

Mathematics

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Lecture 2

جامعة الجميع الذكية

تسعى لتكون الكيان الأول الذي يتبنى مبدأ المجتمع التعليمي
التعاوني ويقدم العلم مقابل العمل لتنمية وبناء المجتمعات.

MAT 122

Scalar and Vector



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- **What is the scalar?**
- **Examples for scalar**
- **What is the vectors?**
- **Examples for vectors**
- **Dot product and cross product concept**
- **How to calculate the dot product ?**

What is Scalar?



Length of a car is 4.5 m

physical quantity

magnitude



Mass of gold bar is 1 kg

physical quantity

magnitude

Lecture 02

Example of scalar



Time is 12.76 s

physical quantity

magnitude



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Lecture 02

Example of scalar



Temperature is 36.8 °C

physical quantity

magnitude



A scalar is a physical quantity that has only a magnitude.

Examples:

- Mass
- Length
- Time
- Temperature

What is Vector?



Lecture 02

Vector Example

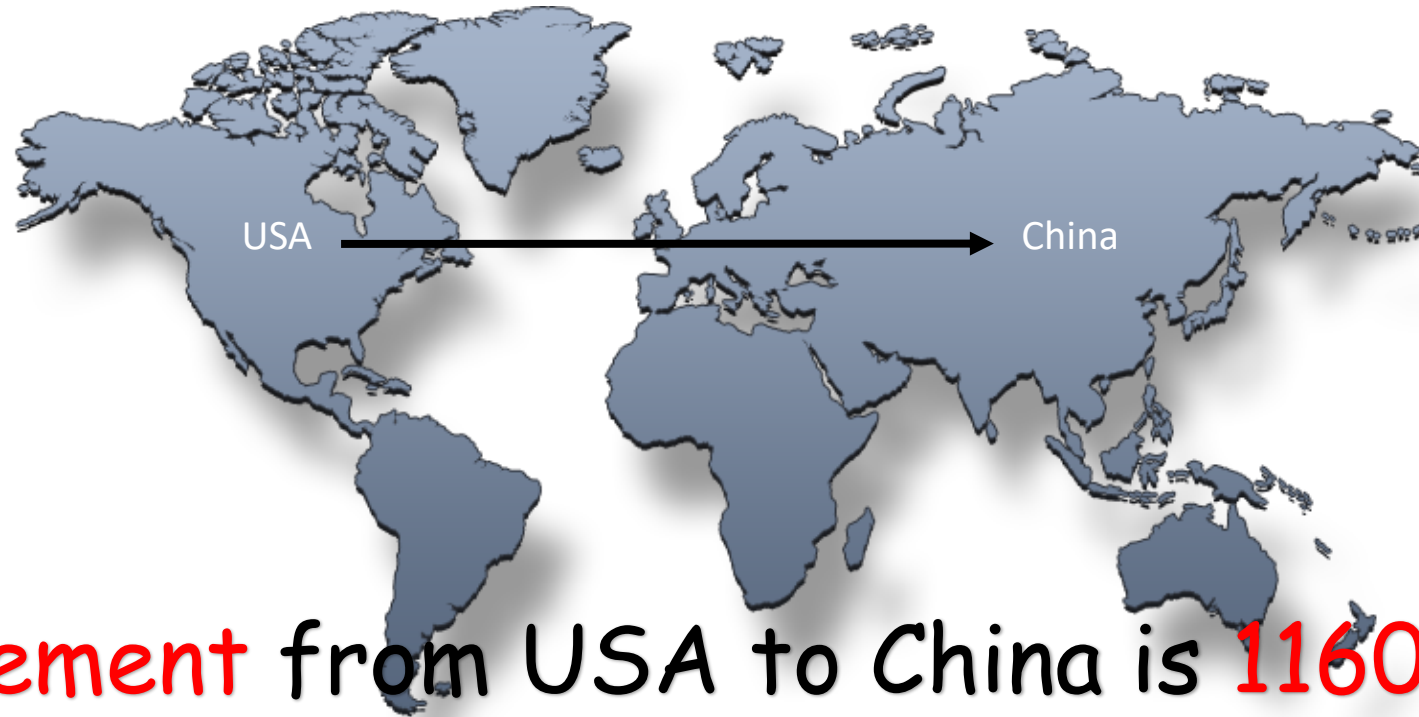


Position of California from North Carolina is **3600 km**
in **west**

physical quantity direction magnitude

Lecture 02

Vector Example



Displacement from USA to China is 11600 km
in east

physical quantity

direction

magnitude



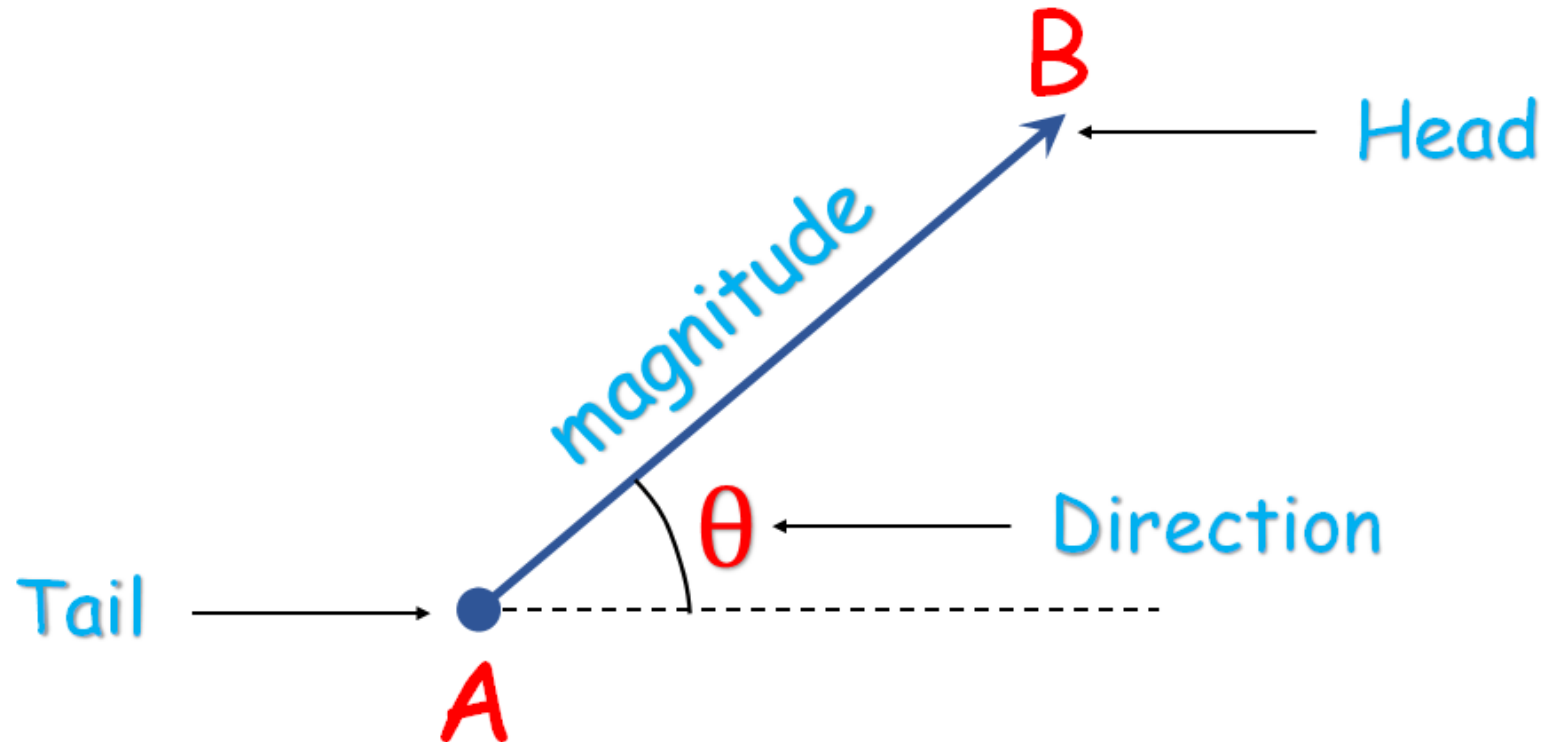
A vector is a physical quantity that has both a magnitude and a direction.

Examples:

- Position
- Displacement
- Velocity
- Acceleration
- Momentum
- Force



Representation of a vector

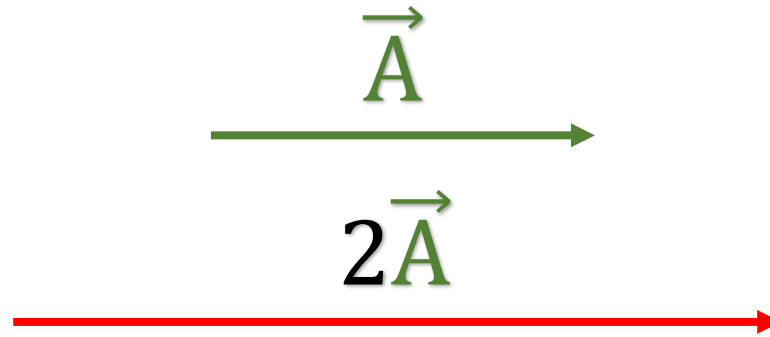


Symbolically it is represented as $\overrightarrow{AB}$

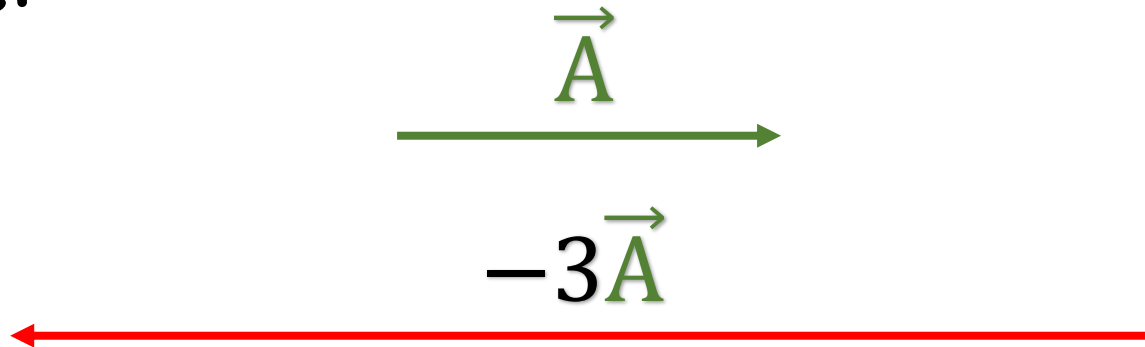
Multiplying a vector by a scalar

- If we multiply a vector $\vec{A}$ by a scalar s , we get a new vector.
- Its direction is the direction of $\vec{A}$ if s is positive but the opposite direction if s is negative.

If s is positive:



If s is negative:

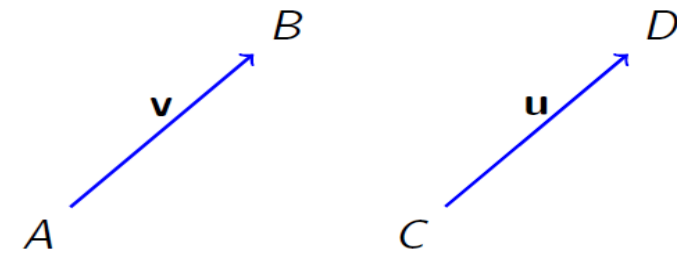


There are two ways to multiply a vector by a vector:

- The first way produces a scalar quantity and called as scalar product (dot product).
- The second way produces a vector quantity and called as vector product (cross product).

- Notes
- Theorem
- Facts

- ▶ A **vector** is something that has *magnitude* and *direction*
- ▶ We denote vectors by boldface ($\mathbf{v}$) or little arrows ($\vec{v}$). One is good for print, one for script
- ▶ Given two points A and B in flatland or spaceland, the vector which starts at A and ends at B is called the **displacement vector** $\overrightarrow{AB}$.
- ▶ Two vectors are equal if they have the same magnitude and direction (they need not overlap)



Theorem

If $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, and c is a scalar, then

- ▶ $\mathbf{a} + \mathbf{b} = \langle a_1 + b_1, a_2 + b_2, a_3 + b_3 \rangle$
- ▶ $\mathbf{a} - \mathbf{b} = \langle a_1 - b_1, a_2 - b_2, a_3 - b_3 \rangle$
- ▶ $c\mathbf{a} = \langle ca_1, ca_2, ca_3 \rangle$



Fact

If $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are vectors and t is a scalar, then

1. $\mathbf{a} \cdot \mathbf{a} = |\mathbf{a}|^2$

2. $\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{a}$

3. $\mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c}$

4. $(t\mathbf{a}) \cdot \mathbf{b} = t(\mathbf{a} \cdot \mathbf{b}) = \mathbf{a} \cdot (t\mathbf{b})$

5. $\mathbf{0} \cdot \mathbf{a} = 0$



Extra Practice



Example 1

Calculate the dot product of

$$\mathbf{a}=(1,2,3) \text{ and } \mathbf{b}=(4,-5,6)$$

Solution:

Using the [component formula](#) for the dot product of three-dimensional vectors,

$$\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3,$$

we calculate the dot product to be

$$\mathbf{a} \cdot \mathbf{b} = 1(4) + 2(-5) + 3(6) = 4 - 10 + 18 = 12.$$

Calculate the dot product of

$$\mathbf{a}=(-4,-9) \text{ and } \mathbf{b}=(-1,2).$$

Solution:

Using the [component formula](#) for the dot product of two-dimensional vectors,

$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2$$

we calculate the dot product to be

$$\mathbf{a} \cdot \mathbf{b} = -4(-1) - 9(2) = 4 - 18 = -14.$$

Determine the dot product, $\vec{a} \cdot \vec{b}$ if $\vec{a} = \langle 9, 5, -4, 2 \rangle$ and $\vec{b} = \langle -3, -2, 7, -1 \rangle$.



We just need to run through the definition of the dot product.

$$\vec{a} \cdot \vec{b} = (9)(-3) + (5)(-2) + (-4)(7) + (2)(-1) = \boxed{-67}$$

Determine the dot product, $\vec{a} \cdot \vec{b}$ if $\vec{a} = \langle 0, 4, -2 \rangle$ and $\vec{b} = 2\vec{i} - \vec{j} + 7\vec{k}$.

We just need to run through the definition of the dot product and do not get excited about the “mixed” notation here. We know that they are equivalent notations!

$$\vec{a} \cdot \vec{b} = (0)(2) + (4)(-1) + (-2)(7) = \boxed{-18}$$

Calculate the dot product of

$$\mathbf{a}=(4,2,1) \text{ and } \mathbf{b}=(-1,5,7).$$



Solution:

Using the [component formula](#) for the dot product of two-dimensional vectors,

$$\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$$

we calculate the dot product to be

$$\mathbf{a} \cdot \mathbf{b} = 4(-1) + 2(5) + 1(7) = -4 + 10 + 7 = 13.$$

Calculate the dot product of

$$\mathbf{a}=(5,2,1) \text{ and } \mathbf{b}=(8,2,2).$$



Solution:

Using the [component formula](#) for the dot product of two-dimensional vectors,

$$\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$$

we calculate the dot product to be

$$\mathbf{a} \cdot \mathbf{b} = 5(8) + 2(2) + 1(2) = 40 + 4 + 2 = 46.$$

Calculate the dot product of

$\mathbf{a}=(6,9,0)$ and $\mathbf{b}=(8,3,1)$.



Solution:

Using the [component formula](#) for the dot product of two-dimensional vectors,

$$\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$$

we calculate the dot product to be

$$\mathbf{a} \cdot \mathbf{b} = 6(8) + 9(3) + 0(1) = 48 + 27 + 0 = 75$$

Calculate the dot product of

$$\mathbf{a}=(1,-2,5,-4) \text{ and } \mathbf{b}=(1,3,-4,5).$$



Solution:

Using the [component formula](#) for the dot product of two-dimensional vectors,

$$\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3 + a_4b_4$$

we calculate the dot product to be

$$\mathbf{a} \cdot \mathbf{b} = 1(1) - 2(3) + 5(-4) - 4(5) = 1 - 6 - 20 - 20 = -45$$

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