

Computer Vision, and Digital Image Processing

Prepared by Dr. Mohamed Mostafa

AI 612

Lecture 7



Filtering in the Frequency Domain Part 2

Lecture 7

AI 612

Prepared by
Dr. Mohamed Mostafa

Chapter
05

Spatial Filtering Image Restoration

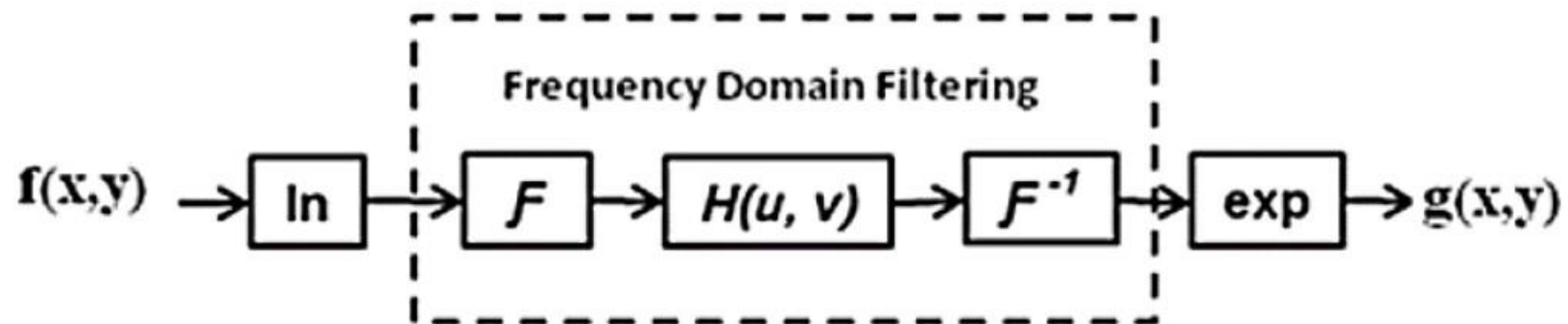
Resources for this
lecture

Chapter 5:

Gonzalez, Rafael C. *Digital image processing*.
Pearson education, India, 2009.



Recap



Filtering in frequency domain

1. Show that the 1D discrete Fourier Transform

$$F(u) = \sum_{x=0}^{M-1} f(x) e^{-i2\pi\mu x/M} \quad u = 0, 1, 2, \dots, M-1$$

and inverse Fourier Transform

$$f(x) = \sum_{u=0}^{M-1} F(u) e^{i2\pi\mu x/M} \quad x = 0, 1, 2, \dots, M-1$$

Recap

$f(0,0)$



$F(x)$

90	240	30	40
----	-----	----	----

$$F(u) = \sum_{x=0}^{M-1} f(x) e^{-i2\pi ux/M} \quad u = 0, 1, 2, \dots, M-1$$

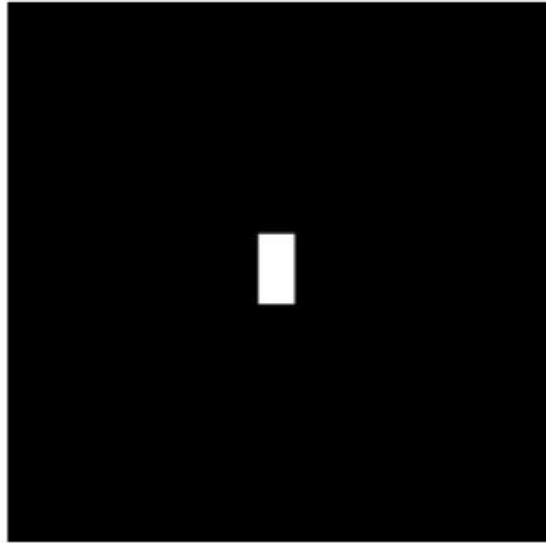
$F(0)$

100			
-----	--	--	--

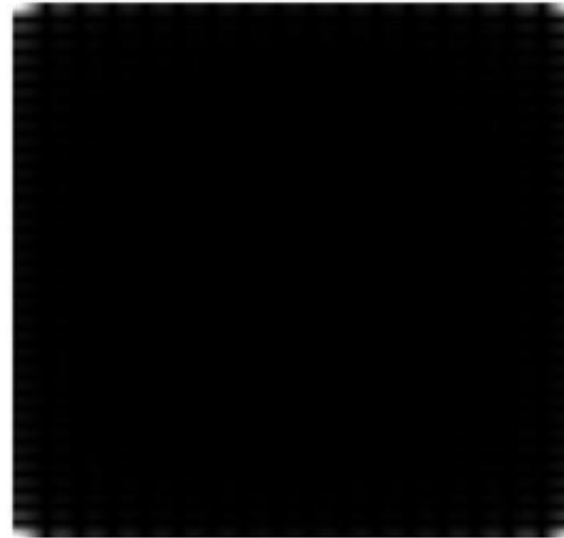
Average of the whole image $f(x)$



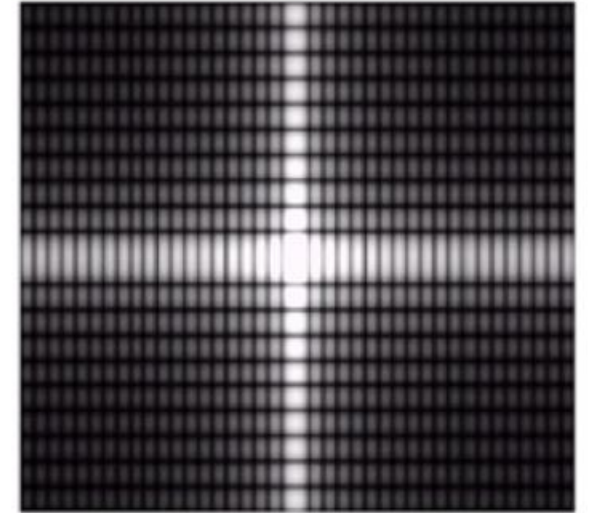
Processes for DFT



Original Image



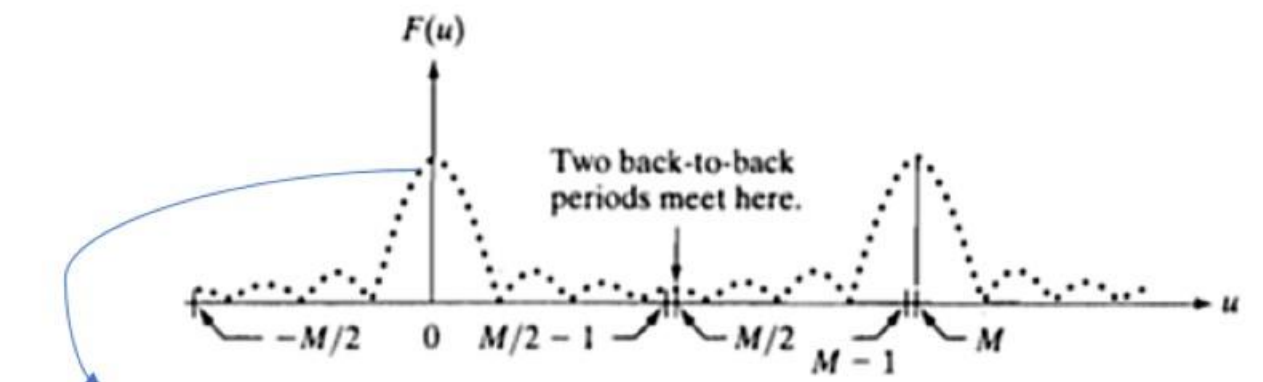
Fourier Transform



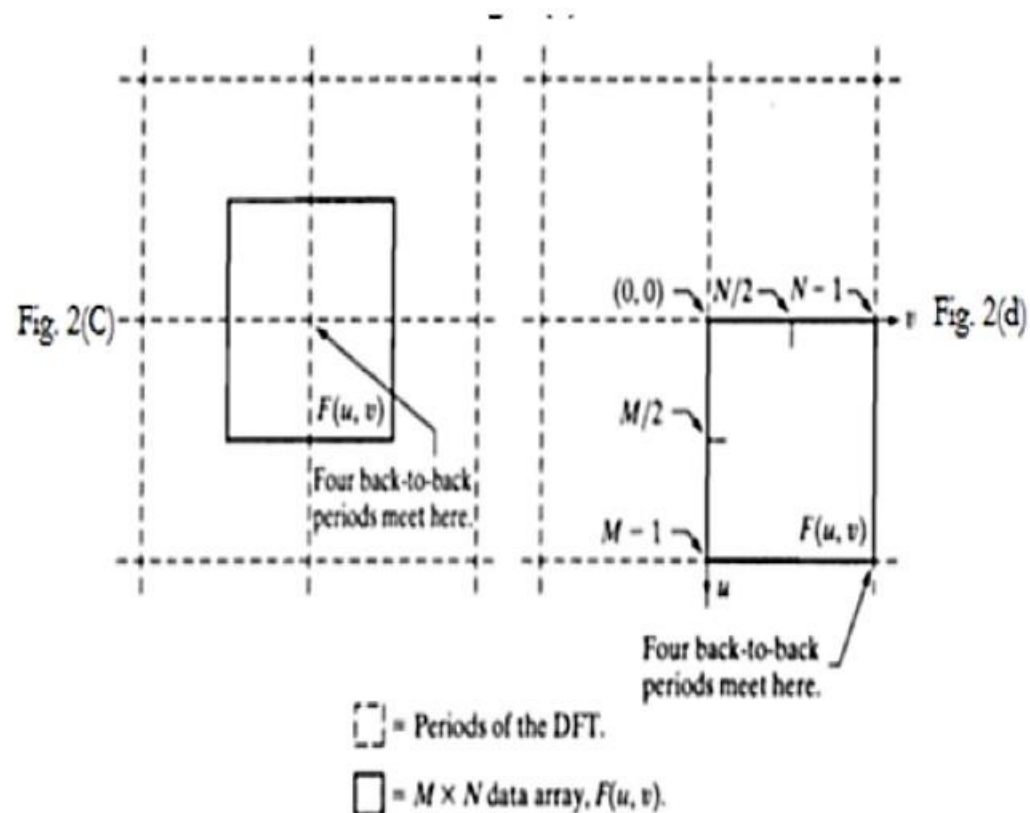
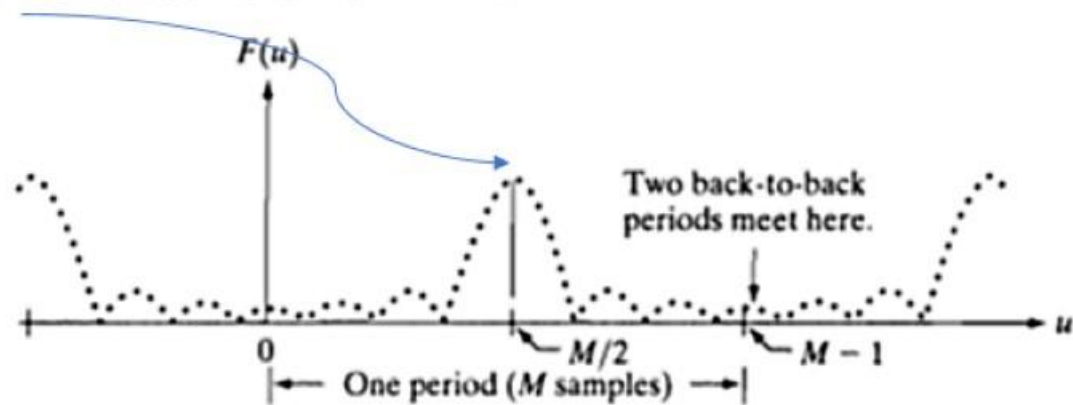
This what you see!

How it is concentrated in the center of the image

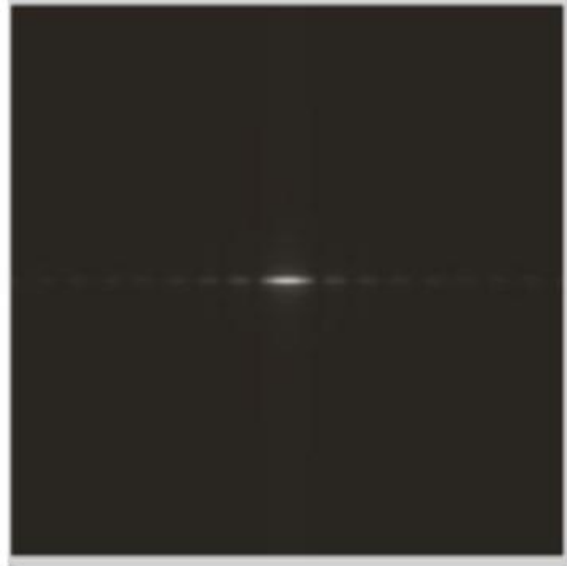
Shifting the origin



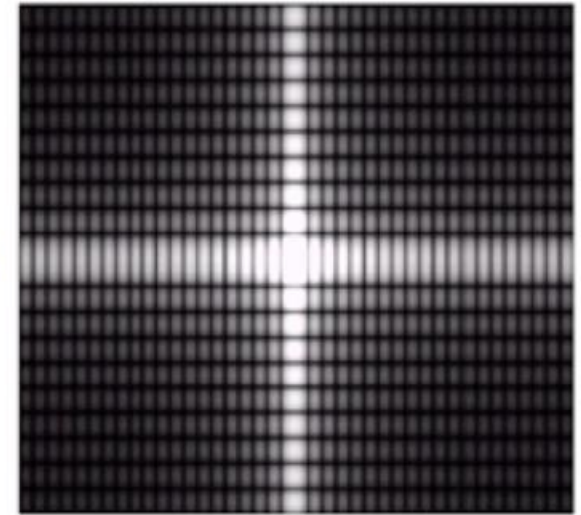
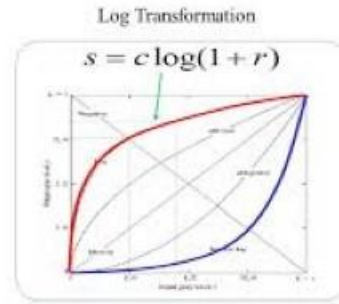
multiplying $f(x,y)$ by $(-1)^{x+y}$ Fig. 2(a)



Log Transform for DFT

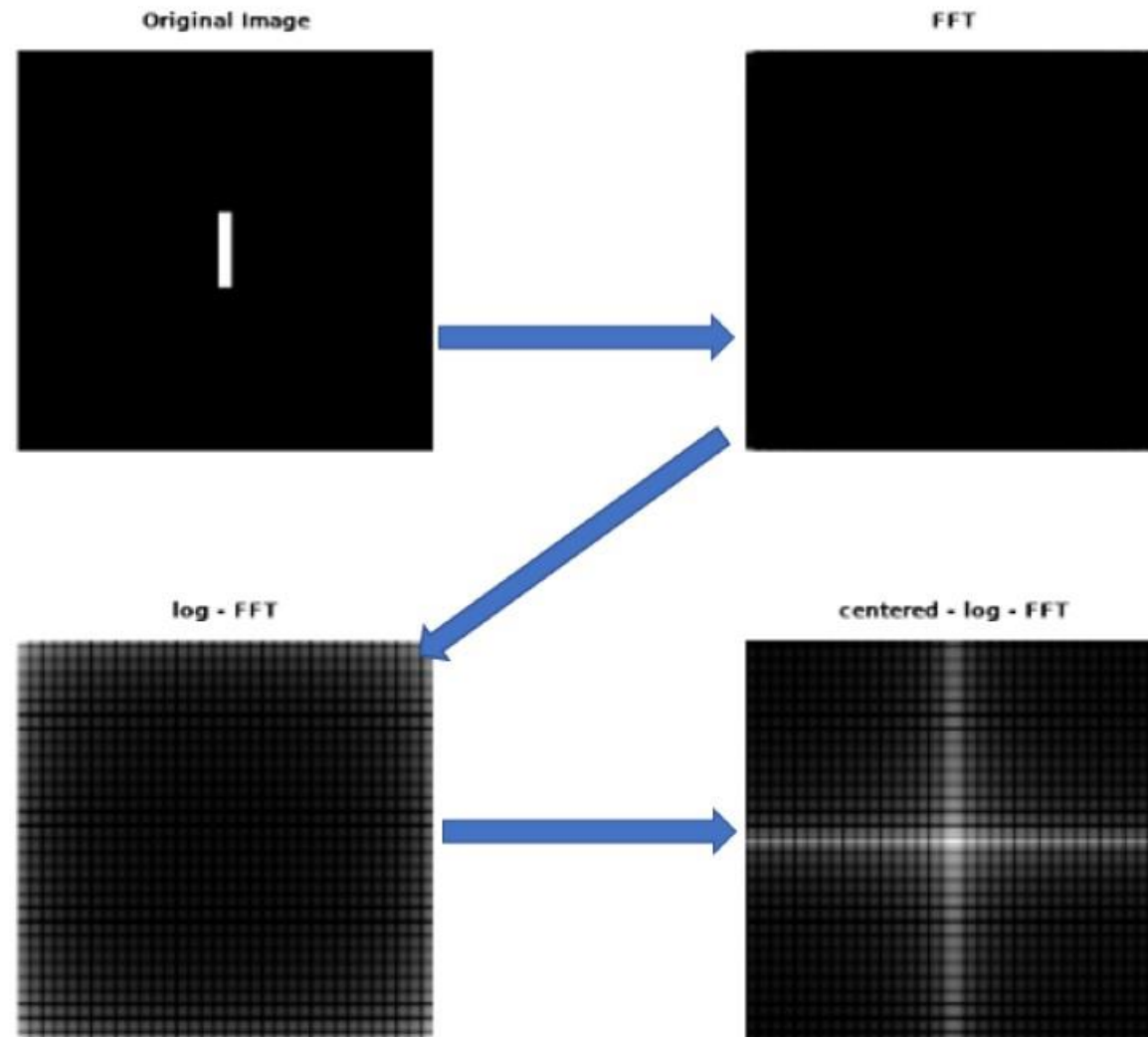


After Shifting the Origin



Final Fourier Transform

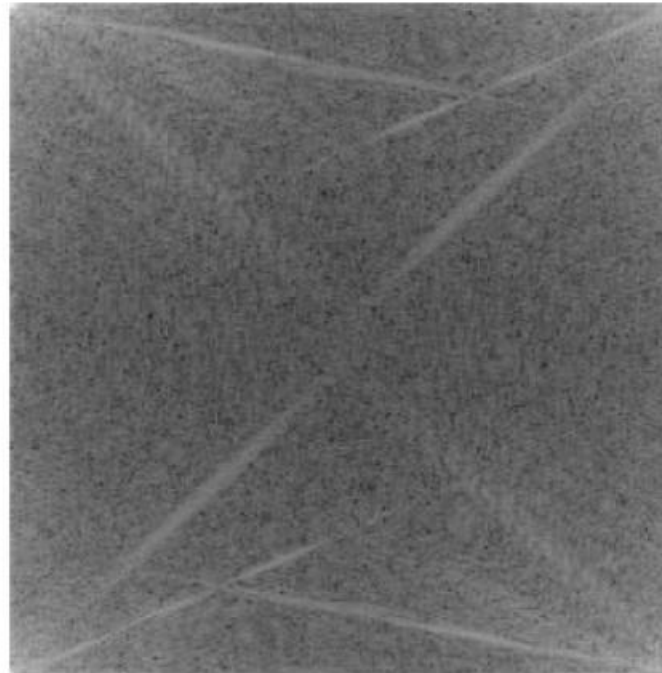
Processes for DFT



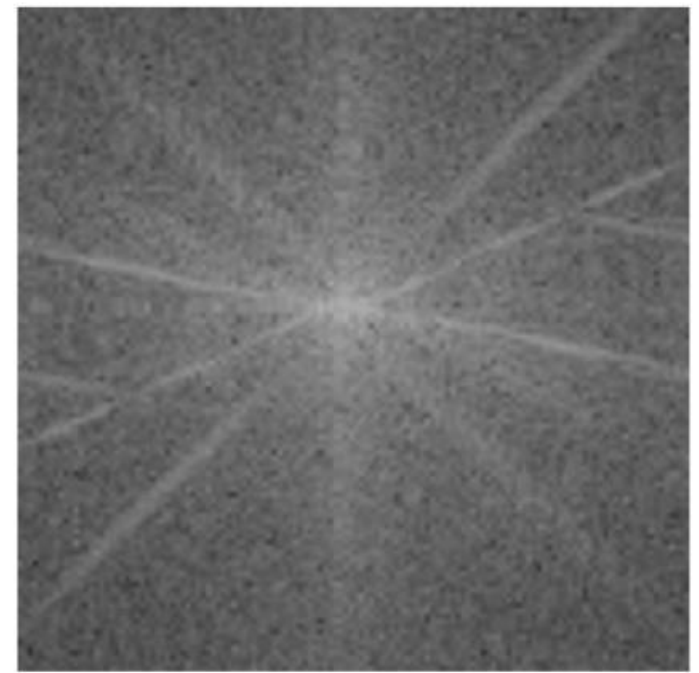
Example



Original Image



**Log + Fourier
Transform**



Shifting the origin

Highpass filters - Sharpening

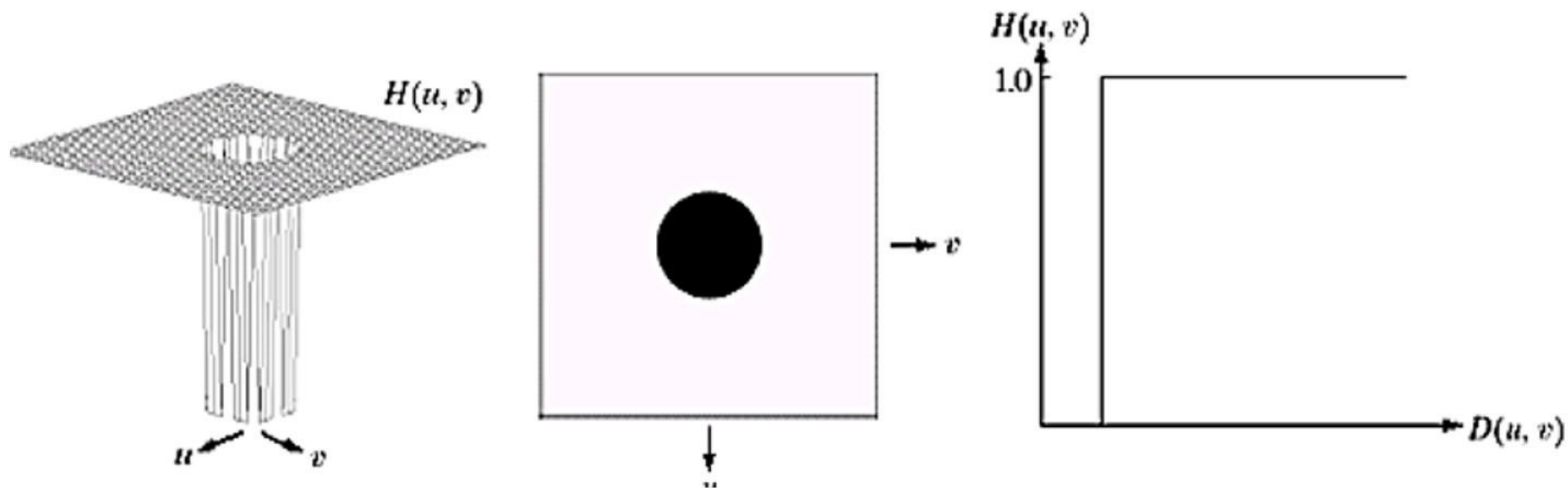
- Edges and complex details in images are linked with high frequency components.
- High pass filters exclusively permit the passage of high frequencies while eliminating the low ones.
- High pass frequencies serve as the complement (opposite) to low pass filters, thereby emphasizing the contrast between regions with significant changes in intensity or color.
- High pass filtering highlights sharp transitions and fine details within an image, accentuating features such as edges, textures, and patterns.
- The primary function of high pass filters is to enhance the clarity and definition of these high frequency components, making them more prominent in the final image output.

$$H_{HP}(u, v) = 1 - H_{LP}(u, v)$$

Highpass filters - Sharpening



Ideal Highpass Filter



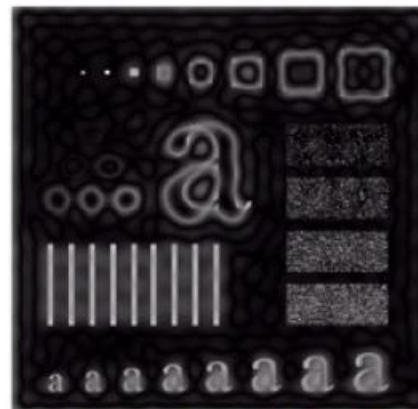
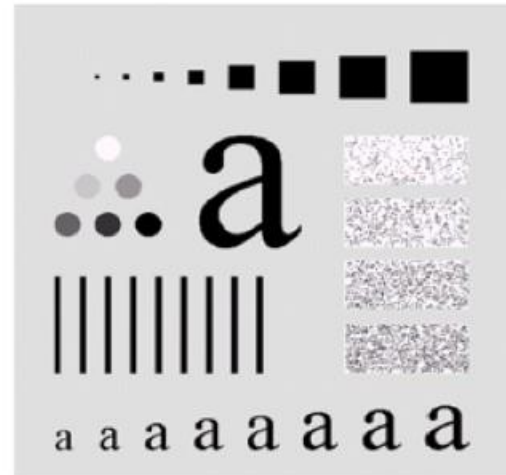
Ideal Highpass Filter

- The ideal high pass filter is given as:
- where D_0 is the cut off distance as before

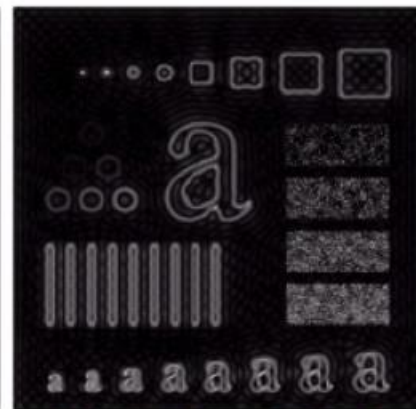
$$H(u, v) = \begin{cases} 0 & \text{if } D(u, v) \leq D_0 \\ 1 & \text{if } D(u, v) > D_0 \end{cases}$$

$$H_{HP}(u, v) = 1 - H_{LP}(u, v)$$

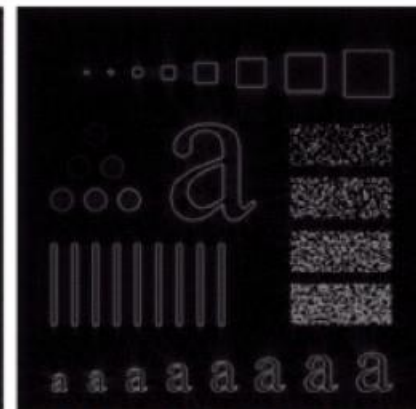
Ideal Highpass Filter



Results of ideal
high pass filtering
with $D_0 = 15$

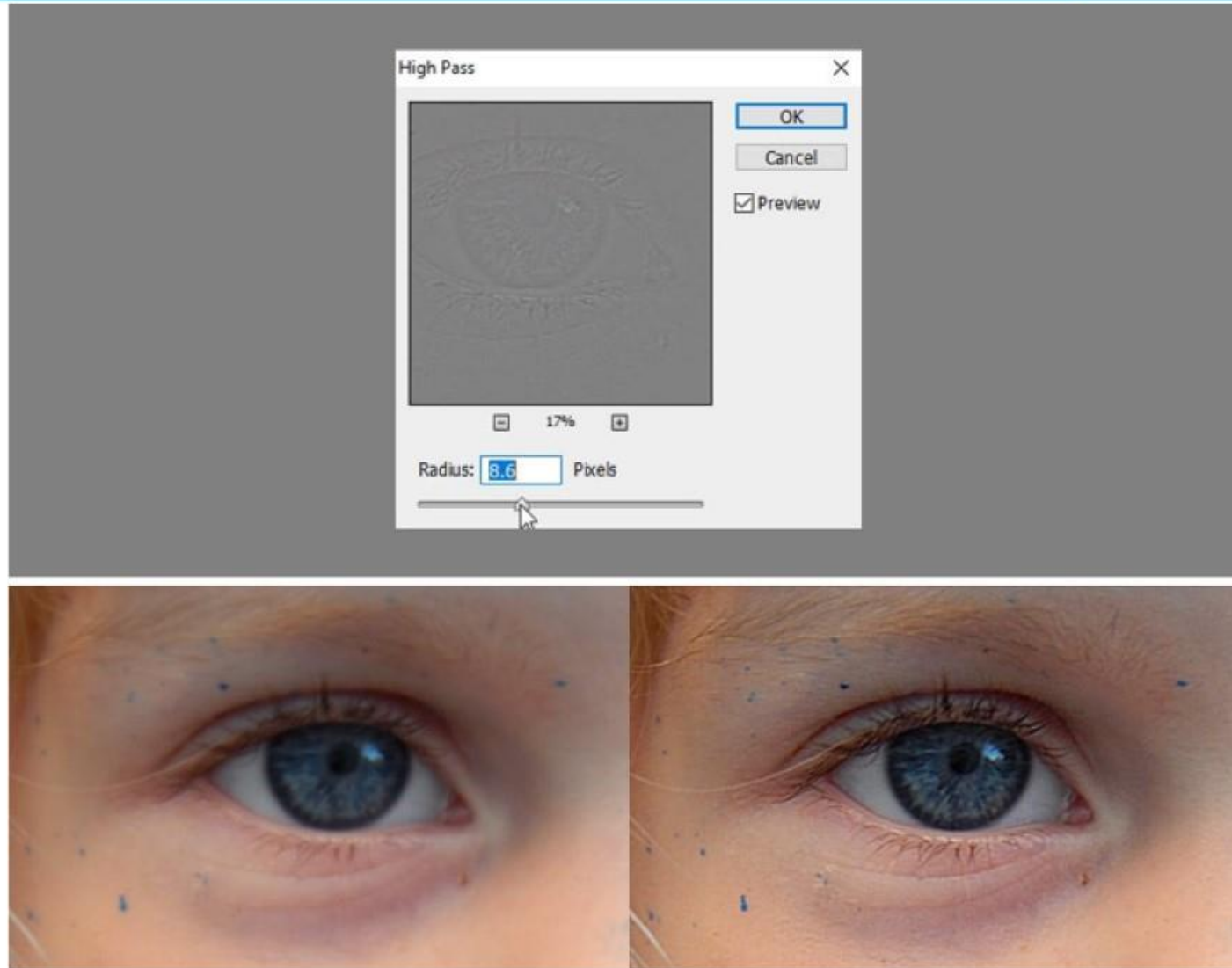


Results of ideal
high pass filtering
with $D_0 = 30$



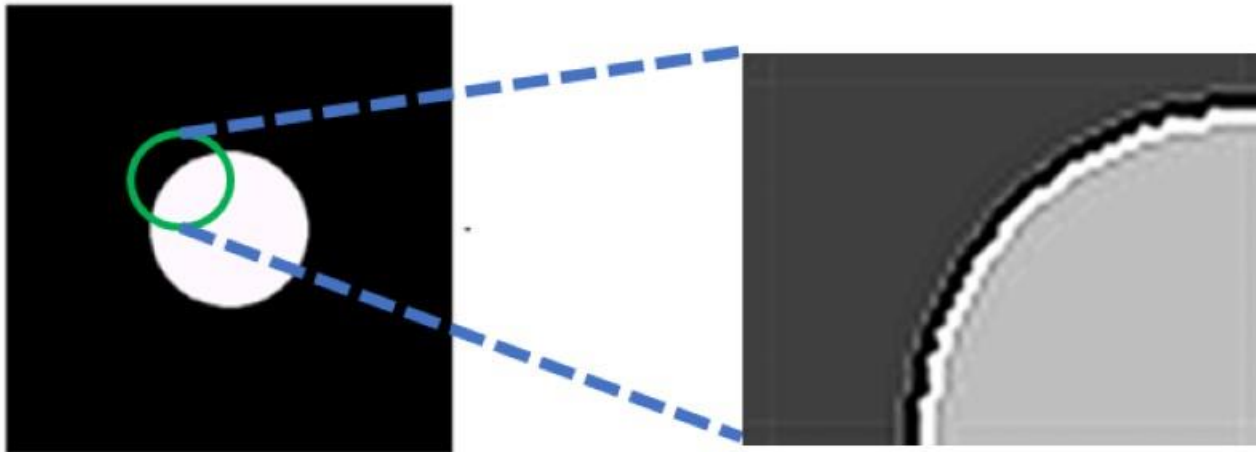
Results of ideal
high pass filtering
with $D_0 = 80$

Ideal Highpass Filter

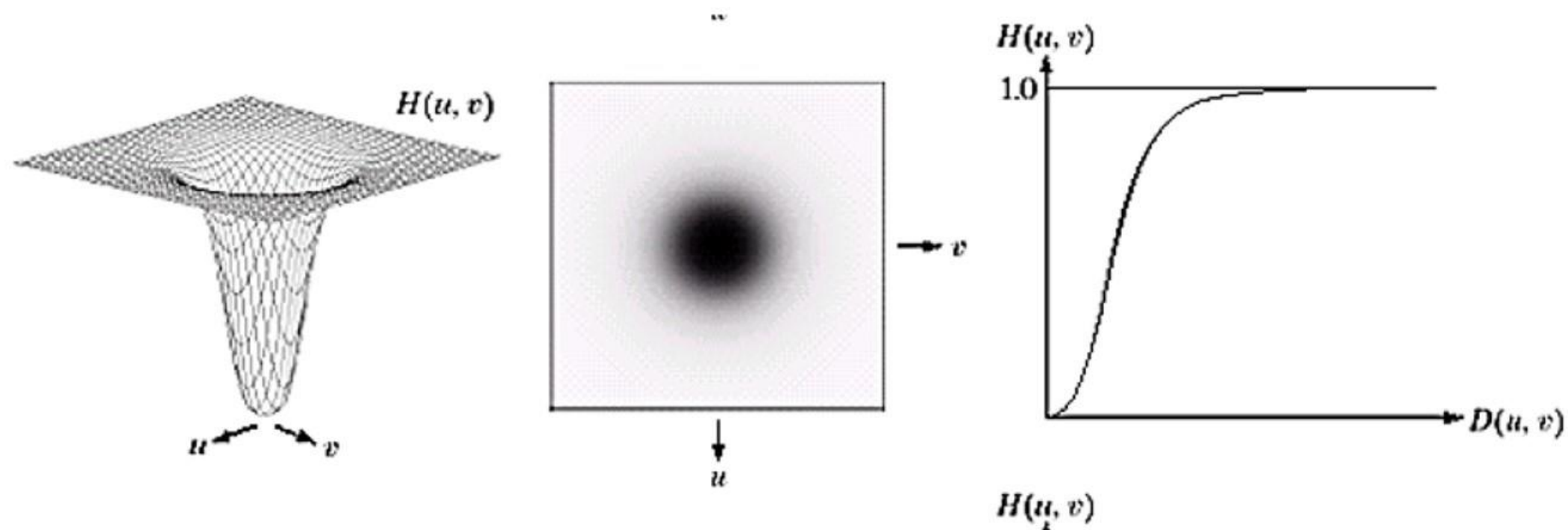


The Ringing Effect

- When using an ideal Highpass/Lowpass filter or Butterworth filter with a sharp cutoff frequency, the abrupt transition from passing to blocking certain frequencies can cause these oscillations or ripples to occur around edges. This phenomenon is called The ringing effect.



Butterworth Highpass Filter



Butterworth Highpass Filter

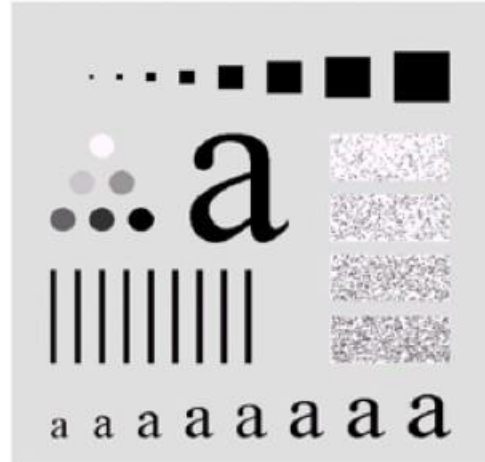
- Butterworth highpass filter is the reverse operation of the Butterworth lowpass filter.
- it introduces a smooth transition from 0 to 1 to reduce ringing artifacts.

$$H_{HP}(u, v) = 1 - H_{LP}(u, v)$$

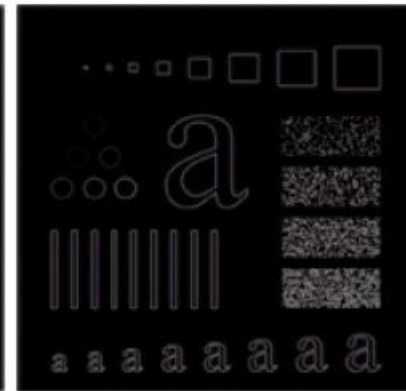
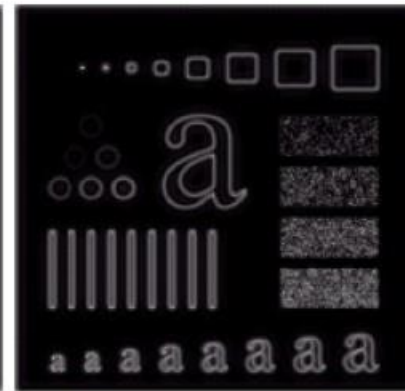
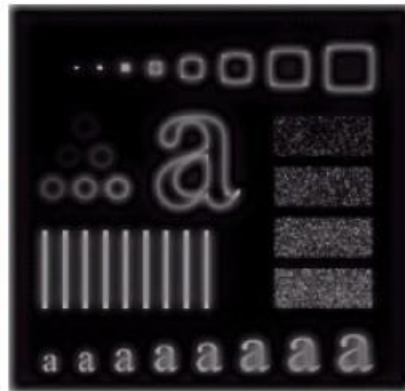
- The transfer function of BHPF of order n is defined as

$$H(u, v) = \frac{1}{1 + [D_0/D(u, v)]^{2n}}$$

Butterworth Highpass Filter



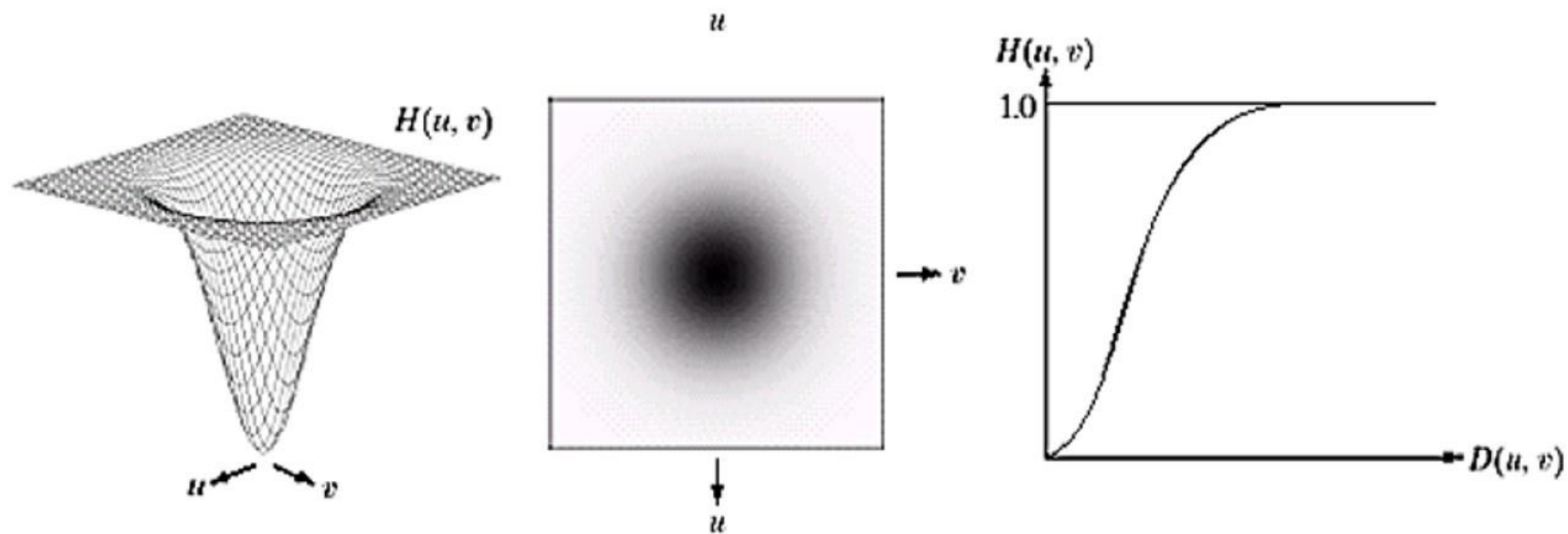
Results of
Butterworth
high pass
filtering of
order 2 with
 $D_0 = 15$



Results of
Butterworth
high pass
filtering of
order 2 with
 $D_0 = 80$

Results of Butterworth high pass
filtering of order 2 with $D_0 = 30$

Gaussian Highpass Filter

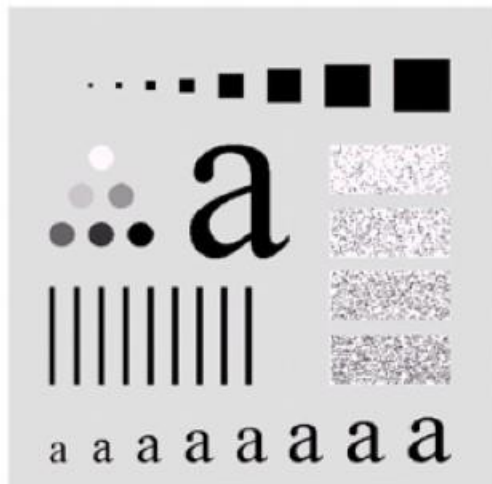


Gaussian Highpass Filter

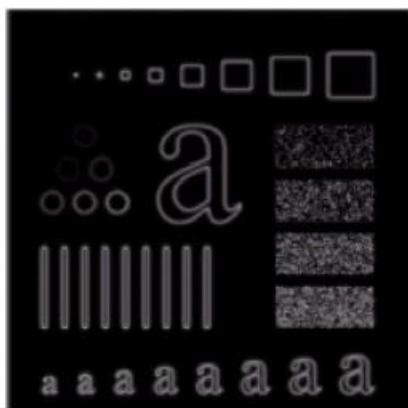
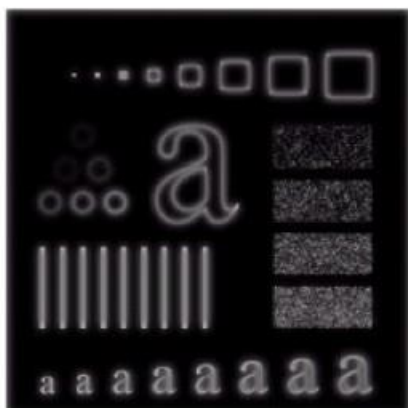
- Unlike traditional highpass filters, which have a uniform frequency response, Gaussian highpass filters have a bell-shaped response curve centered around a specific cutoff frequency.
- The Gaussian high pass filter is given as

$$H(u, v) = 1 - e^{-D^2(u, v) / 2D_0^2}$$

Gaussian Highpass Filter



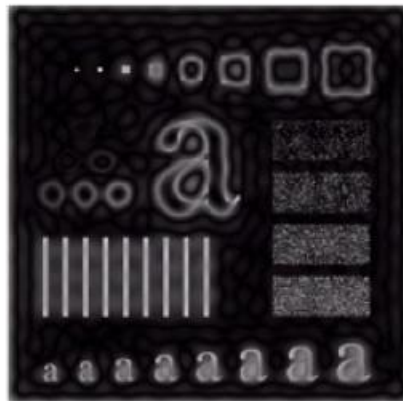
Results of
Gaussian
high pass
filtering with
 $D_0 = 15$



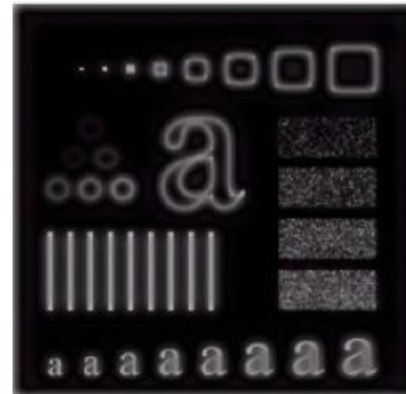
Results of
Gaussian
high pass
filtering with
 $D_0 = 80$

Results of Gaussian high
pass filtering with $D_0 = 30$

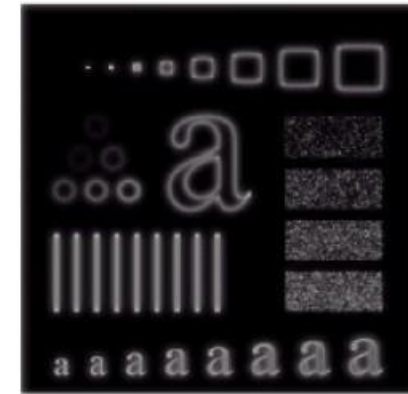
Highpass Filter Which One is Better



Results of ideal
high pass filtering
with $D_0 = 15$



Results of Butterworth
high pass filtering of
order 2 with $D_0 = 15$

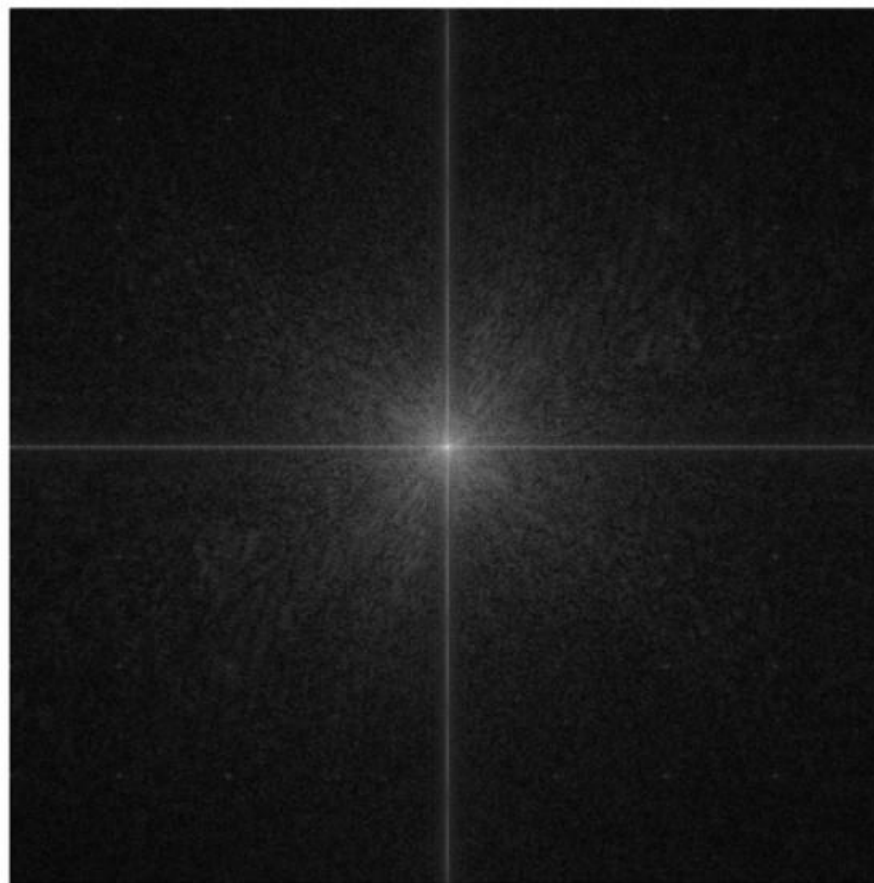


Results of Gaussian
high pass filtering with
 $D_0 = 15$

Examples



FT
→



Examples

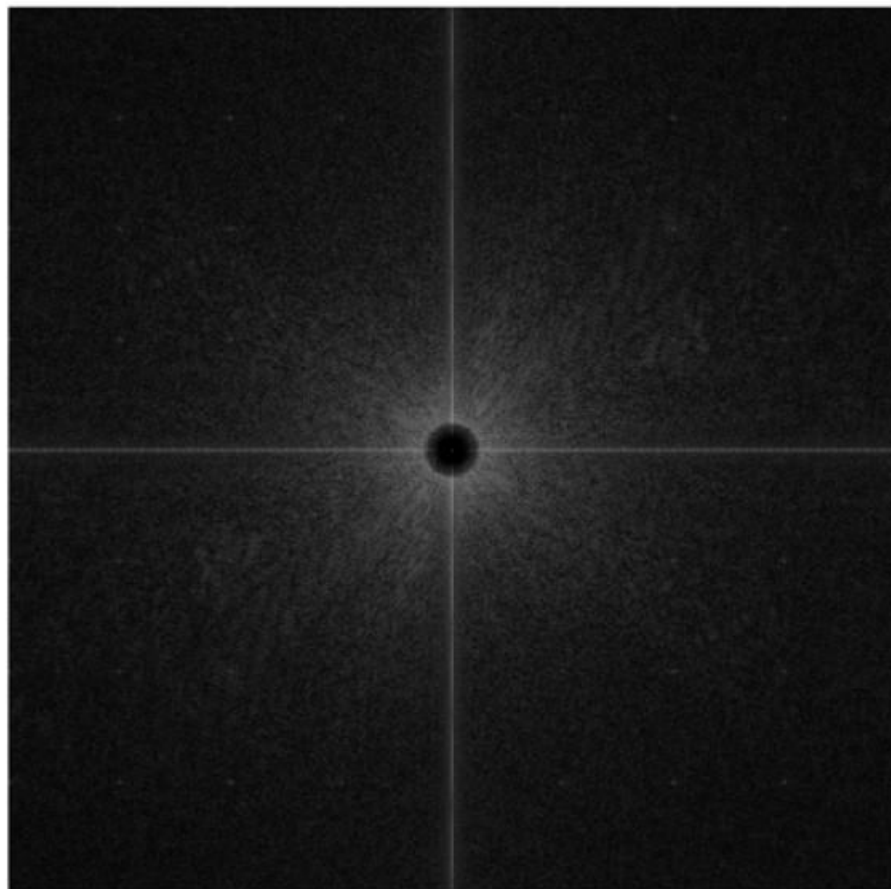


Image Restoration and Reconstruction

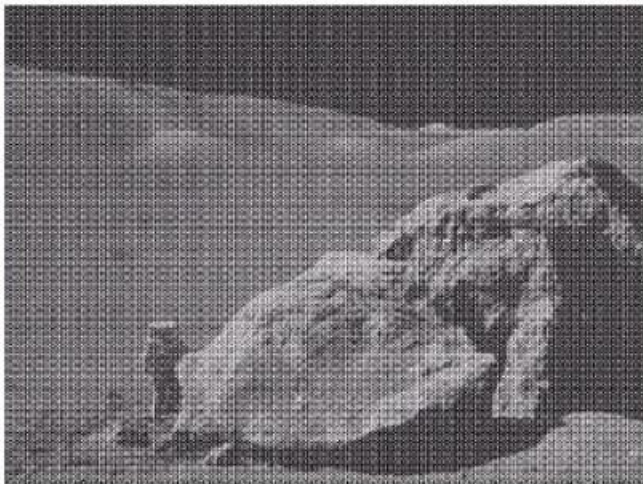
Chapter 5

Image Restoration

- As in image enhancement, the principal goal of restoration techniques is to improve an image in some predefined sense (Image Enhancement).
- The process of taking a corrupted or noisy image and estimating the clean, original image. This restoration involves reversing the effects that blurred the image, often by imaging a point known as the Point Spread Function (PSF) to recover the lost image information due to blurring.

Image Restoration

- Image restoration techniques can be categorized into spatial domain methods, which operate directly on image pixels, and frequency domain methods, which transform images into the frequency domain using techniques like the Fourier transform for restoration operations.



In Brief

- Image restoration aims to rectify or reverse defects that impair image quality.
- Degradation encompasses various forms including motion blur, noise, and camera miss focus.
- For instance, in scenarios of motion blur, it is feasible to generate a precise estimation of the blurring function and reverse the blur to retrieve the original image.
- However, when noise corrupts the image, the most achievable outcome is mitigating the impact of the degradation it induced.

Degradation of images can have many causes

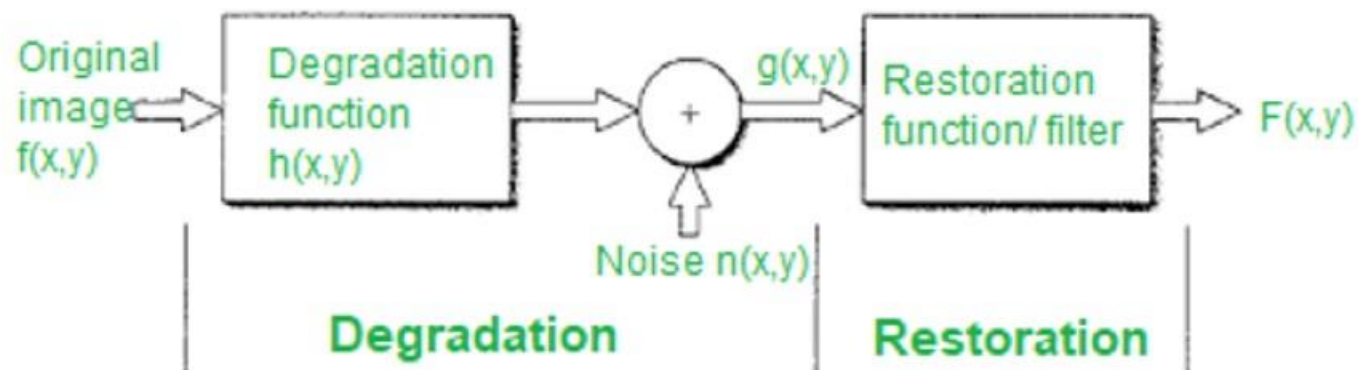
- defects of optical lenses;
- nonlinearity of the electro-optical sensor;
- graininess of the film material;
- relative motion between an object and camera
- wrong focus,
- atmospheric turbulence in remote sensing or astronomy,
- etc.

The objective of image restoration is to reconstruct the original image from its degraded version.

Noise Model

- The principal source of noise in digital images arises during image acquisition and transmission.
- Mathematically Noise can be modeled through the following:

$$g(x, y) = f(x, y) + \eta(x, y)$$



Gaussian noise

In image processing, various types of noise can corrupt an image, affecting its quality and making it challenging to extract useful information.

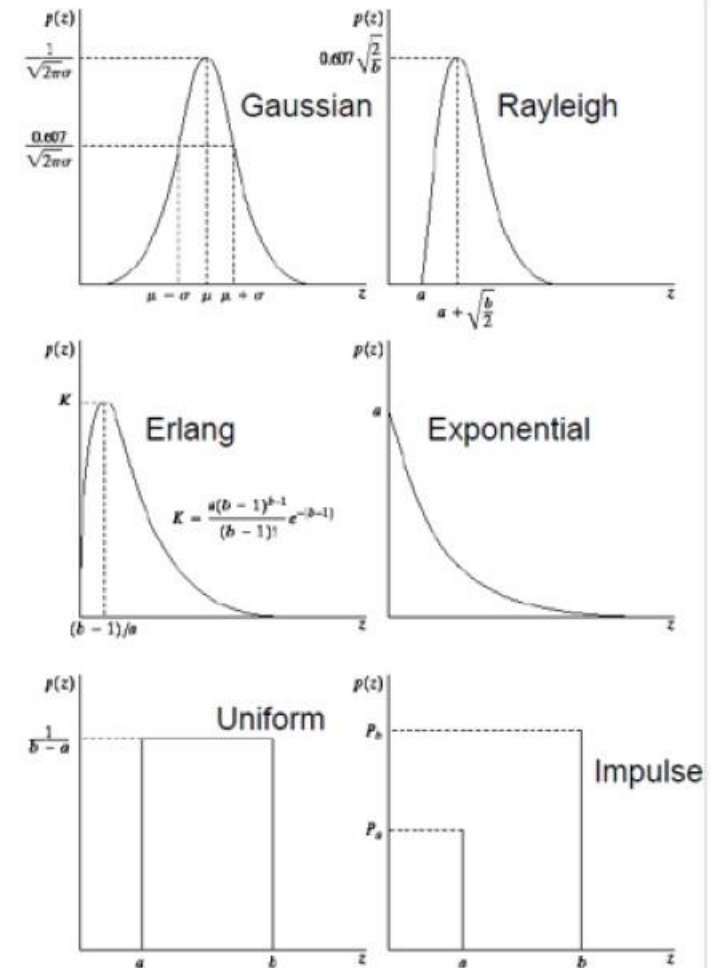
- **Gaussian Noise:**
- Gaussian noise is characterized by a Gaussian probability distribution and typically manifests as random variations in pixel intensity.
- In the frequency domain, Gaussian noise appears as high-frequency components scattered throughout the spectrum.
- Gaussian noise models are used frequently in practice. In fact, this tractability is so convenient that it often results in Gaussian models being used in situations in which they are marginally applicable at best.

The PDF of a *Gaussian* random variable, z , is defined by the following familiar expression:

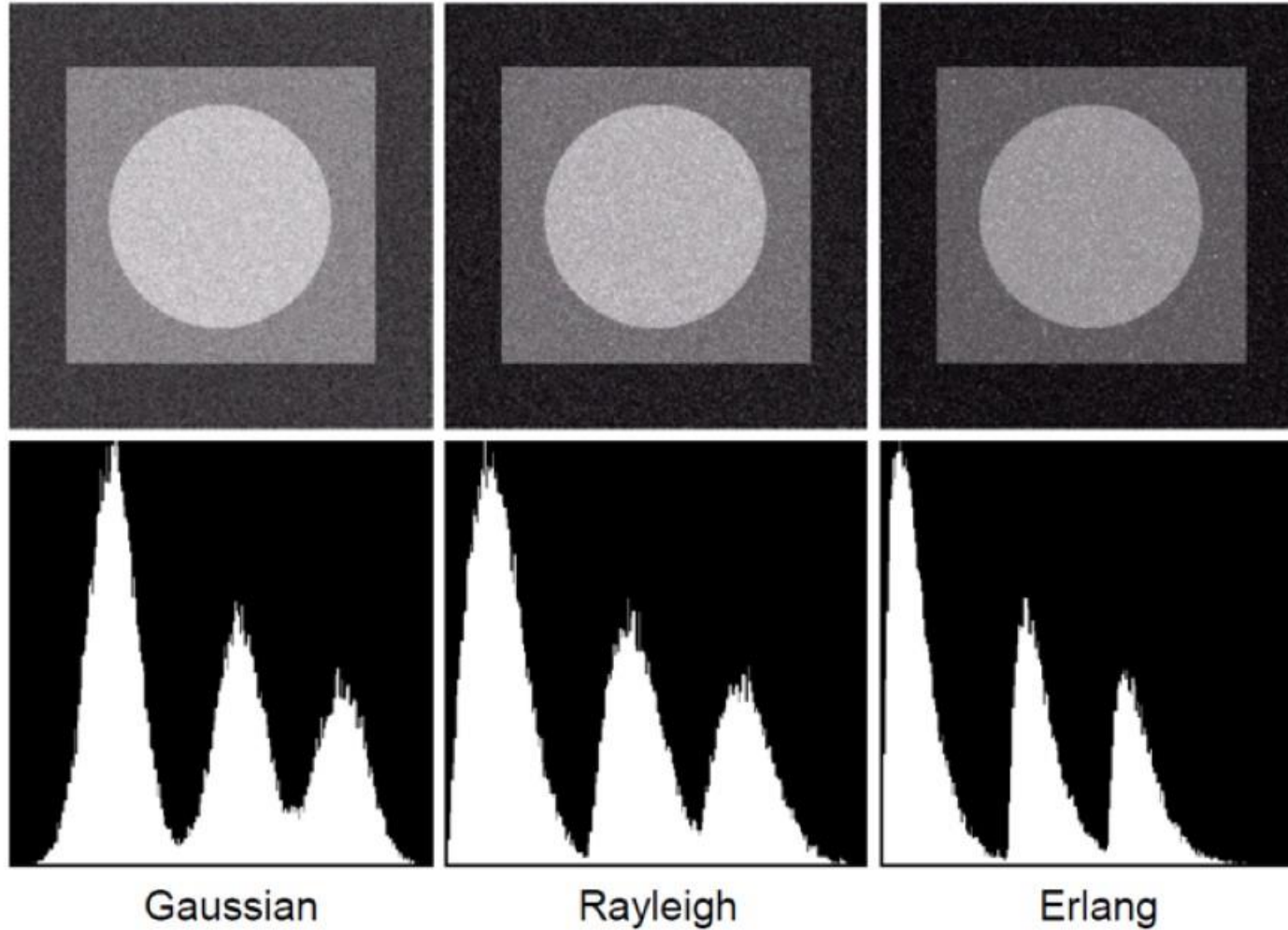
$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z - \bar{z})^2}{2\sigma^2}} \quad -\infty < z < \infty \quad (5-3)$$

Noise Models

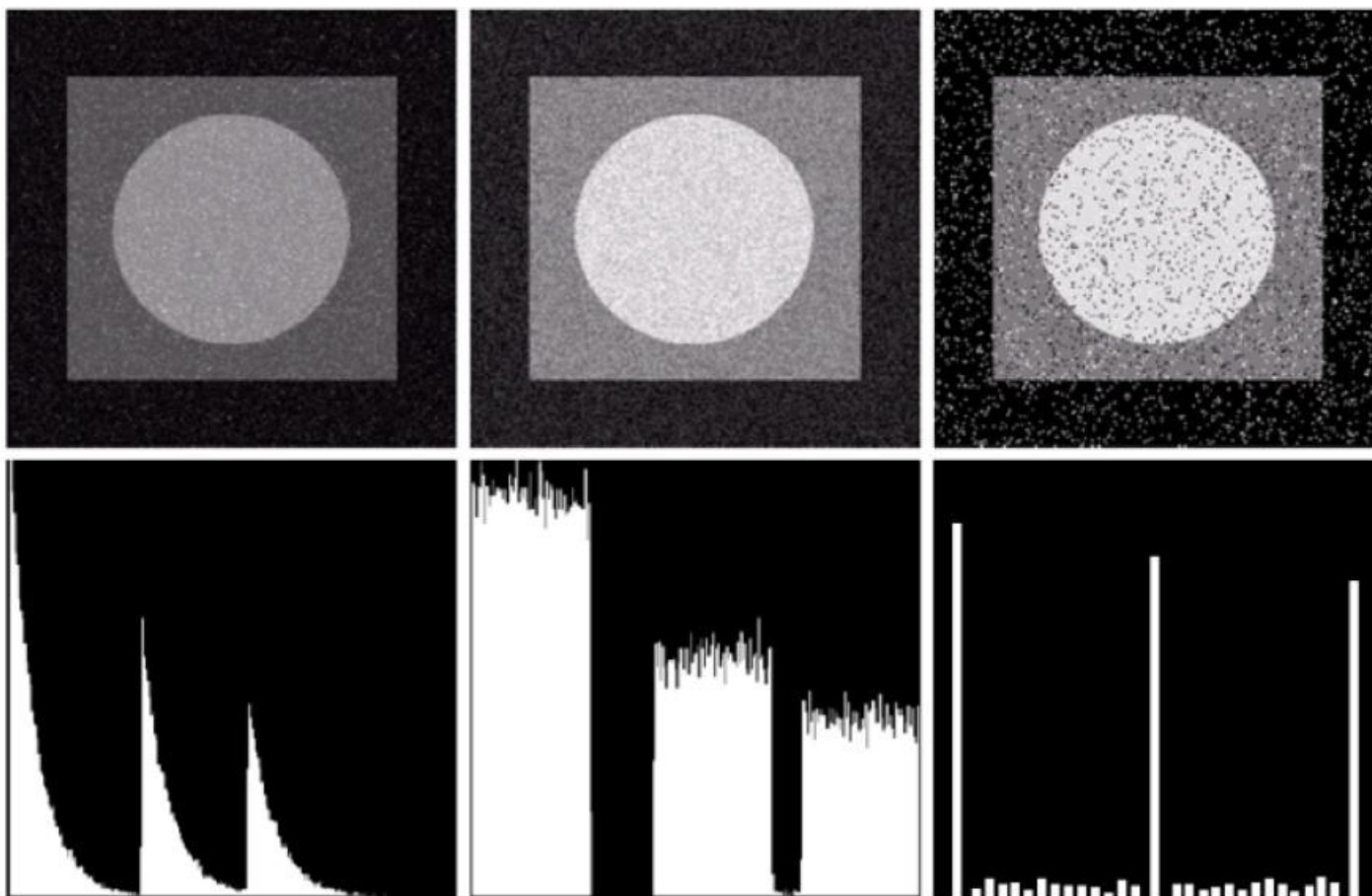
- There are many different models for the image noise term $\eta(x, y)$:
- –Gaussian
 - Most common model
- –Rayleigh
- –Erlang
- –Exponential
- –Uniform
- –Impulse
 - *Salt and pepper* noise



Noise Models



Noise Models



Exponential

Uniform

Impulse

RESTORATION IN THE PRESENCE OF NOISE ONLY—SPATIAL
FILTERING

- When an image is degraded only by additive noise

$$g(x, y) = f(x, y) + \eta(x, y) \quad \text{spatial}$$

$$G(u, v) = F(u, v) + N(u, v) \quad \text{Freq}$$

MEAN FILTERS

- The *arithmetic mean* filter is a very simple one and is calculated as follows:

$$\hat{f}(x, y) = \frac{1}{mn} \sum_{(s,t) \in S_{xy}} g(s, t)$$

- This is implemented as the simple smoothing filter
- Blurs the image to remove noise

$1/9$	$1/9$	$1/9$
$1/9$	$1/9$	$1/9$
$1/9$	$1/9$	$1/9$

MEAN FILTERS

- There are different kinds of mean filters all of which exhibit slightly different behavior:
 - Geometric Mean
 - Harmonic Mean
 - Contraharmonic Mean

Geometric Mean

There are other variants on the mean which can give different performance

Geometric Mean:

$$\hat{f}(x, y) = \left[\prod_{(s, t) \in S_{xy}} g(s, t) \right]^{\frac{1}{mn}}$$

Achieves similar smoothing to the arithmetic mean, but tends to lose less image detail

Harmonic Mean

Harmonic Mean:

$$\hat{f}(x, y) = \frac{mn}{\sum_{(s,t) \in S_{xy}} \frac{1}{g(s,t)}}$$

Works well for salt noise, but fails for pepper noise

Also does well for other kinds of noise such as Gaussian noise

Contraharmonic Mean

$$\hat{f}(x, y) = \frac{\sum_{(s,t) \in S_{xy}} g(s, t)^{Q+1}}{\sum_{(s,t) \in S_{xy}} g(s, t)^Q}$$

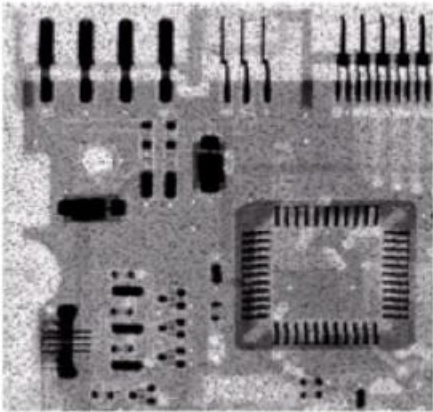
Q is the *order* of the filter and adjusting its value changes the filter's behaviour

Positive values of Q eliminate pepper noise

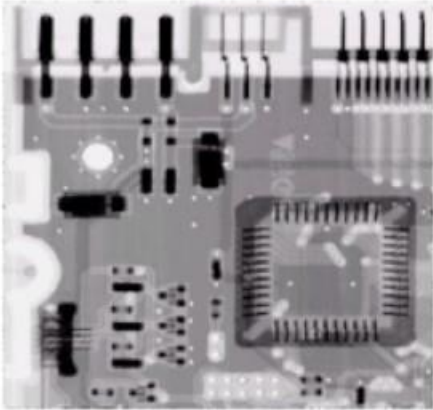
Negative values of Q eliminate salt noise

Examples

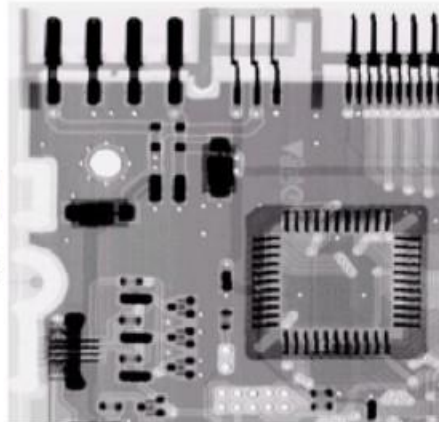
Image
Corrupted
By Pepper
Noise



Result of
Filtering Above
With 3*3
Contra-harmonic
 $Q=1.5$



Original
Image



After A 3*3
Arithmetic
Mean Filter

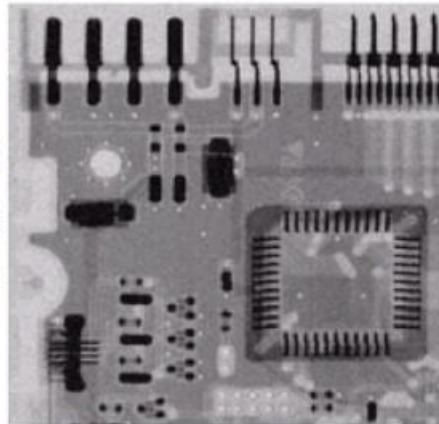
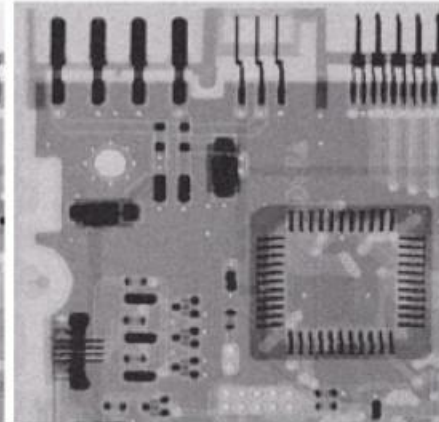
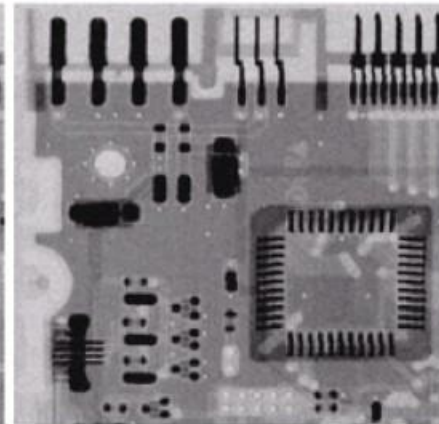


Image
Corrupted
By Gaussian
Noise

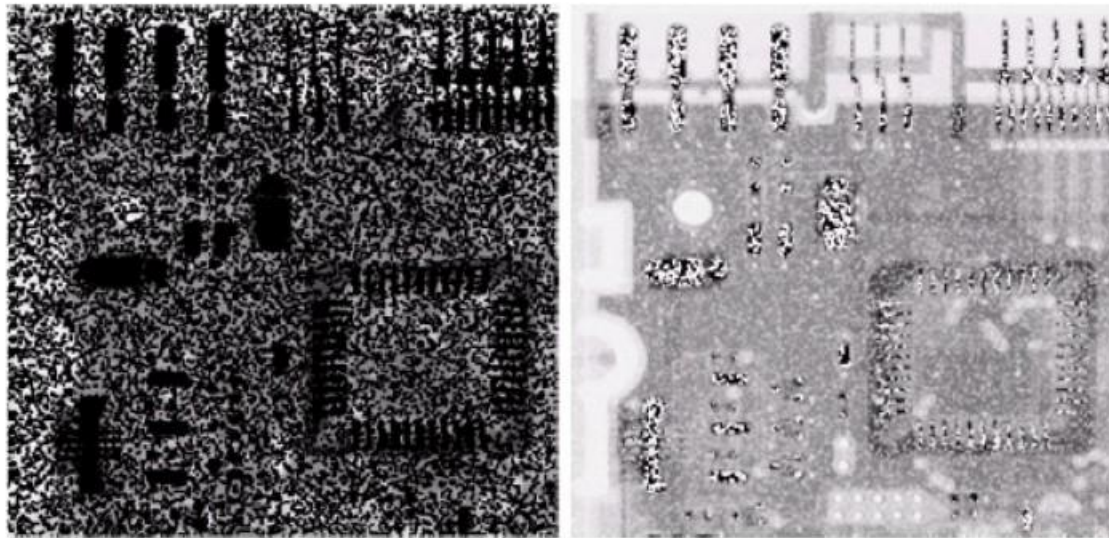


After A 3*3
Geometric
Mean Filter



Examples

- Contraharmonic Filter: Here Be Dragons
- Choosing the wrong value for Q when using the contraharmonic filter can have drastic results

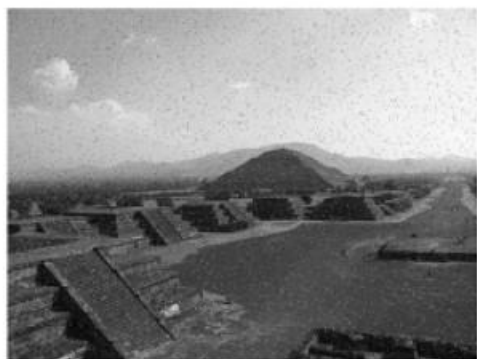


Order Statistics Filters

- Spatial filters that are based on ordering the pixel values that make up the neighborhood operated on by the filter
- Useful spatial filters include
 - Median filter
 - Max and min filter
 - Midpoint filter
 - Alpha trimmed mean filter

Median Filter

- Like the mean filter, the median filter considers each pixel in the image in turn and looks at its nearby neighbors to decide whether or not it is representative of its surroundings. Instead of simply replacing the pixel value with the *mean* of neighboring pixel values, it replaces it with the *median* of those values.



123	125	126	130	140
122	124	126	127	135
118	120	150	125	134
119	115	119	123	133
111	116	110	120	130

Neighbourhood values:

115, 119, 120, 123, 124,
125, 126, 127, 150

Median value: 124

Max/Min Filters

- The max filter replaces each pixel with the maximum value in its neighborhood, enhancing bright regions and edges. In contrast, the min filter replaces pixels with the minimum value, emphasizing dark regions and edges. These filters are useful for enhancing local contrast and detecting edges in images

Max filter is good for pepper noise and
min is good for salt noise

Max Filter:

$$\hat{f}(x, y) = \max_{(s,t) \in S_{xy}} \{g(s, t)\}$$

Min Filter:

$$\hat{f}(x, y) = \min_{(s,t) \in S_{xy}} \{g(s, t)\}$$

Max/Min Filters

	67	66	55	10
	57	255	56	70
	57	50	60	70
	67	60	75	80

67	66	55	10
57	255	56	70
57	50	60	70
67	60	75	80

0	66	55	10
57	255	10	70
57	50	60	70
67	60	75	80

Midpoint Filter

- The midpoint filter calculates the midpoint between the maximum and minimum pixel values in a neighborhood, providing a balance between the max and min filters. It is effective at reducing noise while preserving image details and is particularly useful for images with varying intensity levels

$$\hat{f}(x, y) = \frac{1}{2} \left[\max_{(s,t) \in S_{xy}} \{g(s, t)\} + \min_{(s,t) \in S_{xy}} \{g(s, t)\} \right]$$

Good for random Gaussian and uniform noise

Alpha-Trimmed Mean Filter

- The alpha trimmed mean filter removes a specified number of extreme pixel values (outliers) from a neighborhood and then calculates the mean of the remaining values. This filter is robust against outliers and noise, making it suitable for images with impulsive noise or extreme pixel values

$$\hat{f}(x, y) = \frac{1}{mn - d} \sum_{(s, t) \in S_{xy}} g_r(s, t)$$

Alpha-Trimmed Mean Filter

67	66	55	10
57	255	56	70
57	50	60	70
67	60	75	80

66, 55, ~~10~~, ~~255~~, 56, 70, 50, 60, 70

$$\text{Mean} = 66 + 55 + 56 + 70 + 50 + 60 + 70 = 427 / 7 = 61$$



Thank You