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Introduction to Information Systems – IT111
Numbering Systems

Lecture 1 (Practical)



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1-The decimal system (base 10)

- The word decimal is derived from the Latin root **decimal**. In this system the **base $b = 10$** and we use ten symbols.

$$S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

- The symbols in this system are often referred to as **decimal digits** or just **digits**.

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Integer Number

$$N = \pm S_{k-1} \times 10^{k-1} + S_{k-2} \times 10^{k-2} + \dots + S_2 \times 10^2 + S_1 \times 10^1 + S_0 \times 10^0$$

10^{k-1}	10^{k-2}	$\dots$	10^2	10^1	10^0	Place values	
$\pm$	S_{k-1}	S_{k-2}	$\dots$	S_2	S_1	S_0	Number
	↓	↓		↓	↓	↓	
$N = \pm S_{k-1} \times 10^{k-1} + S_{k-2} \times 10^{k-2} + \dots + S_2 \times 10^2 + S_1 \times 10^1 + S_0 \times 10^0$							Values

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Integer Number Example

The following shows the place values for the integer +224 in the decimal system.

		10^2		10^1		10^0	Place values
		2		2		4	Number
N	$=$	$+$	2×10^2	$+$	2×10^1	$+$	4×10^0 Values

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Reals Number

$$R = \pm \left(S_{k-1} \times 10^{k-1} + \dots + S_1 \times 10^1 + S_0 \times 10^0 \right) + \left(S_{-1} \times 10^{-1} + \dots + S_{-l} \times 10^{-l} \right)$$

The following shows the place values for the real number +24.13.

	10^1	10^0		10^{-1}	10^{-2}	Place values
	2	4	•	1	3	Number
$R = +$	2×10	$+ 4 \times 1$	$+$	1×0.1	$+ 3 \times 0.01$	Values

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2-The binary system (base 2)

In this system the base $b = 2$ and we use only two symbols,

$$S = \{0, 1\}$$

The symbols in this system are often referred to as binary digits or bits (binary digit).

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binary system (Example)

The following shows that the number $(11001)_2$ in binary is the same as 25 in decimal. The subscript 2 shows that the base is 2.

	2^4	2^3	2^2	2^1	2^0	Place values
	1	1	0	0	1	Number
$N =$	1×2^4	$+ 1 \times 2^3$	$+ 0 \times 2^2$	$+ 0 \times 2^1$	$+ 1 \times 2^0$	Decimal

The equivalent decimal number is $N = 16 + 8 + 0 + 0 + 1 = 25$.

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Reals Number (binary system)

The following shows that the number $(101.11)_2$ in binary is equal to the number 5.75 in decimal.

	2^2	2^1	2^0	2^{-1}	2^{-2}	Place values
	1	0	1	• 1	1	Number
R =	1×2^2	$+ 0 \times 2^1$	$+ 1 \times 2^0$	$+ 1 \times 2^{-1}$	$+ 1 \times 2^{-2}$	Values

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3-The Hexadecimal system (base 16)

In this system the **base $b = 16$** and we use sixteen symbols to represent a number. The set of symbols is

$$S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F\}$$

Note that the symbols **A, B, C, D, E, F** are equivalent to **10, 11, 12, 13, 14, and 15** respectively. The symbols in this system are often referred to as **hexadecimal digits**.

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Hexadecimal system(Example)

The following shows that the number $(2AE)_{16}$ in hexadecimal is equivalent to 686 in decimal.

	16^2	16^1	16^0	Place values
	2	A	E	Number
N =	2×16^2	$+ 10 \times 16^1$	$+ 14 \times 16^0$	Values

The equivalent decimal number is $N = 512 + 160 + 14 = \mathbf{686}$.

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4-The Octal system (base 8)

In this system the **base $b = 8$** and we use eight symbols to represent a number. The set of symbols is

$$S = \{0, 1, 2, 3, 4, 5, 6, 7\}$$

Octal system(Example)

The following shows that the number $(1256)_8$ in octal is the same as 686 in decimal.

	8^3	8^2	8^1	8^0	Place values
	1	2	5	6	Number
N =	1×8^3	$+$ 2×8^2	$+$ 5×8^1	$+$ 6×8^0	Values

Note that the decimal number is $N = 512 + 128 + 40 + 6 = 686$.

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summary of the four positional number systems

summary of the four positional number systems discussed in this chapter.

<i>System</i>	<i>Base</i>	<i>Symbols</i>	<i>Examples</i>
Decimal	10	0, 1, 2, 3, 4, 5, 6, 7, 8, 9	2345.56
Binary	2	0, 1	$(1001.11)_2$
Octal	8	0, 1, 2, 3, 4, 5, 6, 7	$(156.23)_8$
Hexadecimal	16	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F	$(A2C.A1)_{16}$

Any Base to Decimal Conversion

- The column weights of binary numbers are powers of two that increase from right to left beginning with $2^0 = 1$:

$$\dots 2^5 \ 2^4 \ 2^3 \ 2^2 \ 2^1 \ 2^0 \cdot$$

- For fractional binary numbers, the column weights are negative powers of two that decrease from left to right:

$$2^2 \ 2^1 \ 2^0 \cdot 2^{-1} \ 2^{-2} \ 2^{-3} \ 2^{-4} \dots$$

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Any Base to Decimal Conversion (Example)

Example

Convert the hexadecimal number A01.3 to decimal.

Solution

Start by writing the column weights; then add the weights that correspond to each number.

$$16^2 \ 16^1 \ 16^0 \ . \ 16^{-1}$$

$$256 \ 16 \ 1 \ . \ 1/16$$

$$10 \ 0 \ 1 \ . \ 1$$

$$2560+0+1 +1/16= 2561.0625$$

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Any Base to Decimal Conversion (Example)

• $(111001.011)_2 = (57.375)_{10}$

Binary			1	1	1	0	0	1	0	1	1	
weight	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}	2^{-4}
Value	128	64	32	16	8	4	2	1	0.5	0.25	0.125	0.0625

$$(1*2^5) + (1*2^4) + (1*2^3) + (1*2^0) + (1*2^{-2}) + (1*2^{-3}) = 51.375$$

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Any Base to Decimal Conversion (Example)

Octal -----> Decimal

Example

Convert the **octal number 174.1** to decimal.

Solution

Start by writing the column weights; then add the weights that correspond to each number.

$$\begin{array}{cccc}
 8^2 & 8^1 & 8^0 & 8^{-1} \\
 64 & 8 & 1 & 1/8 \\
 1 & 7 & 4 & 1 \\
 64+56+4 & +1/8= & \mathbf{124.125}
 \end{array}$$

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Convert the following Octal to Decimal

$8^2 \ 8^1 \ 8^0$

Octal

----->

Decimal

Ex1) 272

$$(2 \cdot 8^2) + (7 \cdot 8^1) + (2 \cdot 8^0) = 186$$

Ex2) 61

$$(6 \cdot 8^1) + (1 \cdot 8^0) = 49$$

Ex3) 126

$$(1 \cdot 8^2) + (2 \cdot 8^1) + (6 \cdot 8^0) = 86$$

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Convert the following Hexadecimal to Decimal

Hexadecimal -----> Decimal

Example

Convert the hexadecimal number A01.3 to decimal.

Solution

Start by writing the column weights; then add the weights that correspond to each number.

$$16^2 \quad 16^1 \quad 16^0 \quad 16^{-1}$$

$$256 \quad 16 \quad 1 \quad 1/16$$

$$10 \quad 0 \quad 1 \quad 1$$

$$2560 + 0 + 1 + 1/16 = 2561.0625$$

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Convert the following Hexadecimal to Decimal by using the *sum-of-weighted method*

 $16^2 \ 16^1 \ 16^0$

Ex1) B02

$$(11 * 16^2) + (0 * 16^1) + (2 * 16^0) = 2818$$

 $16^1 \ 16^0$

Ex2) EC

$$(14 * 16^1) + (12 * 16^0) = 236$$

 $16^2 \ 16^1 \ 16^0$

Ex3) 296

$$(2 * 16^2) + (9 * 16^1) + (6 * 16^0) = 662$$

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Decimal to Binary Conversion

Decimal to Binary Conversion

sum-of-weighted

1. Choose largest weight $\leq$ the given number to convert
2. Subtract the given number from the weight
3. Repeat step 1 & 2 till the subtraction results = 0

repeated division by 2.

1. Divide the whole number by 2
2. Check if there is a remainder put 1
3. Repeat the division processes for the whole number till the division results reaches 0.5

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Convert the *following decimal numbers to binary* by using the *sum-of-weighted method*

weight	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
Value	128	64	32	16	8	4	2	1
Output				1	0	0	0	1

Choose the largest weight $\leq 17 \rightarrow 16$

Subtract $17 - 16 = 1$

Put 1 under the weight that has been chosen (16)

Choose the largest weight $\leq 1 \rightarrow 1$

Put 1 under the weight that has been chosen (1)

Subtract $1 - 1 = 0$

Put 0s under the other weights in between till weight (2^0)

Done

Output = 10001

Choose the largest weight $\leq 186 \rightarrow 128$

Subtract $186 - 128 = 58$

Put 1 under the weight that has been chosen (128)

Choose the largest weight $\leq 58 \rightarrow 32$

Put 1 under the weight that has been chosen (32)

Subtract $58 - 32 = 26$

Choose the largest weight $\leq 26 \rightarrow 16$

Put 1 under the weight that has been chosen (16)

Subtract $26 - 16 = 10$

Choose the largest weight $\leq 10 \rightarrow 8$

Put 1 under the weight that has been chosen (8)

Subtract $10 - 8 = 2$

Choose the largest weight $\leq 2 \rightarrow 2$

Put 1 under the weight that has been chosen (2)

Subtract $2 - 2 = 0$

Put 0s under the other weights in between till weight (2^0)

Done

Output = 10111010

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Convert the *following decimal numbers to binary* by using the *sum-of-weighted method*

➤ Conversion from decimal to binary :

convert from binary to decimal: Sum of weights method

1)Sum of weights method :

Examples :

$$(234)_{10} = (11101010)_2$$

128	64	32	16	8	4	2	1
1	1	1	0	1	0	1	0

$$(148)_{10} = (10010100)_2$$

128	64	32	16	8	4	2	1
1	0	0	1	0	1	0	0

Value	Weight
1	2^0
2	2^1
4	2^2
8	2^3
16	2^4
32	2^5
64	2^6
128	2^7

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$$(467)_{10} = (111010011)_2$$

256	128	64	32	16	8	4	2	1
1	1	1	0	1	0	0	1	1

→

$$(23)_{10} = (10111)_2$$

128	64	32	16	8	4	2	1
0	0	0	1	0	1	1	1

→

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Convert the following *decimal numbers to binary* by using the *division by 2*

a) 17

2	17	8.5	R	1
2	8	4		0
2	4	2		0
2	2	1		0
2	1	0.5		1

Stopping Condition

Result : 10001

c) 186

2	186	93	R	0
2	93	46.5		1
2	46	23		0
2	23	11.5		1
2	11	5.5		1
2	5	2.5		1
2	2	1		0
2	1	0.5		1

Stopping Condition

Result : 10111010

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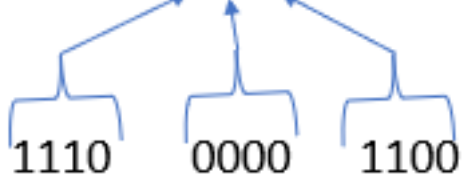
Convert the following *Hexadecimal numbers to binary*

- Note: Every digit in Hexa-decimal has to be represented by **4 bits** in **Binary** as the maximum digit in Hexa-decimal (F=15) represented by (1111) in binary

Ex1) (61)₁₆ = ()₂



Ex2) (E0C)₁₆ = ()₂



Hexa-decimal	Binary
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
A	1010
B	1011
C	1100
D	1101
E	1110
F	1111

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Convert the following *Hexadecimal numbers to binary*

What is the binary equivalent of $(24C)_{16}$?

Solution

Each hexadecimal digit is converted to 4-bit patterns:

$2 \rightarrow 0010$, $4 \rightarrow 0100$, and $C \rightarrow 1100$

The result is $(001001001100)_2$.

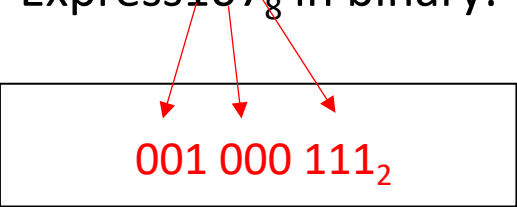
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Convert the following *Octal numbers to binary*

- Octal number can easily be converted to binary by expanding each digit into **3 bits at a time**

Example Express 107_8 in binary:

Solution



001 000 111₂

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Convert the following *Octal numbers to binary*

Octal $\rightarrow$ Binary

- Octal number can easily be converted to binary by expanding each digit into **3 bits at a time**

What is the binary equivalent of for $(24)_8$?

Solution

Write each octal digit as its equivalent bit pattern to get

$2 \rightarrow 010$ and $4 \rightarrow 100$

The result is $(010100)_2$.

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Numbering system

Decimal	Binary	Octal	Hexadecimal
0	0000	0	0
1	0001	1	1
2	0010	2	2
3	0011	3	3
4	0100	4	4
5	0101	5	5
6	0110	6	6
7	0111	7	7
8	1000		8
9	1001		9
10	1010		A
11	1011		B
12	1100		C
13	1101		D
14	1110		E
15	1111		F

Numbering System	Base	Digits Range
Binary	2	0,1
Octal	8	0-7
Decimal	10	0-9
HexaDecimal	16	0-9 ,A,B,C,D,E,F



Thank You