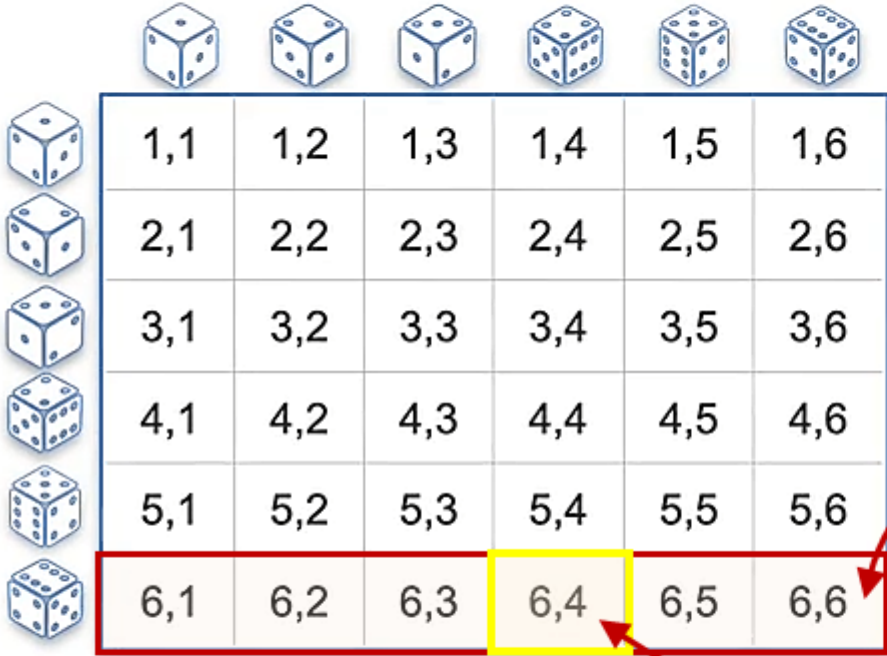
*What is the probability of obtaining the first dice is (6) **and** the sum of two dice is (10)?*

A 6x6 grid representing the possible outcomes of two dice. Each row and column is headed by a small blue die icon. The grid contains pairs of numbers from (1,1) to (6,6). The cell containing (6,4) is highlighted with a red border.

	1,1	1,2	1,3	1,4	1,5	1,6
	2,1	2,2	2,3	2,4	2,5	2,6
	3,1	3,2	3,3	3,4	3,5	3,6
	4,1	4,2	4,3	4,4	4,5	4,6
	5,1	5,2	5,3	5,4	5,5	5,6
	6,1	6,2	6,3	6,4	6,5	6,6

$$P_{(6, \Sigma 10)} = \frac{1}{36}$$

Conditional Probability

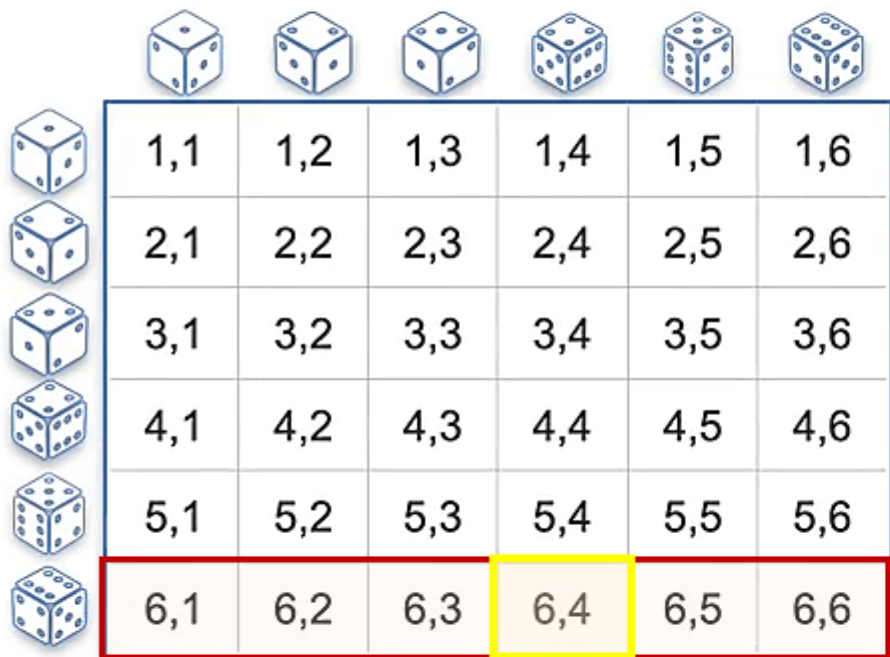


	1,1	1,2	1,3	1,4	1,5	1,6
	2,1	2,2	2,3	2,4	2,5	2,6
	3,1	3,2	3,3	3,4	3,5	3,6
	4,1	4,2	4,3	4,4	4,5	4,6
	5,1	5,2	5,3	5,4	5,5	5,6
	6,1	6,2	6,3	6,4	6,5	6,6

$$P(1^{\text{st}} = 6) = \frac{6}{36} = \frac{1}{6}$$

$$P(\sum 10) = \frac{1}{6}$$

$$P(1^{\text{st}} = 6 \cap \sum 10) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$



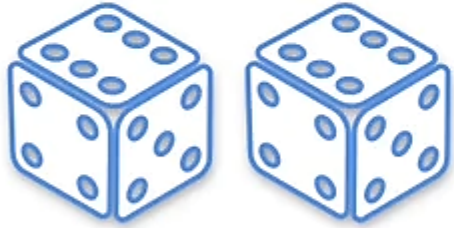
	1,1	1,2	1,3	1,4	1,5	1,6
	2,1	2,2	2,3	2,4	2,5	2,6
	3,1	3,2	3,3	3,4	3,5	3,6
	4,1	4,2	4,3	4,4	4,5	4,6
	5,1	5,2	5,3	5,4	5,5	5,6
	6,1	6,2	6,3	6,4	6,5	6,6

$$P(1^{\text{st}} = 6 \cap \sum 10) = P(1^{\text{st}} = 6) \times P(\sum 10 | 1^{\text{st}} = 6)$$

$$P(A \cap B) = P(A) \times P(B|A)$$

When Independent

$$P(B|A) = P(B)$$



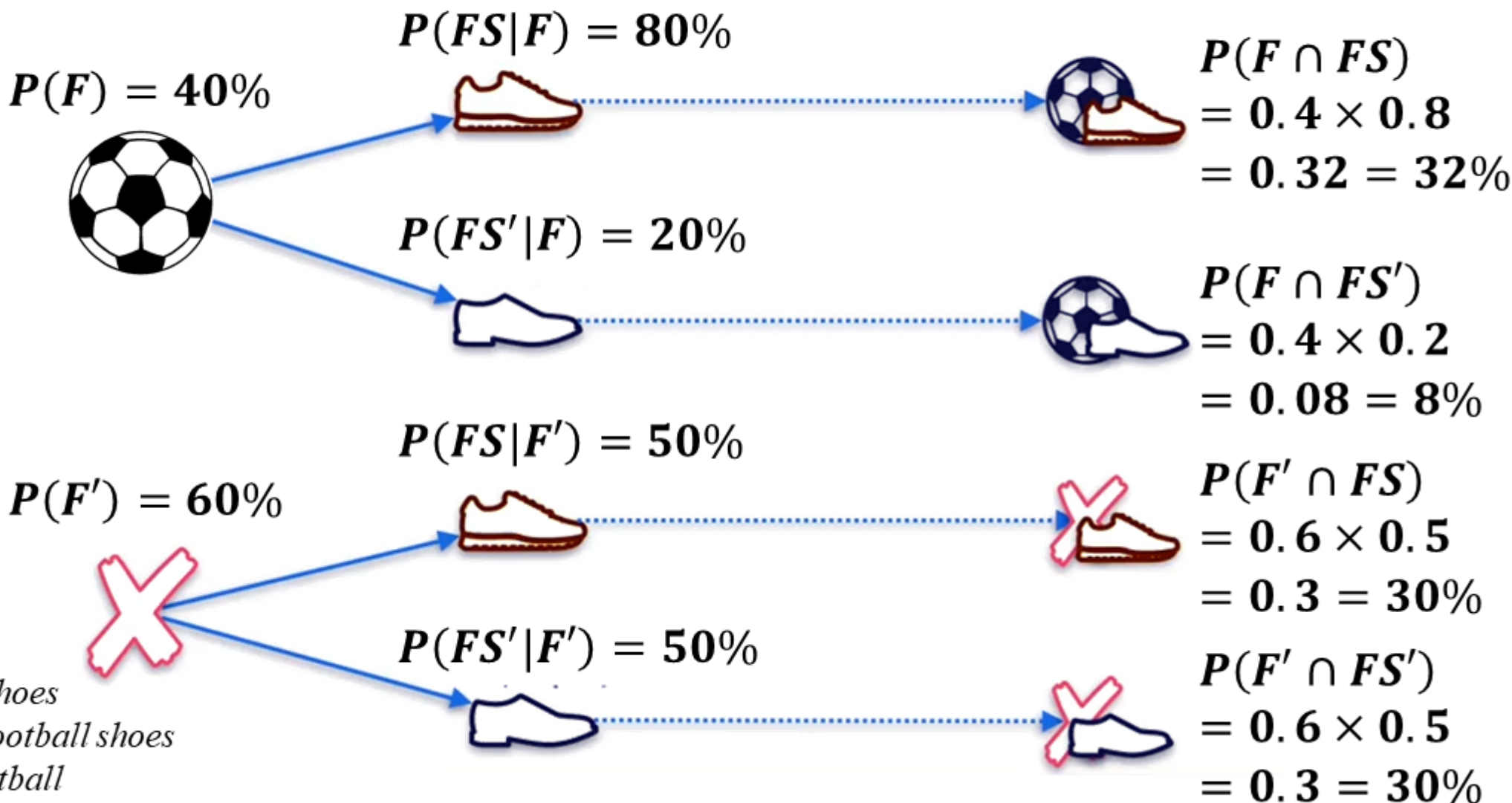
*What is the probability of obtaining the sum of (10) **given that the first dice is (6)**?*

	1,1	1,2	1,3	1,4	1,5	1,6
	2,1	2,2	2,3	2,4	2,5	2,6
	3,1	3,2	3,3	3,4	3,5	3,6
	4,1	4,2	4,3	4,4	4,5	4,6
	5,1	5,2	5,3	5,4	5,5	5,6
	6,1	6,2	6,3	6,4	6,5	6,6

$$P(\sum 10) = \frac{3}{36} = \frac{1}{12}$$

$$P(\sum 10 \mid 1^{\text{st}} = 6) = \frac{1}{6}$$

Conditional Probability



Population

1,000,000 people

Sick ratio

1 / 10,000 people



99% Effective

Test effective

Tested Sick

*Test is +ve**Sick*

100 people

Diagnosed Sick

99

Diagnosed Healthy

1

Healthy

100 people



1

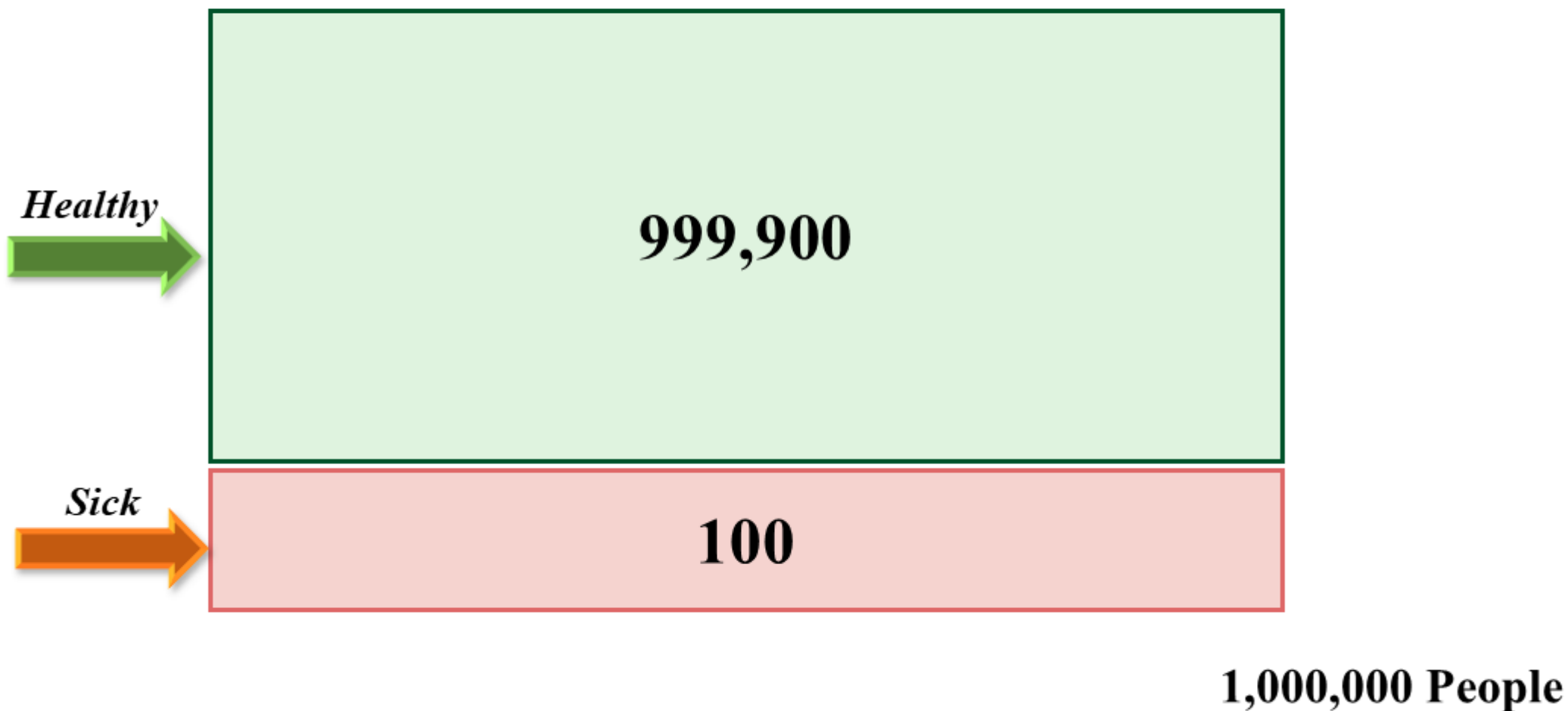


99

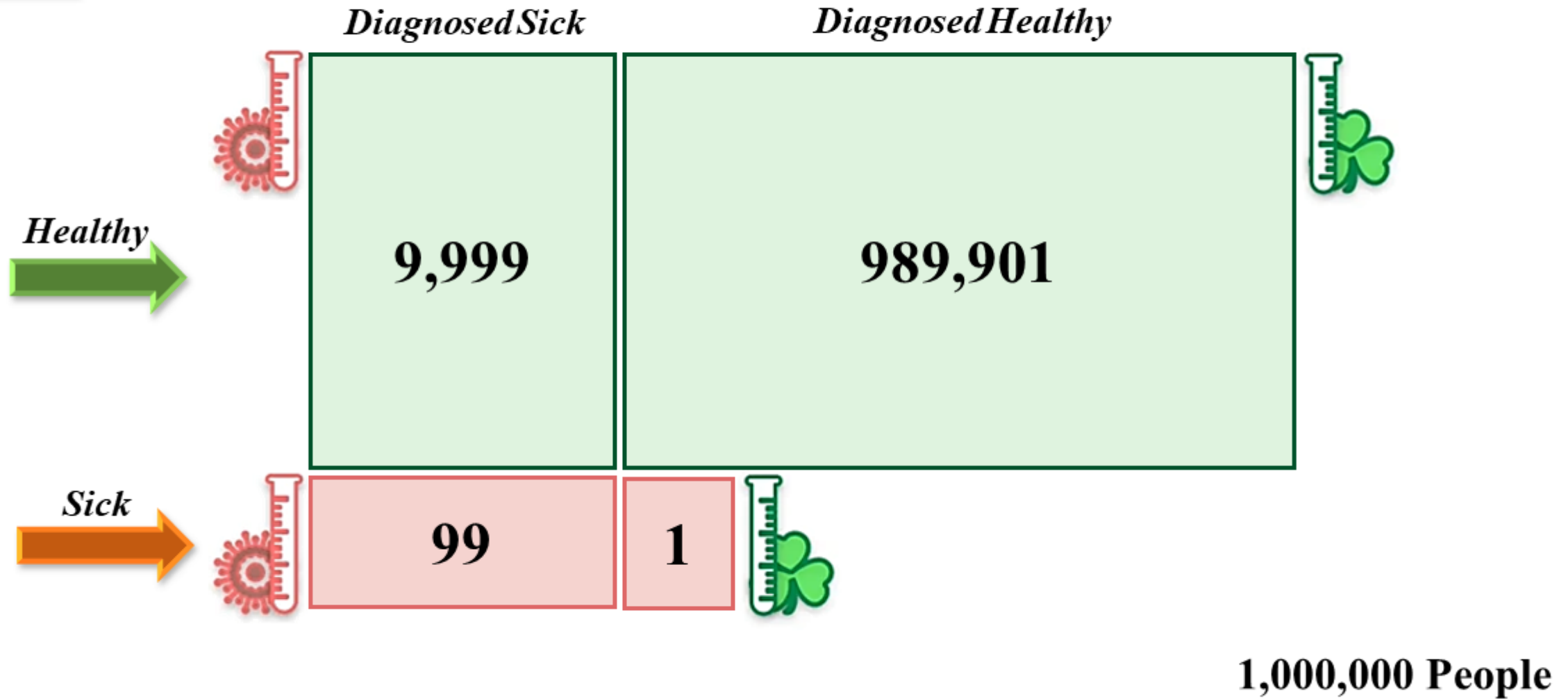
What is the probability that the person is sick given that he tested sick?



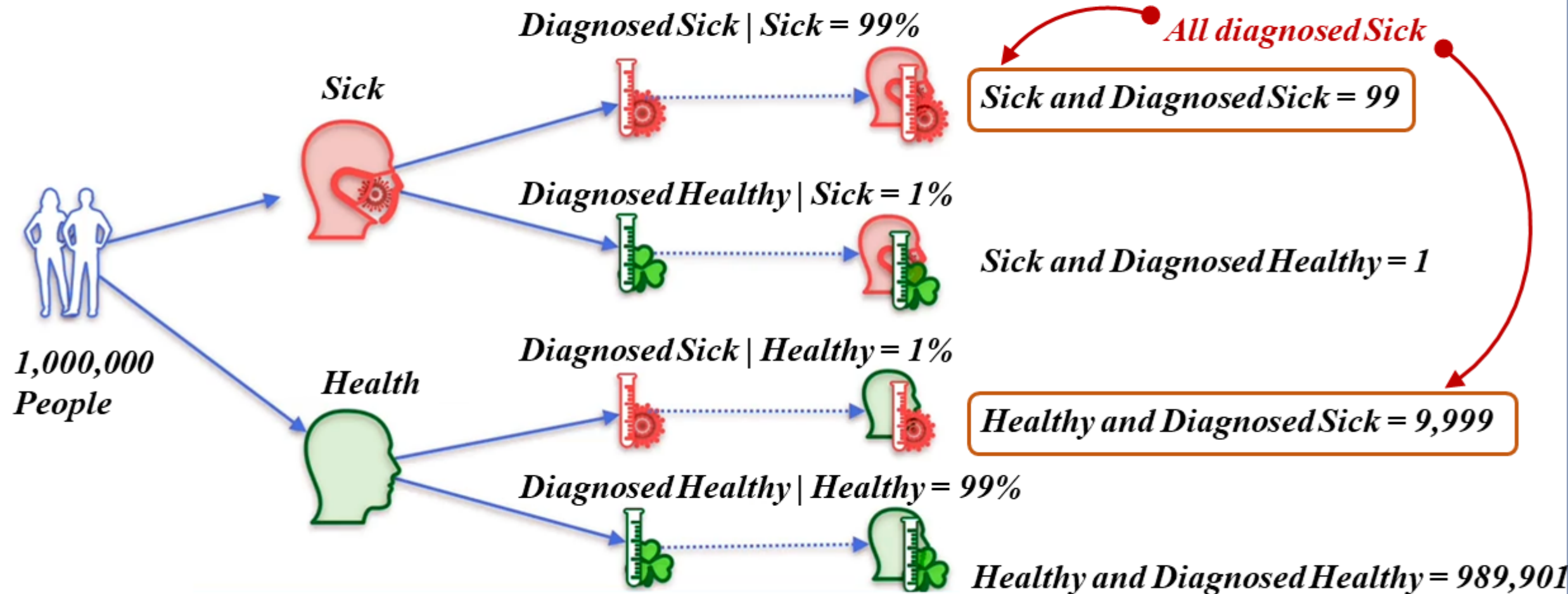
$$P(A)|P(B)$$



Bayes Theorem



Bayes Theorem



Diagnosed Sick



9,999

Healthy



Sick



99



$$P(\text{Sick} | P(\text{Diagnosed Sick})) = \frac{99}{99 + 9999}$$

$$= \frac{99}{10098} = 0.0098$$

$$= \frac{\text{Sick and Diagnosed Sick}}{\text{Sick and Diagnosed Sick} + \text{Healthy and Diagnosed Sick}}$$

From Conditional Probability

If $A=Sick$, and $B=Diagnosed Sick$, then from conditional probability:

$$P(A \cap B) = P(B) \times P(A|B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A|B) = \frac{P(Sick \text{ and Diagnosed Sick})}{P(Diagnosed Sick)}$$

$P(Sick \text{ and Diagnosed Sick})=?$

$P(Diagnosed Sick)=?$

Bayes Theorem describes the probability of an event, based on prior knowledge of conditions that might be related to the event.

Bayes Theorem formula:

$$P(A|B) = \frac{P(\text{Sick and Diagnosed Sick})}{P(\text{Diagnosed Sick})}$$

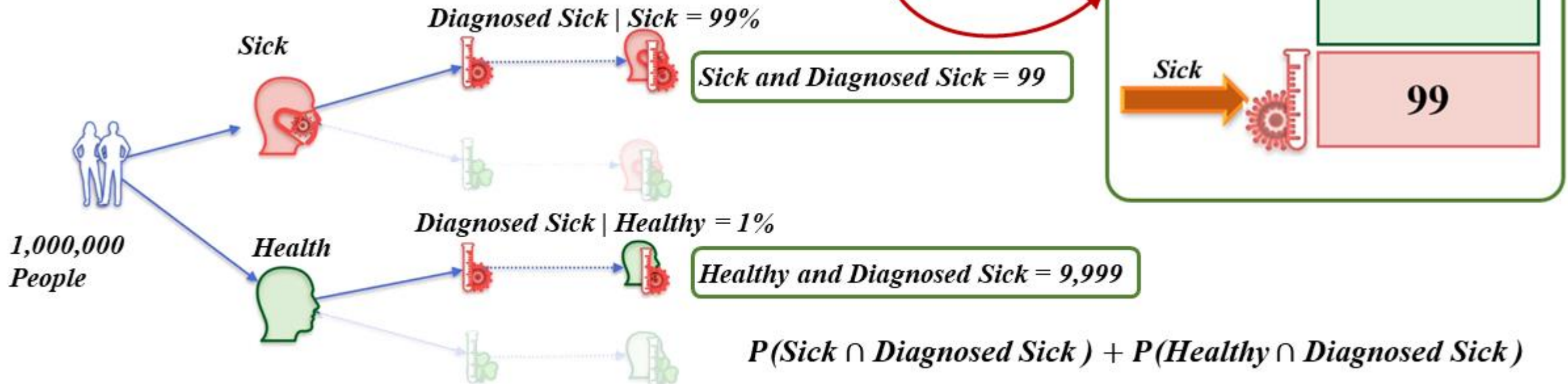
from conditional probability;

$$P(A \cap B) = P(A) \times P(B|A)$$

$$P(\text{Sick and Diagnosed Sick}) = P(\text{Sick}) \times P(\text{Diagnosed Sick} | \text{Sick})$$

Bayes Theorem formula:

$$P(A|B) = \frac{P(\text{Sick}) \times P(\text{Diagnosed Sick} | \text{Sick})}{P(\text{Diagnosed Sick})}$$



Bayes Theorem formula:

$$P(A|B) = \frac{P(\text{Sick}) \times P(\text{Diagnosed Sick} | \text{Sick})}{P(\text{Sick} \cap \text{Diagnosed Sick}) + P(\text{Healthy} \cap \text{Diagnosed Sick})}$$

$$P(A|B) = \frac{P(\text{Sick}) \times P(\text{Diagnosed Sick} | \text{Sick})}{P(\text{Sick}) \times P(\text{Diagnosed Sick} | \text{Sick}) + P(\text{Healthy}) \times P(\text{Diagnosed Sick} | \text{Healthy})}$$

$$P(\text{Sick}) = 0.01\%$$

$$P(\text{Healthy}) = 99.99\%$$

$$P(\text{Diagnosed Sick} | \text{Sick}) = 99\%$$

$$P(\text{Diagnosed Sick} | \text{Healthy}) = 1\%$$

Bayes Theorem formula:

$$P(A|B) = \frac{P(\text{Sick}) \times P(\text{Diagnosed Sick} | \text{Sick})}{P(\text{Sick} \cap \text{Diagnosed Sick}) + P(\text{Healthy} \cap \text{Diagnosed Sick})}$$

$$P(A|B) = \frac{P(\text{Sick}) \times P(\text{Diagnosed Sick} | \text{Sick})}{P(\text{Sick}) \times P(\text{Diagnosed Sick} | \text{Sick}) + P(\text{Healthy}) \times P(\text{Diagnosed Sick} | \text{Healthy})}$$

$$P(A|B) = \frac{P(A) \times P(B|A)}{P(A) \times P(B|A) + P(A') \times P(B|A')} \quad \text{Bayes Theorem formula}$$

$$P(A) = 0.01\%$$

$$P(A') = 99.99\%$$

$$P(B|A) = 99\%$$

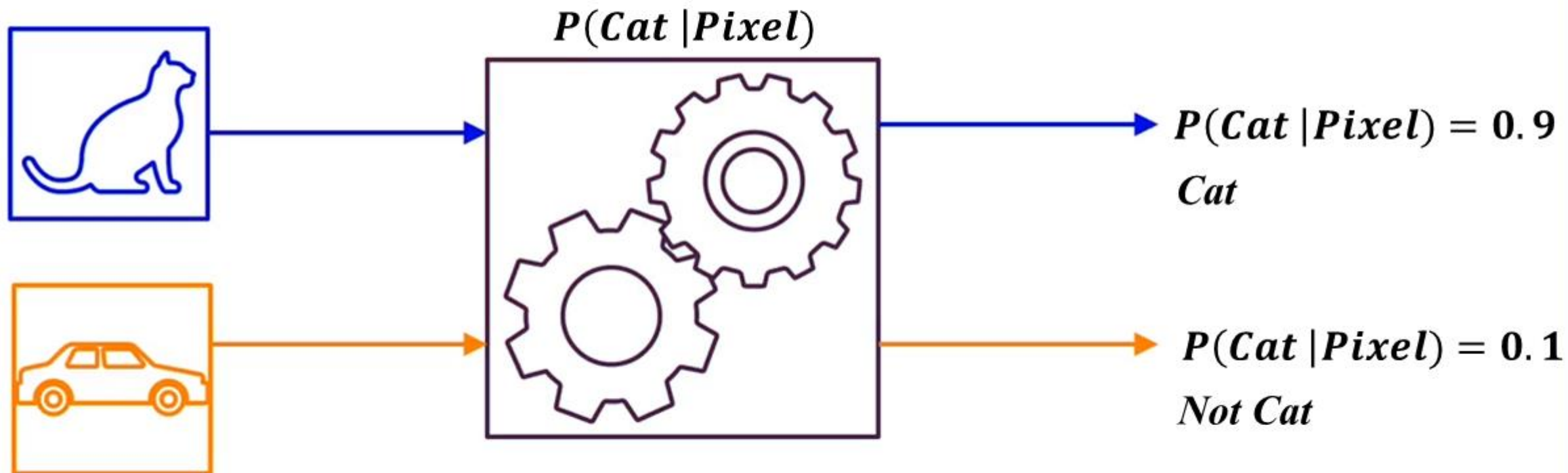
$$P(B|A') = 1\%$$

$$= \frac{0.0001 \times 0.99}{0.0001 \times 0.99 + 0.9999 \times 0.01} = 0.0098$$

In Image recognition

What is the probability that there is a cat in the image?

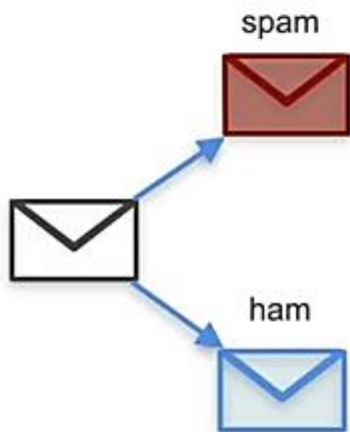


In Image recognition

Probability in Machine Learning

Spam filter

PRIOR

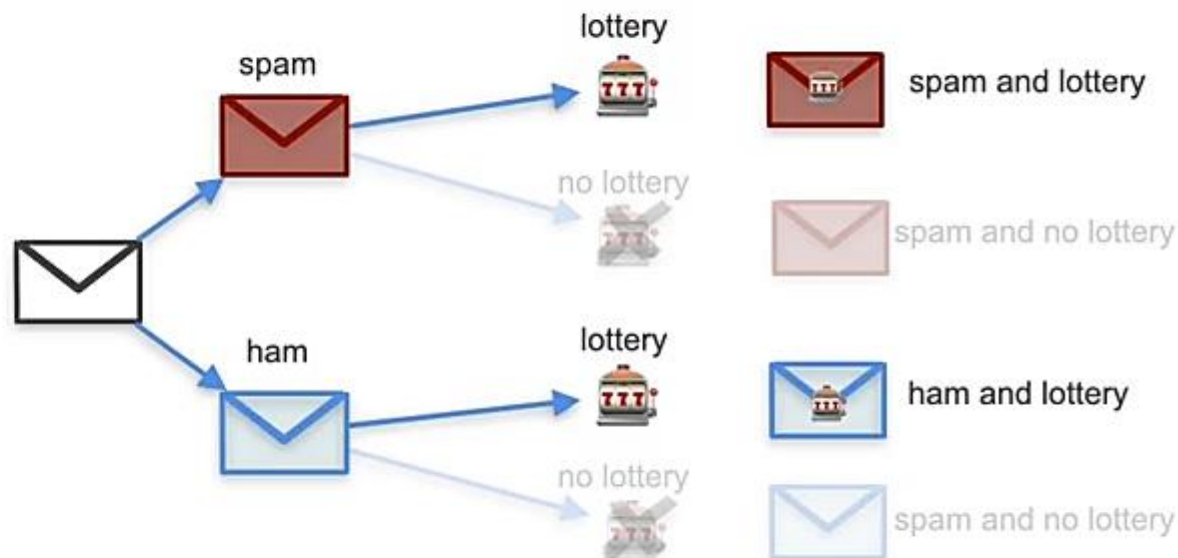


$$P(\text{spam}) = \frac{\text{spam icon}}{\text{spam icon} + \text{ham icon}}$$

EVENT



POSTERIOR



$$P(\text{spam} \mid \text{lottery}) = \frac{\text{spam and lottery icon}}{\text{spam and lottery icon} + \text{ham and lottery icon}}$$

