

Operations Research

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IT 262
Operations Research

Lecture 3



Linear Programming

Graphical Solution

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Chapter

02

Lecture 3 Topics

- ❖ **Linear Programming**
- ❖ **mathematical model**
- ❖ **Graphical Solution**
- ❖ **Optimal Solution**

Graphical Solution

Graphical Solution Can be used when there are two decision variables by Plot the constraint equations at their limits by converting each equation to equality, then identifying the feasible solution space and getting the optimal solution.

Graphical Solution

Steps in the graphical method :

- 1- Formulate the objective and constraint functions.
- 2- Draw a graph with one variable on the horizontal axis and one on the vertical axis.
- 3- Plot each of the constraints as if they were lines.
- 4- Outline the feasible solution space
- 5- Circle the potential solution points .These are the intersections of the constraints or axes on the inner (minimization) or outer (maximization) perimeter of the feasible solution space.
- 6- Substitute each of the potential solution point values of the two decision variables into the objective function and solve for Z .
- 7- Select the solution point that optimizes Z (Optimal Solution).

Problem 1

A manager wants to know how many units of each product to produce on a daily basis in order to achieve the highest contribution to profit. Production requirement for the products are shown in the following table.

The profit is £8 for each unit of the first product and £6 for each unit of the second product

	Product	Processing time required for the first product (Hours)	Processing time required for the second product (Hours)	Profit (£/unit)
x_1	I	4	2	8
x_2	II	2	4	6

Production capacity
available (Hours)

60

48

Problem 1

- The objective function for the LP is:

$$\text{Maximize } Z = 8X_1 + 6X_2$$

- The two constraints can be expressed as:

$$4X_1 + 2X_2 \leq 60 \quad \text{constraint I}$$

$$2X_1 + 4X_2 \leq 48 \quad \text{constraint II}$$

- The no. negatively restriction is all

$$X_1, X_2 \geq 0$$

Problem 1

Solution:

1- Plot the Constraints (shown the following figure) Change constraints to equalities:

$$4X_1 + 2X_2 \leq 60$$

Line 1

$$4X_1 + 2X_2 = 60$$



$$\begin{aligned} \text{Let } X_1 &= 0 \\ 4(0) + 2X_2 &= 60 \\ X_2 &= 30 \end{aligned}$$

$X_1 \quad X_2$
The Point (0,30)

$$\begin{aligned} \text{Let } X_2 &= 0 \\ 4X_1 + 2(0) &= 60 \\ X_1 &= 15 \end{aligned}$$

$X_1 \quad X_2$
The Point (15,0)

$$2X_1 + 4X_2 \leq 48$$

Line 2

$$2X_1 + 4X_2 = 48$$



$$\begin{aligned} \text{Let } X_1 &= 0 \\ 2(0) + 4X_2 &= 48 \\ X_2 &= 12 \end{aligned}$$

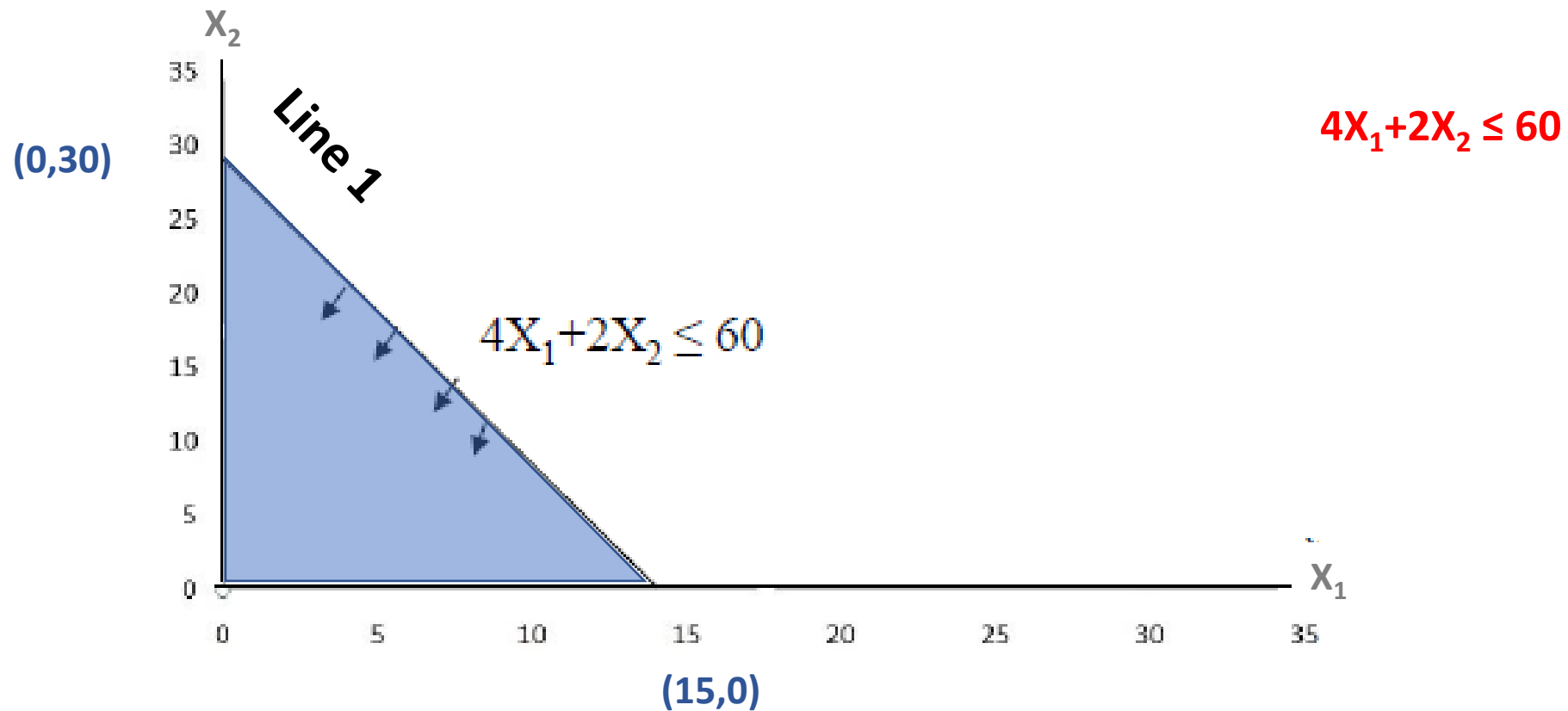
$X_1 \quad X_2$
The Point (0,12)

$$\begin{aligned} \text{Let } X_2 &= 0 \\ 2X_1 + 4(0) &= 48 \\ X_1 &= 24 \end{aligned}$$

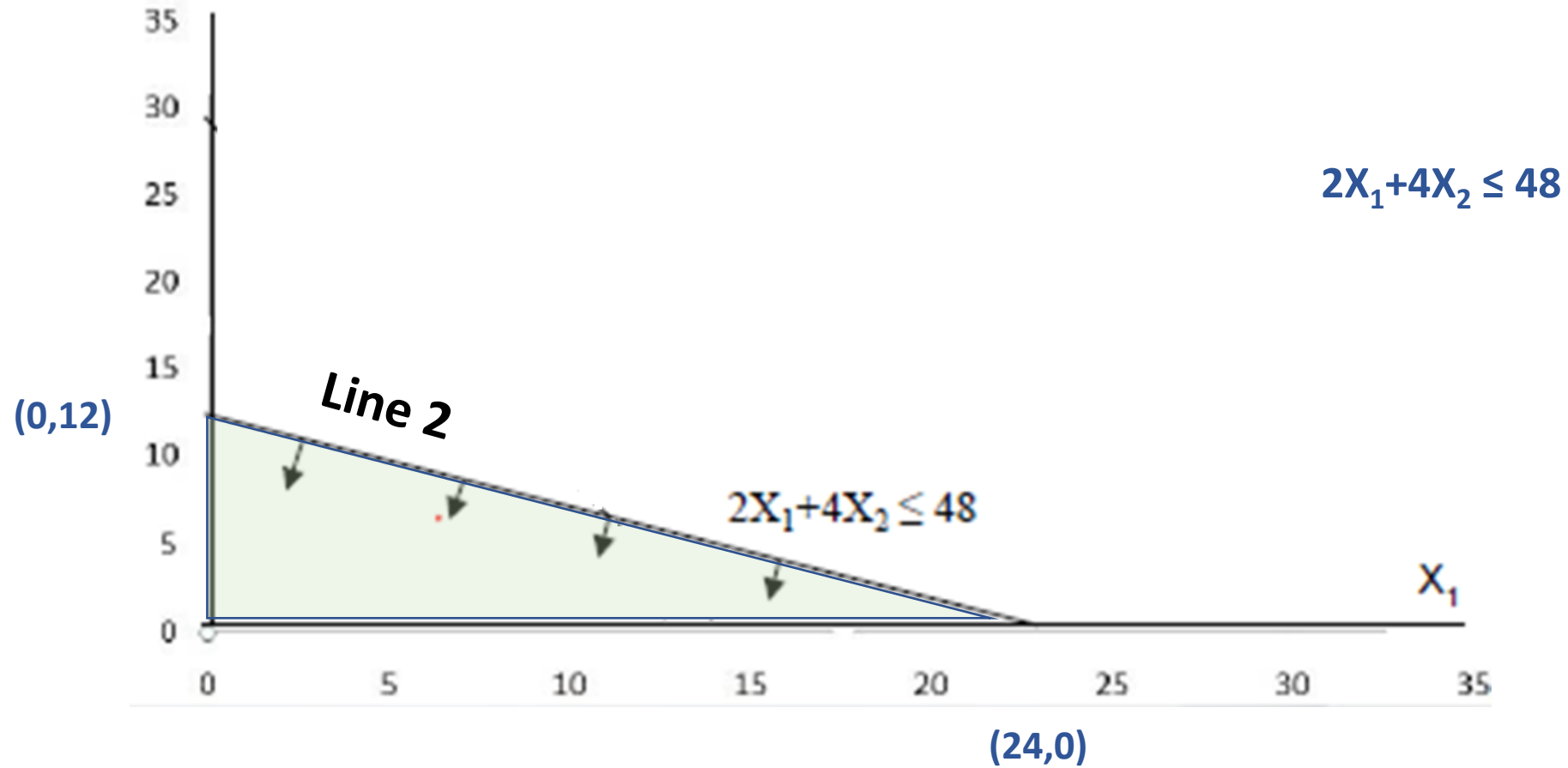
$X_1 \quad X_2$
The Point (24,0)

Problem 1

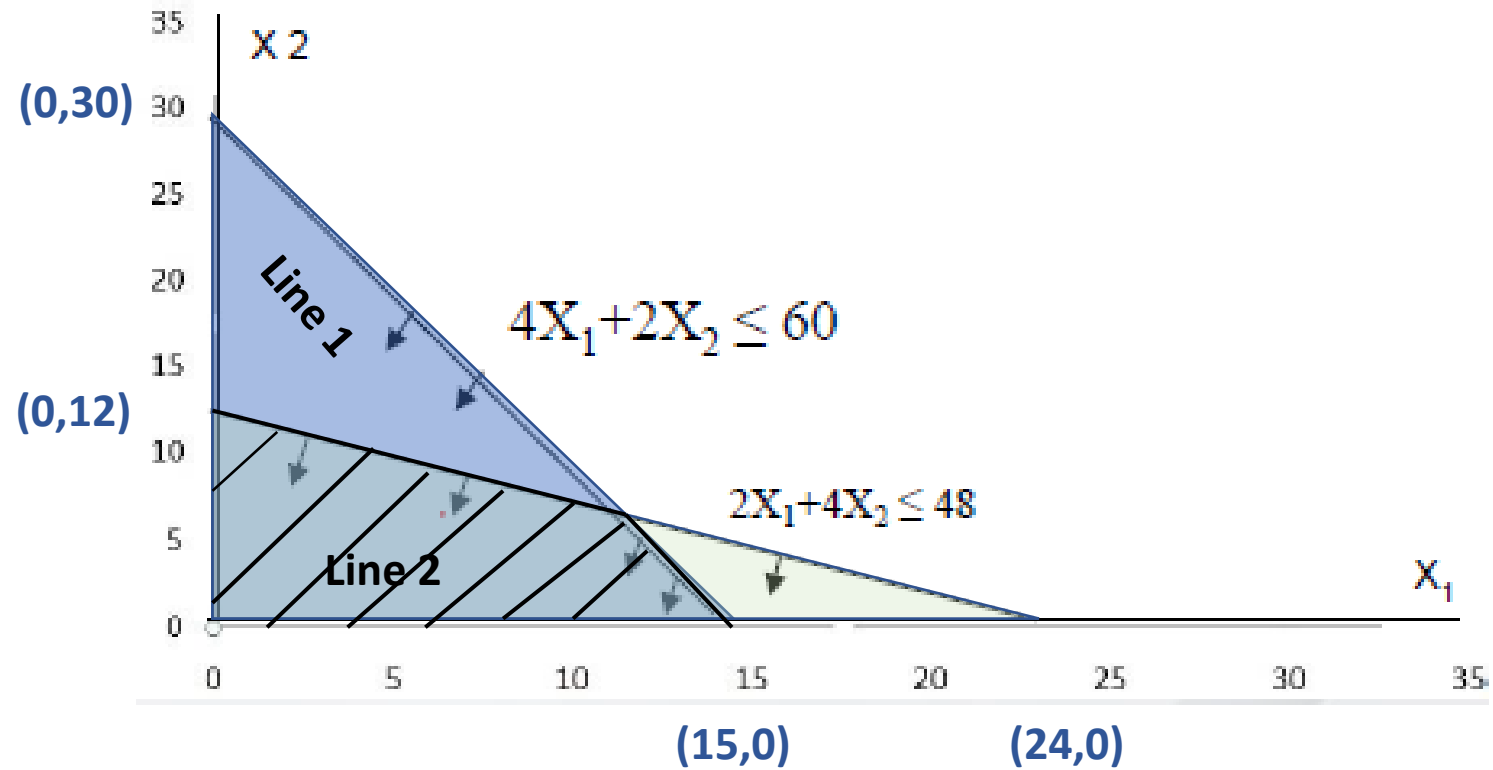
2- Outline the feasible solution space the values of X_1 and X_2 at points M,A,B and C are four potential solutions to problem.



Problem 1

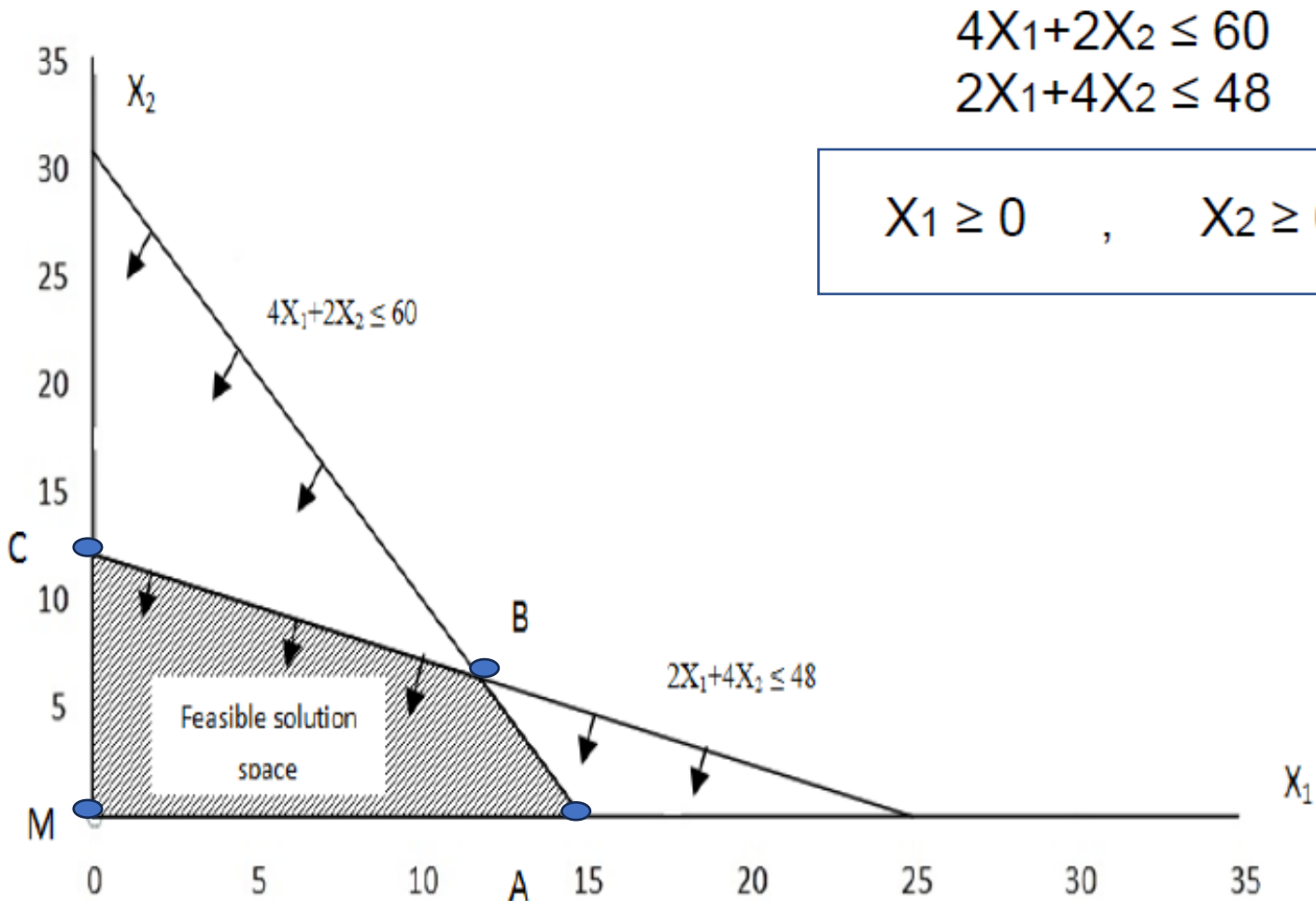


Problem 1



Problem 1

Outline the feasible solution space the values of X_1 and X_2 at points M, A, B and C are four potential solutions to problem.



Problem 1

Corner Point Method

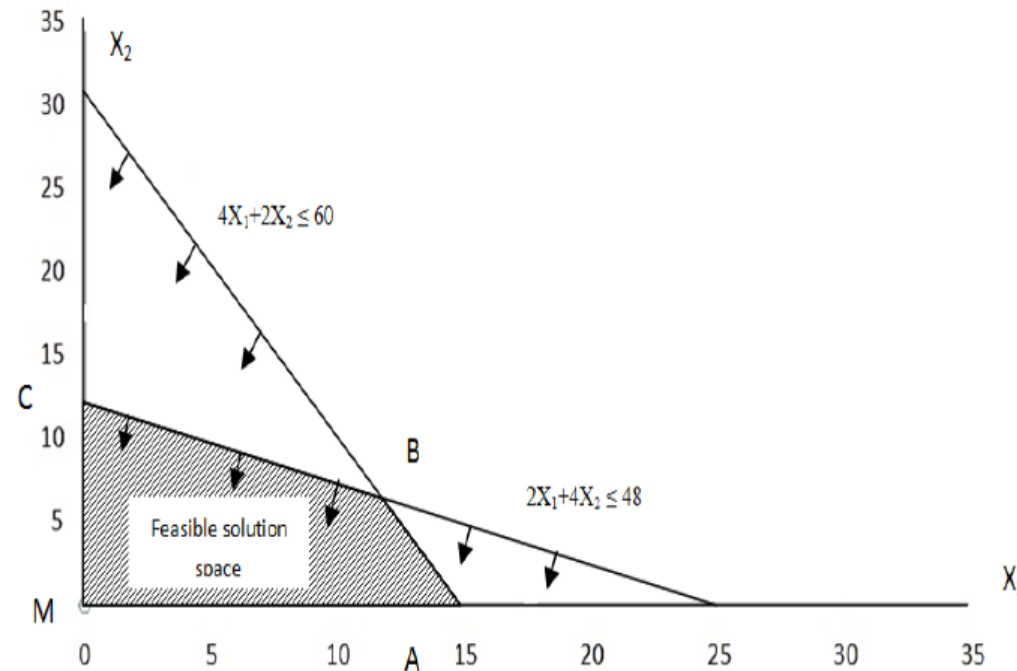
- ❑ The optimal value will always be at a corner point.
- ❑ Find the objective function value at each corner point and choose the one with the highest profit.

Point M: $(x_1=0, x_2=0)$

Point A: $(x_1=15, x_2=0)$

Point C: $(x_1=0, x_2=12)$

Point B: ?



Point B can be determined as follow: Solve for the intersection of two constraints

$$(2) \quad 2X_1 + 4X_2 = 48$$

$$4X_1 + 2X_2 = 60$$

$$4X_1 + 8X_2 = 96$$

$$- 4X_1 - 2X_2 = - 60$$

$$6X_2 = 36$$

$$X_2 = 36/6 = 6$$

Then: $4X_1 + 2X_2 = 60$

$$4X_1 + 2(6) = 60$$

$$4X_1 = 48 \quad X_1 = 48/4 = 12 \quad \text{Point B : } (X_1 = 12, X_2 = 6)$$

Problem 1

$$\text{Maximize } Z = 8X_1 + 6X_2$$

Point M: $(x_1=0, x_2=0)$

Point A: $(x_1=15, x_2=0)$

Point C: $(x_1=0, x_2=12)$

Point B: $(x_1=12, x_2=6)$

Points	x_1	x_2	Z
M	0	0	$Z=8(0) + 6(0) = 0$
A	15	0	$Z=8(15) + 6(0) = 120$
C	0	12	$Z=8(0) + 6(12) = 72$
B	12	6	$Z=8(12) + 6(6) = 132$

To maximize Z, the optimal solution is at point B, where $x_1=12$ and $x_2=6$ Then the $Z = 132$ £

The optimal solution is $Z = 132$ £

Problem 2

Dorian Auto Problem

- Dorian Auto manufactures luxury cars and trucks.
- The company believes that its most likely customers are high-income women and men.
- To reach these groups, Dorian Auto has embarked on an ambitious TV advertising campaign and will purchase 1-minute commercial spots on two type of programs: comedy shows and football games.

Problem 2

Dorian Auto Problem

- Each comedy commercial is seen by 7 million high income women and 2 million high-income men and costs \$50,000.
- Each football game is seen by 2 million high-income women and 12 million high-income men and costs \$100,000.
- Dorian Auto would like for commercials to be seen by at least 28 million high-income women and 24 million high-income men.
- Use LP to determine how Dorian Auto can meet its advertising requirements at minimum cost.

Problem 2

Dorian Auto Problem

number of 1-minute	Women	Men	Cost (\$/unit)
comedy ads	7	2	50
football ads	2	12	100

Target

28

24

Problem 2

Dorian Auto Problem

- Dorian must decide how many comedy and football ads should be purchased, so the decision variables are

x_1 = number of 1-minute comedy ads

x_2 = number of 1-minute football ads

- Dorian wants to minimize total advertising cost.
- Dorian's objective functions is

$$\min z = 50 x_1 + 100x_2$$

- Dorian faces the following the constraints

Commercials must reach at least 28 million high-income women.

$$(7x_1 + 2x_2 \geq 28)$$

Commercials must reach at least 24 million high-income men.

$$(2x_1 + 12x_2 \geq 24)$$

- The sign restrictions are necessary, so $x_1, x_2 \geq 0$.

Problem 2

Dorian Auto Problem

1- The Mathematical solution to get the objective function and constraints equation

$$\text{Min } Z = 50 x_1 + 100 x_2$$

Subject to

$$7 x_1 + 2 x_2 \geq 28$$

$$2 x_1 + 12 x_2 \geq 24$$

$$x_1 \geq 0$$

$$x_2 \geq 0$$

Problem 2

Solution:

1- Plot the Constraints (shown the following figure) Change constraints to equalities:

$$7X_1 + 2X_2 \geq 28$$

Line 1

$$7X_1 + 2X_2 = 28$$



$$\begin{aligned} \text{Let } X_1 &= 0 \\ 7(0) + 2X_2 &= 28 \\ X_2 &= 14 \end{aligned}$$

$X_1 \quad X_2$

The Point (0,14)

$$\begin{aligned} \text{Let } X_2 &= 0 \\ 7X_1 + 2(0) &= 28 \\ X_1 &= 4 \end{aligned}$$

$X_1 \quad X_2$

The Point (4,0)

$$2X_1 + 12X_2 \geq 24$$

Line 2

$$2X_1 + 12X_2 = 24$$



$$\begin{aligned} \text{Let } X_1 &= 0 \\ 2(0) + 12X_2 &= 24 \\ X_2 &= 2 \end{aligned}$$

$X_1 \quad X_2$

The Point (0,2)

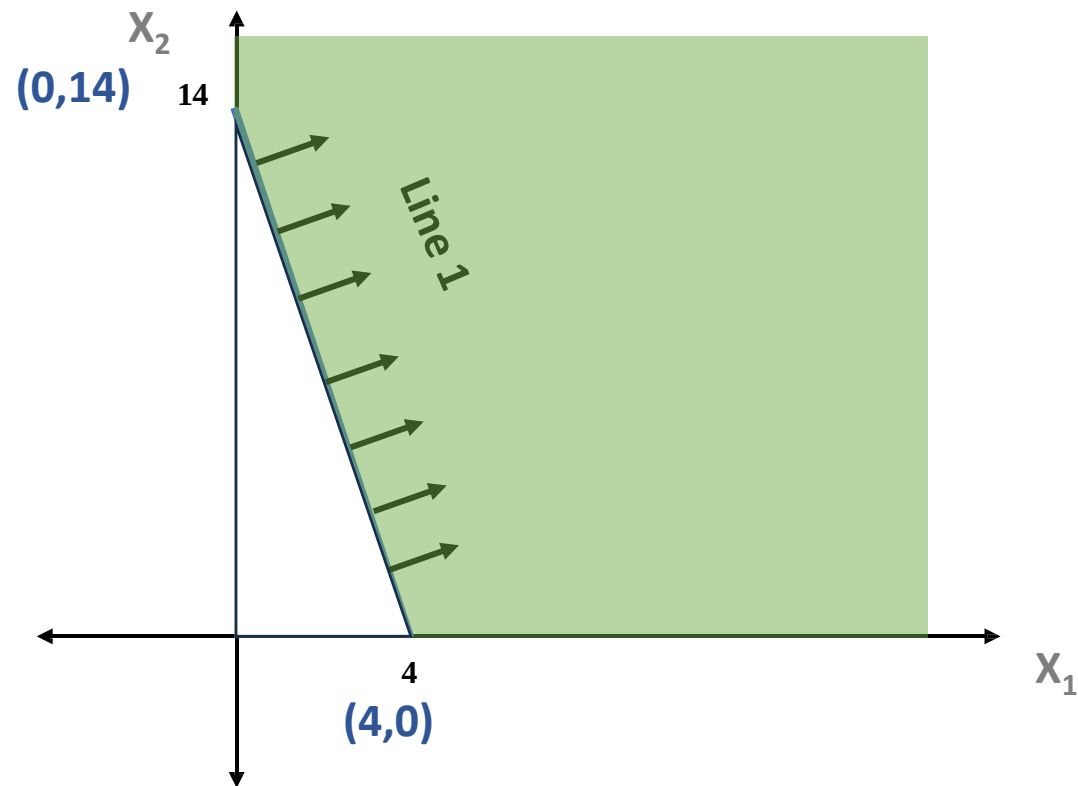
$$\begin{aligned} \text{Let } X_2 &= 0 \\ 2X_1 + 12(0) &= 24 \\ X_1 &= 12 \end{aligned}$$

$X_1 \quad X_2$

The Point (12,0)

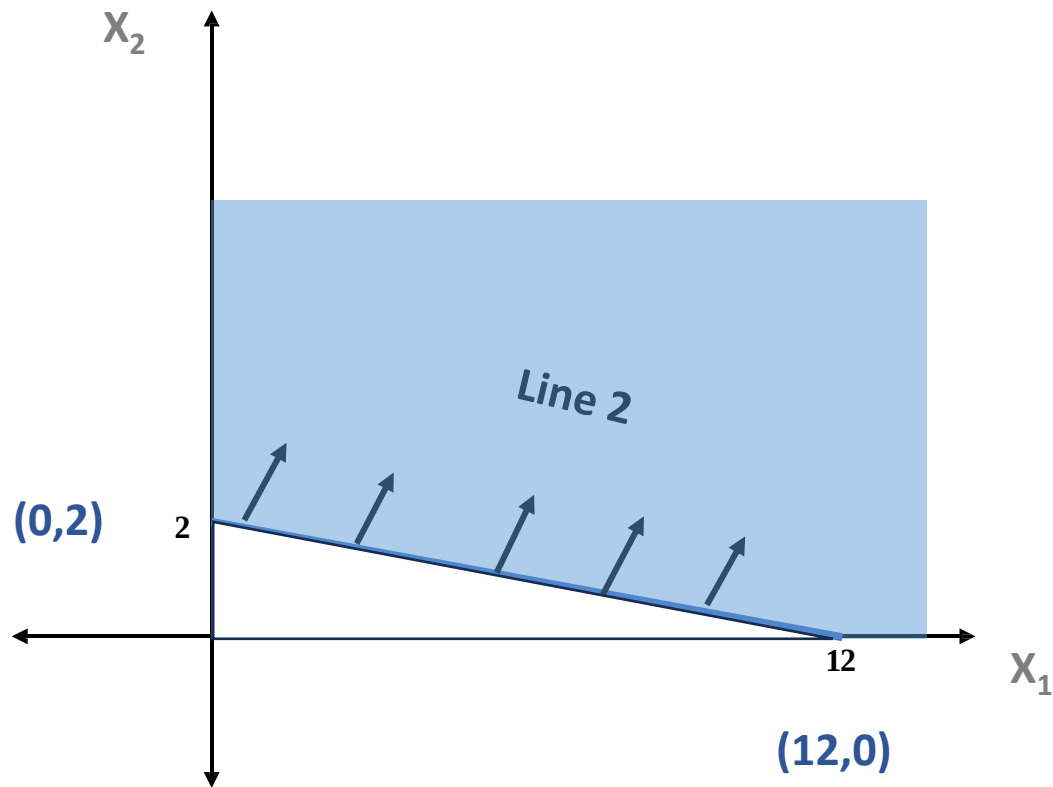
Problem 2

2- Graphical solution to get the Optimal Solution



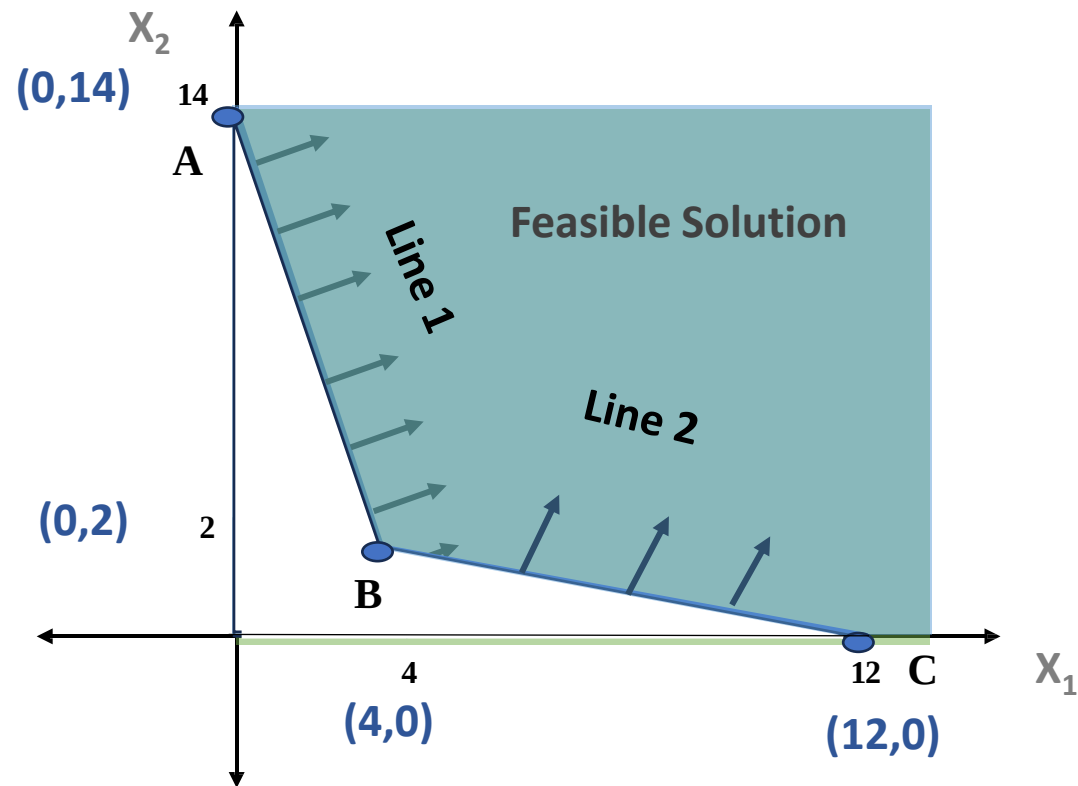
Problem 2

2- Graphical solution to get the Optimal Solution



Problem 2

2- Graphical solution to get the Optimal Solution



Problem 2

To get the coordinates of point B

Line 1

$$7X_1 + 2X_2 \geq 28$$

Line 2

$$2X_1 + 12X_2 \geq 24$$



$$7X_1 + 2X_2 = 28 \quad (1)$$



$$7X_1 + 2X_2 = 28$$

$$\frac{2}{6}X_1 + 2X_2 = 4$$

$$\frac{40}{6}X_1 = 24$$

$$X_1 = 3.6$$

From equation 1:

$$7(3.6) + 2X_2 = 28$$

$$X_2 = 1.4$$

Problem 2

$$\text{Minimize } Z = 50X_1 + 100X_2$$

Point A: $(x_1=0, x_2=14)$

Point B: $(x_1=3.6, x_2=1.4)$

Point C: $(x_1=12, x_2=0)$

Points	x_1	x_2	Z
A	0	14	$Z=50(0) + 100(14) = 1400$
B	3.6	1.4	$Z=50(3.6) + 100(1.4) = 320$
C	12	0	$Z=50(12) + 100(0) = 600$

To minimize Z, the optimal solution is at point B, where $x_1=3.6$ and $x_2=1.4$ Then the $Z = 320$ \$

The optimal solution is $Z = 320$ \$

Cases in Linear Programming

Unique Optimal Solution

Alternative Optimal Solutions

Unbounded Optimal Solutions

Infeasible Optimal Solutions

Cases in Graphical Solution

- **Unique Optimal Solution:** Occurs when one point optimizes the objective function.
- **Infeasibility Solution:** Occurs when a model has no feasible point.
- **Unbounded Solution:** Occurs when the objective can become infinitely large (max), or infinitely small (min).
- **Alternate solution:** Occurs when more than one point optimizes the objective function.

Unique Optimal Solution

- **Unique Optimal Solution:** Occurs when one point optimizes the objective function.

For example:

$$\text{Min } Z = 50 x_1 + 100 x_2$$

Subject to

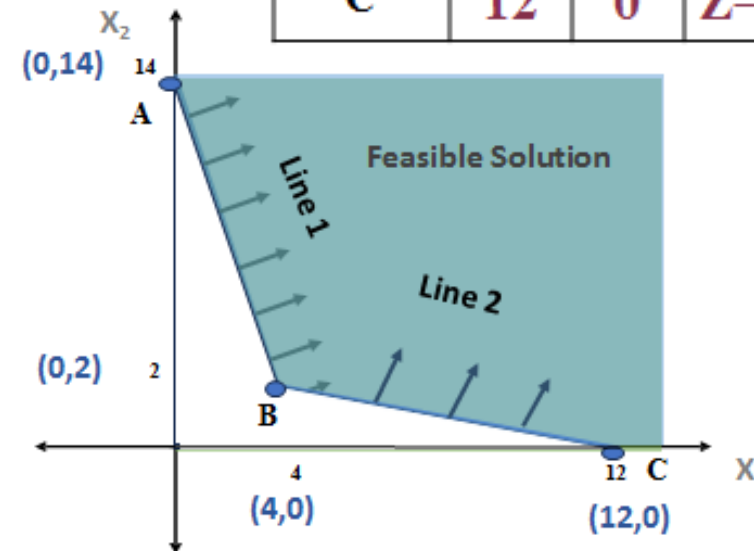
$$7 x_1 + 2 x_2 \geq 28$$

$$2 x_1 + 12 x_2 \geq 24$$

$$x_1 \geq 0$$

$$x_2 \geq 0$$

Points	x_1	x_2	Z
A	0	14	$Z=1400$
B	3.6	1.4	$Z=320$
C	12	0	$Z=600$



The optimal solution is at point B, where $x_1=3.6$ and $x_2=1.4$ Then the $Z = 320$ \$

Infeasibility Solution

- **Infeasibility Solution:** Occurs when a model has no feasible point.

consider the following three constraints:

$$\text{Max } Z = 2x_1 + 4x_2$$

Subject to:

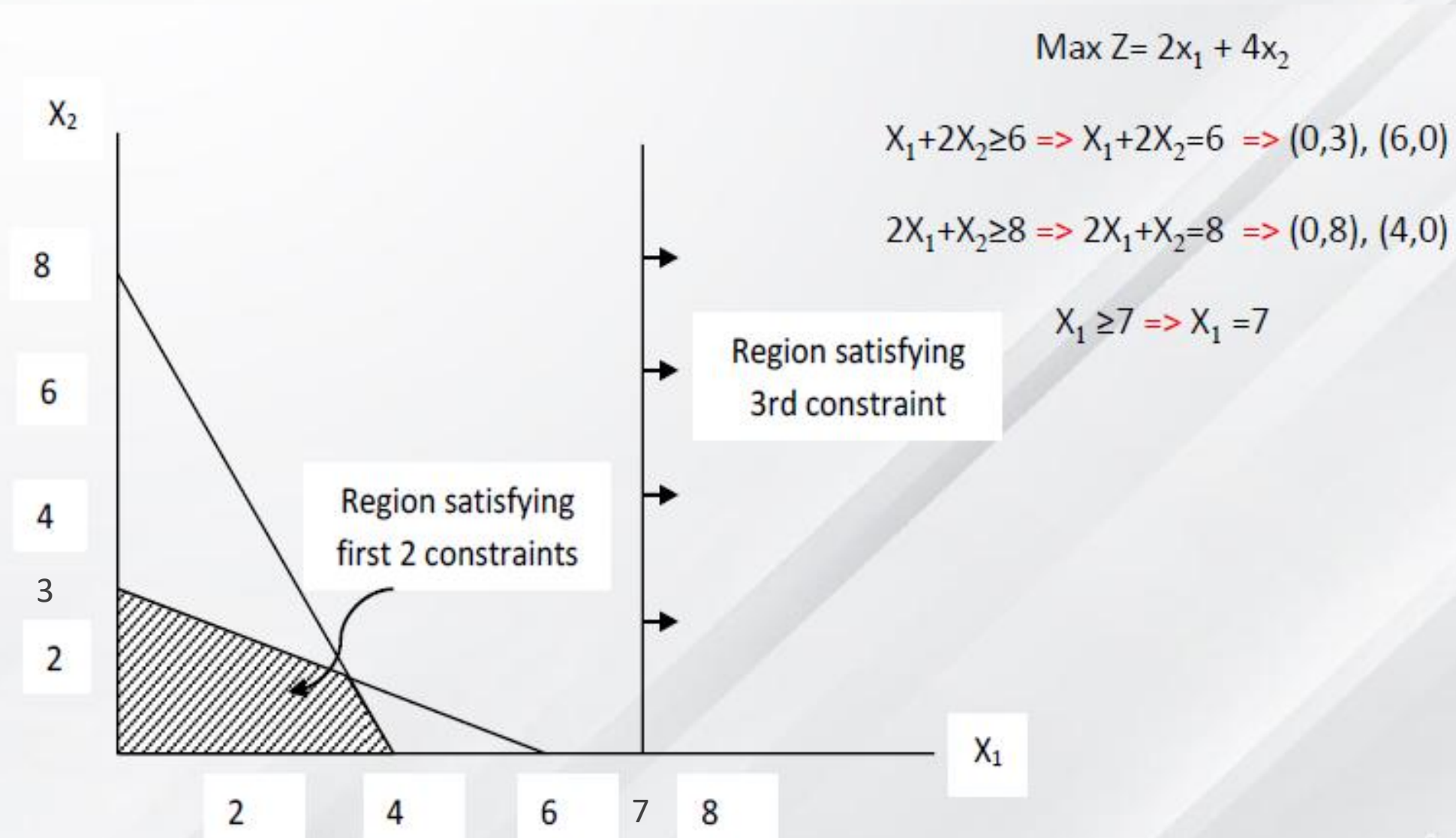
$$x_1 + 2x_2 \geq 6$$

$$2x_1 + x_2 \geq 8$$

$$x_1 \geq 7$$

$$x_1, x_2 \geq 0$$

Infeasibility Solution



Unbounded Solution

- **Unbounded Solution** : Occurs when the objective can become infinitely large (max), or infinitely small (min).

consider the following three constraints:

$$\text{Max } Z = 3x_1 + 5x_2$$

Subject to:

$$x_1 \geq 6$$

$$x_2 \leq 10$$

$$x_1 + 2x_2 \geq 10$$

$$x_1, x_2 \geq 0$$

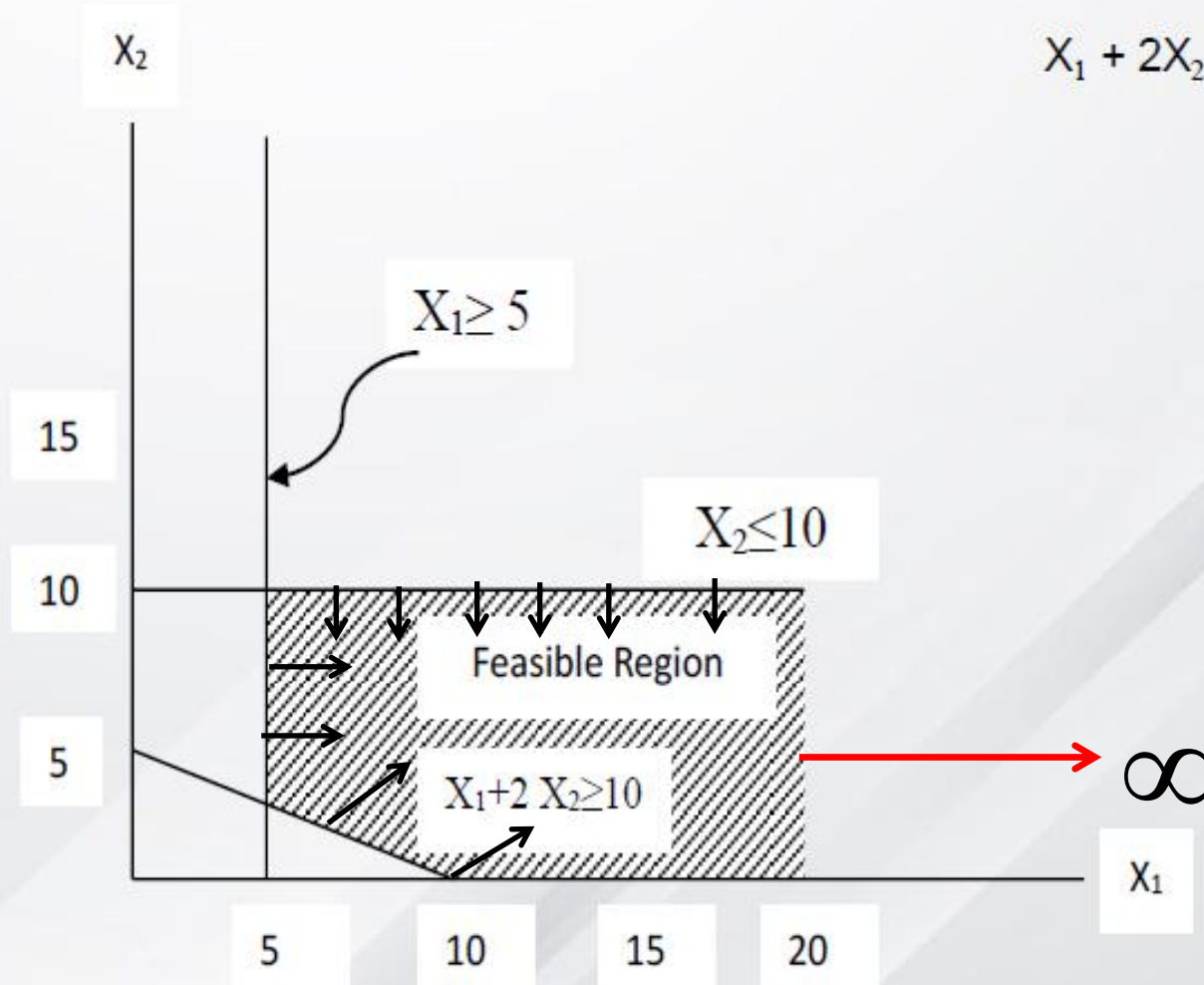
Unbounded Solution

$$X_1 \geq 5 \Rightarrow X_1 = 5$$

$$X_2 \leq 10 \Rightarrow X_2 = 10$$

$$X_1 + 2X_2 \geq 10 \Rightarrow X_1 + 2X_2 = 10 \Rightarrow (0, 5), (10, 0)$$

$$X_1, X_2 \geq 0$$



Unbounded Solution

Alternative Solution

- **Alternate solution:** Occurs when more than one point optimizes the objective function.

consider the following three constraints:

$$\text{Max } Z = 3x_1 + 2x_2$$

Subject to:

$$6x_1 + 4x_2 \leq 24$$

$$x_1 \leq 3$$

$$x_1, x_2 \geq 0$$

Alternative Solution

$$\text{Max } Z = 3x_1 + 2x_2$$

$$6x_1 + 4x_2 \leq 24$$



$$(0,6), (4,0)$$

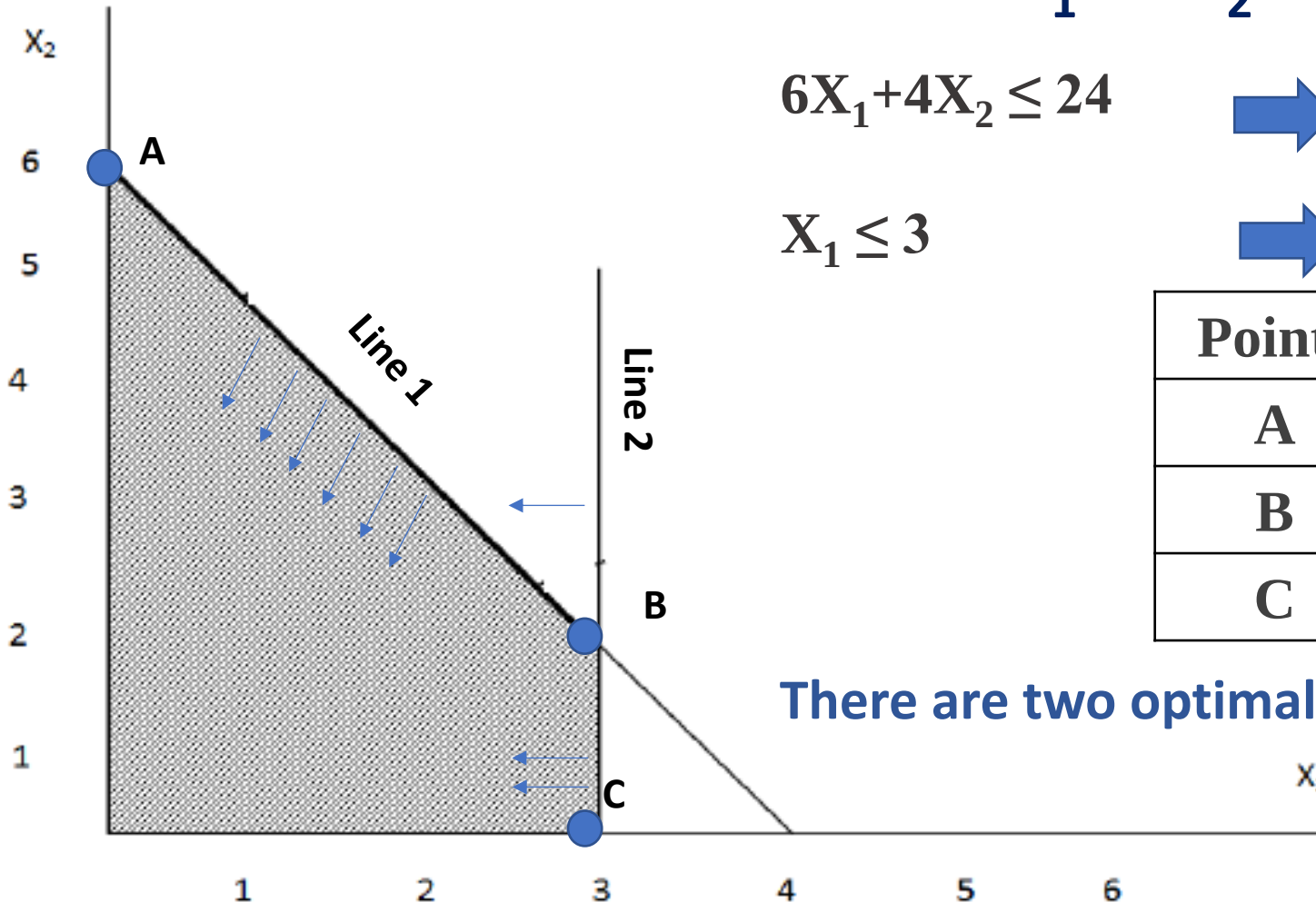
Line 1

$$x_1 \leq 3$$



$$(3,0)$$

Line 2



Points	x_1	x_2	Z
A	0	6	12
B	3	1.5	12
C	3	0	9

There are two optimal solutions at point A and point B

Graphical Solution

- Graphical Solution:
 - Corner Point Method: solving for intersection of constraints.
 - Cases in Graphical Solution .
- Cases in Graphical Solution:
 - Unique Optimal Solution
 - Infeasibility Solution
 - Unbounded Solution
 - Alternate solution



Thank You