

# Operations Research

**Prepared by Dr. Mohamed Eassa**

IT 262  
Operations Research

## Lecture 6



# Transportation Applications

## Lecture 6

IT 262

Operations Research

Prepared by

***Dr. Mohamed Eassa***



*Lecture 6 Topics*

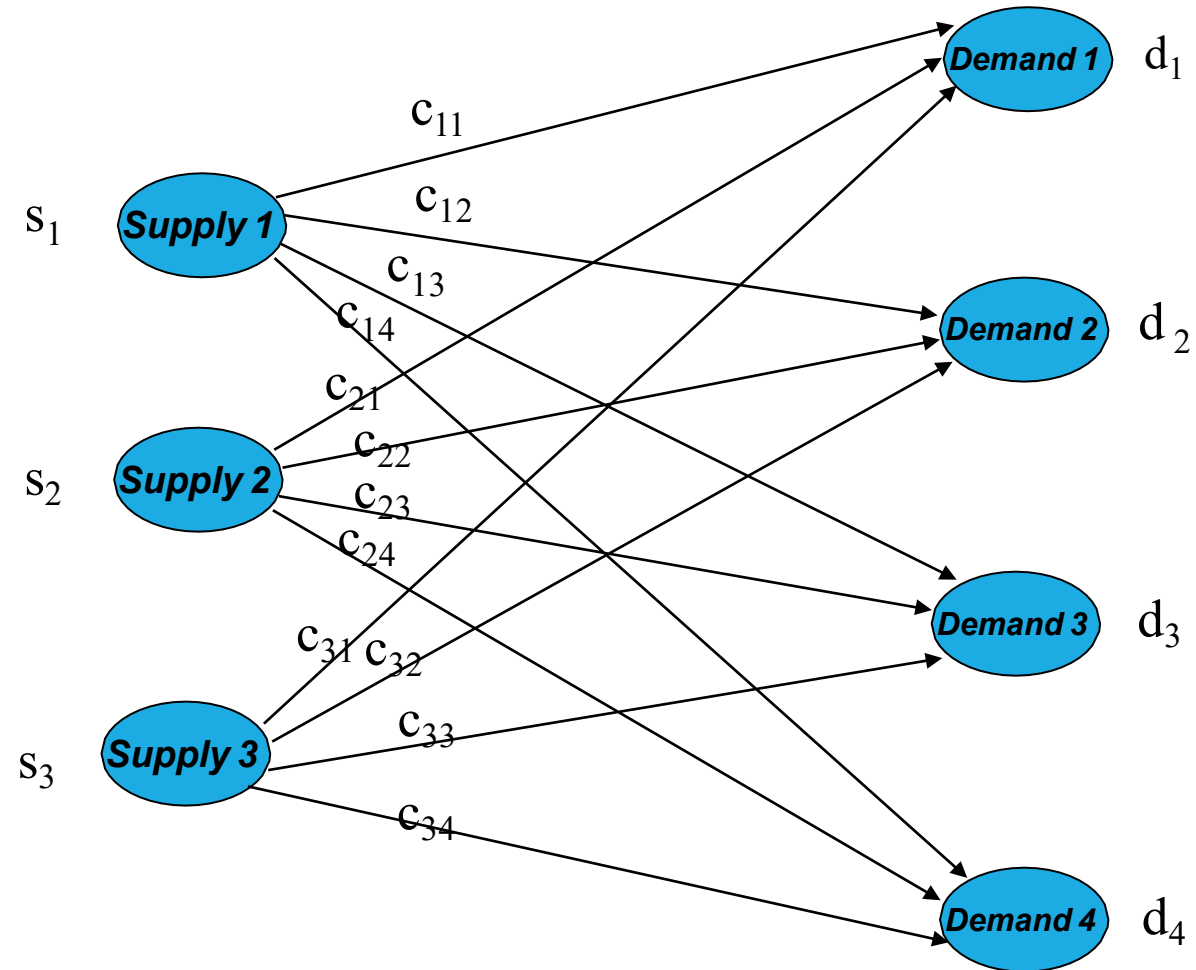
- ❖ **Transportation Model**
- ❖ **Balanced Problem**
- ❖ **Evaluation the total cost of transportation**

# Transportation Applications

## Basic elements in transport applications

1. A set of supply nodes  $\rightarrow n$  nodes
2. A set of demand nodes  $\rightarrow m$  nodes
3. Maximum total supply at the supply node  $i = s_i$  for all  $i$
4. The total demand at the demand node  $j = d_j$  for all  $j$
5. The cost of transport a unit from the supply node  $i$  to the demand node  $j = c_{ij}$

# Transportation Applications



**What is the optimal method to transport units from supply nodes to demand nodes?**

# Transportation Applications

## The general formula of this mathematical model

$x_{ij}$  = number of units shipped from supply node  $i$  to demand node  $j$  .

$$i = 1, 2, \dots, m \quad \text{and} \quad j = 1, 2, \dots, n$$

*Min*

- Cost to shipment  $x_{ij}$  from supply  $i$  to demand  $j$  is  $= c_{ij} x_{ij}$
- Each demand node must get at least what covers its demand
- Each supply node must send its maximum supply capacity at most

# Transportation Applications

## The general formula of this mathematical model

$x_{ij}$  = number of units shipped from supply point  $i$  to demand point  $j$  .

$i = 1, 2, \dots, m$                       and                       $j = 1, 2, \dots, n$

$$Z = \sum_{i=1}^n \sum_{j=1}^m c_{ij} x_{ij}$$

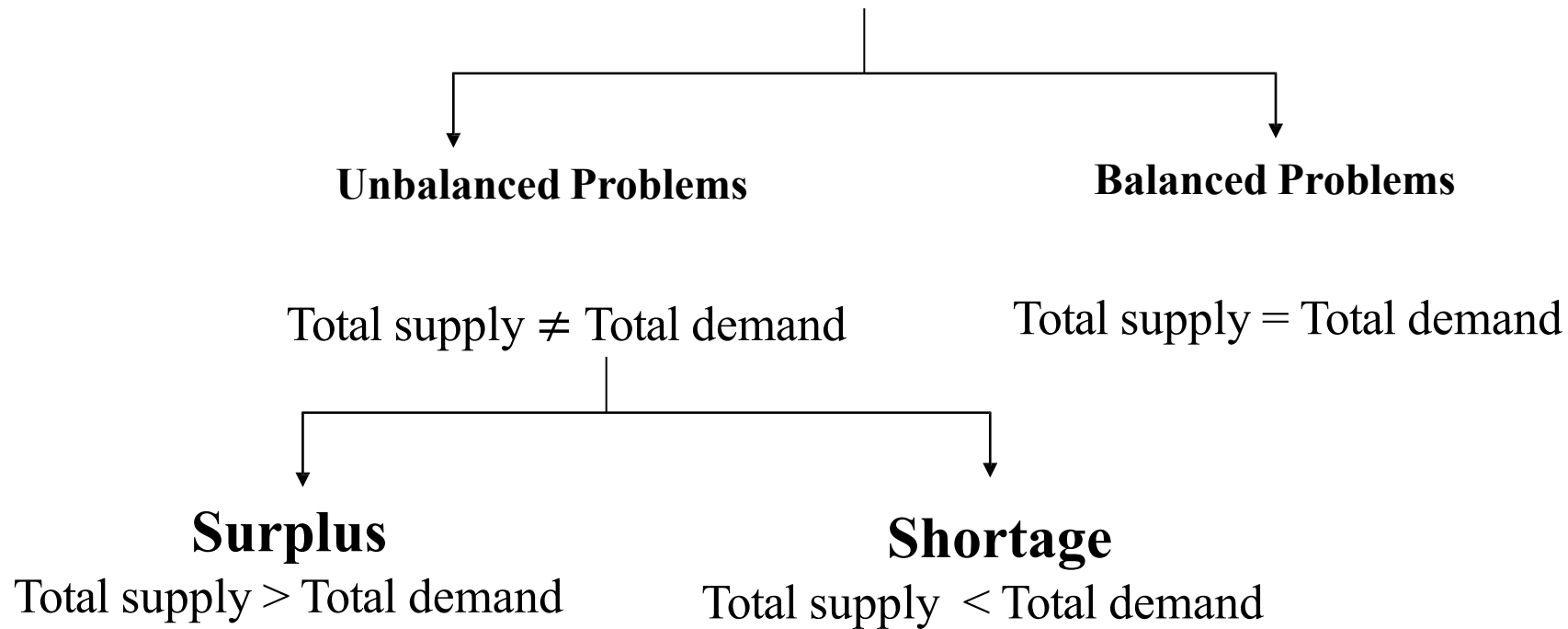
## Example

Electricity Company has three power plants in different areas to secure the demand for electricity consumption of four cities. The production capacity of electricity from the station-1, 35 million kilowatts (kwh) per day and the capacity of electricity from the plant Station -2, 50 million kwh per day, while the capacity of electricity from the station -3, 40 million kwh per day. The previous data show that the peak daily consumption of the city-1 is 45 million kwh and the daily peak-day consumption of the city -2 is 20 million kwh and the daily peak-day consumption of the city -3 is 30 million kwh and the peak daily consumption of the city-4 is 30 million kwh. Cause to the distance from the station sites to the cities, there exist a cost to secure ship every million kwh from any station to any city. These costs are shown in the following table:

# Transportation Applications

|           | To     |        |        |        |
|-----------|--------|--------|--------|--------|
| From      | City-1 | City-2 | City-3 | City-4 |
| Station-1 | 8      | 6      | 10     | 9      |
| Station-2 | 9      | 12     | 13     | 7      |
| Station-3 | 14     | 9      | 16     | 5      |

## Transportation Models



# Transportation Applications

## Balanced Problem

Total supply = Total demand

$$\sum_{i=1}^m s_i = \sum_{j=1}^n d_j$$

All constraints are binding:

$$\sum_{j=1}^n x_{ij} = s_i \quad i = 1, 2, \dots, m$$

$$\sum_{i=1}^m x_{ij} = d_j \quad j = 1, 2, \dots, n$$

$$x_{ij} \geq 0$$

# Transportation Applications

## Standard Form

$$\begin{aligned}
 & \text{Min} \quad Z = \sum_{i=1}^n \sum_{j=1}^m c_{ij} x_{ij} \\
 & \text{s.t.} \quad \sum_{j=1}^n x_{ij} = s_i \quad i = 1, 2, \dots, m \\
 & \quad \quad \sum_{i=1}^m x_{ij} = d_i \quad j = 1, 2, \dots, n \\
 & \quad \quad x_{ij} \geq 0
 \end{aligned}$$

# Transportation Applications

## TP Table

|               |          |          |     |          |               |
|---------------|----------|----------|-----|----------|---------------|
|               | $c_{11}$ | $c_{12}$ | ... | $c_{1n}$ | <i>Supply</i> |
| $x_{11}$      |          | $x_{12}$ |     | $x_{1n}$ | $s_1$         |
|               | $c_{21}$ | $c_{22}$ | ... | $c_{2n}$ | $s_2$         |
| $x_{21}$      |          | $x_{22}$ |     | $x_{2n}$ |               |
| $\vdots$      |          | $\vdots$ |     | $\vdots$ | $\vdots$      |
|               | $c_{m1}$ | $c_{m2}$ | ... | $c_{mn}$ | $s_m$         |
| $x_{m1}$      |          | $x_{m2}$ |     | $x_{mn}$ |               |
| <i>Demand</i> | $d_1$    | $d_2$    | ... | $d_n$    | <b>Equal</b>  |

**Balanced Problems**

# Transportation Applications

## Initial solution of transportation problems

- The cell  $(i, j)$  in transportation table represents the value of the variable  $x_{ij}$ .
- The initial table must contain **(m+n-1) cell filled** and the rest of the cells to remain blank.
- Any cell not filled in the initial solution means that the value of the variable associated with this cell is equal to **zero** but is **not written** in the cell.
- Among the filled cells **may exist** that with the **value zero**.
- The sum of the **values** of filled cells in column (j) = the demand on the column (j).
- The sum of the **values** of filled cells in row (i) = the supply on the row (i).

# Transportation Applications

## Example

|               |    |          |          |          |               |
|---------------|----|----------|----------|----------|---------------|
|               | 8  | 6        | 10       | 9        | <i>Supply</i> |
| $x_{11}$      |    | $x_{12}$ | $x_{13}$ | $x_{14}$ | 35            |
|               | 9  | 12       | 13       | 7        | 50            |
| $x_{21}$      |    | $x_{22}$ | $x_{23}$ | $x_{24}$ |               |
|               | 14 | 9        | 16       | 5        | 40            |
| $x_{31}$      |    | $x_{32}$ | $x_{33}$ | $x_{34}$ |               |
| <i>Demand</i> | 45 | 20       | 30       | 30       |               |

# Transportation Applications

## Example

|               |    |    |    |    |    |    |   |
|---------------|----|----|----|----|----|----|---|
|               | 8  |    | 6  |    | 10 |    | 9 |
|               |    | 10 |    | 25 |    |    |   |
| 45            | 9  |    | 12 |    | 13 |    | 7 |
|               |    |    |    | 5  |    |    |   |
|               | 14 |    | 9  |    | 16 |    | 5 |
|               |    | 10 |    |    |    | 30 |   |
| <b>Demand</b> | 45 | 20 | 30 | 30 |    |    |   |

**Supply**

35

50

40

# of cells =  $6 = 3 + 4 - 1 = m + n - 1$

**Solution:**

$x_{21} = 45$  ,  $x_{12} = 10$  ,  $x_{32} = 10$  ,  $x_{13} = 25$  ,  $x_{23} = 5$  ,  $x_{34} = 30$

$x_{11} = x_{14} = x_{22} = x_{34} = x_{31} = x_{33} = 0$

|          |    |          |    |          |    |          |   |
|----------|----|----------|----|----------|----|----------|---|
|          | 8  |          | 6  |          | 10 |          | 9 |
| $x_{11}$ |    | $x_{12}$ |    | $x_{13}$ |    | $x_{14}$ |   |
|          | 9  |          | 12 |          | 13 |          | 7 |
| $x_{21}$ |    | $x_{22}$ |    | $x_{23}$ |    | $x_{24}$ |   |
|          | 14 |          | 9  |          | 16 |          | 5 |
| $x_{31}$ |    | $x_{32}$ |    | $x_{33}$ |    | $x_{34}$ |   |

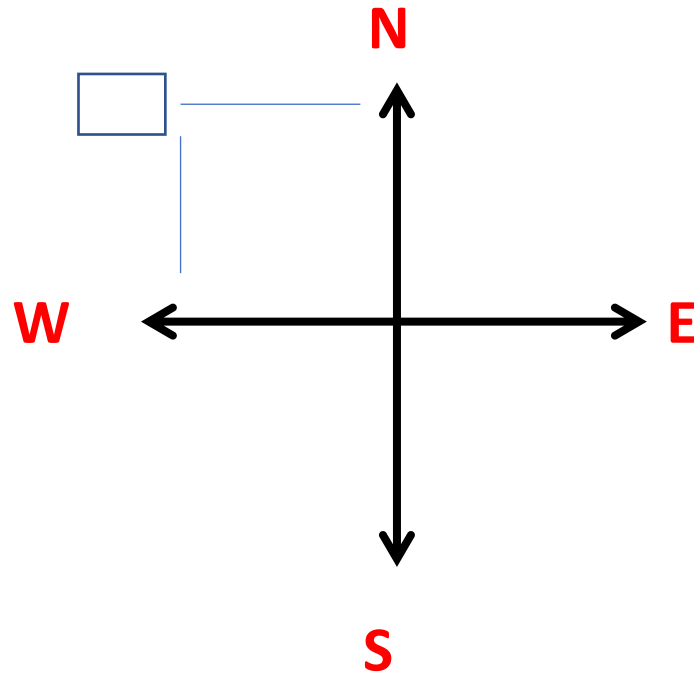
# Transportation Applications

## Methods to determine the initial solution:

1. North-West Corner Method
2. Least Cost Method
3. Vogel approximation Method

# Transportation Applications

## 1- North-West Corner Method:



# Transportation Applications

## Initial Basic Feasible Solution (North-West Corner)

|  |    |  |    |  |    |  |   |               |
|--|----|--|----|--|----|--|---|---------------|
|  | 8  |  | 6  |  | 10 |  | 9 | <i>Supply</i> |
|  |    |  |    |  |    |  |   | 35            |
|  | 9  |  | 12 |  | 13 |  | 7 | 50            |
|  |    |  |    |  |    |  |   |               |
|  | 14 |  | 9  |  | 16 |  | 5 | 40            |
|  |    |  |    |  |    |  |   |               |

**Supply = 125**

*Demand*

45

20

30

30

**Demand = 125**

**Supply = Demand = 125**



**Balanced Transportation**

# Transportation Applications

## Initial Basic Feasible Solution (North-West Corner)

|               |    |    |    |    |               |
|---------------|----|----|----|----|---------------|
|               | 8  | 6  | 10 | 9  | <i>Supply</i> |
| <b>35</b>     |    |    |    |    | 35            |
|               | 9  | 12 | 13 | 7  | 50            |
|               |    |    |    |    |               |
|               | 14 | 9  | 16 | 5  | 40            |
|               |    |    |    |    |               |
| <i>Demand</i> | 45 | 20 | 30 | 30 |               |

# Transportation Applications

## Initial Basic Feasible Solution (North-West Corner)

|               |                     |    |    |    |                 |
|---------------|---------------------|----|----|----|-----------------|
|               | 8                   | 6  | 10 | 9  | <i>Supply</i>   |
| <b>35</b>     |                     |    |    |    | <del>35</del> 0 |
|               | 9                   | 12 | 13 | 7  | 50              |
|               | 14                  | 9  | 16 | 5  | 40              |
| <i>Demand</i> | <del>45</del><br>10 | 20 | 30 | 30 |                 |

# Transportation Applications

## Initial Basic Feasible Solution (North-West Corner)

|               |               |    |    |    |                 |
|---------------|---------------|----|----|----|-----------------|
|               | 8             | 6  | 10 | 9  | <i>Supply</i>   |
| <b>35</b>     |               |    |    |    | <del>35</del> 0 |
|               | 9             | 12 | 13 | 7  | 50              |
|               | 14            | 9  | 16 | 5  | 40              |
| <i>Demand</i> | <del>45</del> | 20 | 30 | 30 |                 |
|               | 10            |    |    |    |                 |

# Transportation Applications

## Initial Basic Feasible Solution (North-West Corner)

|               |               |    |    |    |                  |
|---------------|---------------|----|----|----|------------------|
|               | 8             | 6  | 10 | 9  | <i>Supply</i>    |
| <b>35</b>     |               |    |    |    | <del>35</del> 0  |
| <b>10</b>     | 9             | 12 | 13 | 7  | <del>50</del> 40 |
|               | 14            | 9  | 16 | 5  | 40               |
| <i>Demand</i> | <del>45</del> | 20 | 30 | 30 |                  |
|               | <del>10</del> |    |    |    |                  |
|               | 0             |    |    |    |                  |

# Transportation Applications

## Initial Basic Feasible Solution (North-West Corner)

|                  |    |    |    |   |                  |
|------------------|----|----|----|---|------------------|
|                  | 8  | 6  | 10 | 9 | <i>Supply</i>    |
| <b>35</b>        |    |    |    |   | <del>35</del> 0  |
| <b>10</b>        | 9  | 12 | 13 | 7 | <del>50</del> 40 |
|                  | 14 | 9  | 16 | 5 | 40               |
| <del>45</del> 10 | 20 | 30 | 30 |   | 0                |

# Transportation Applications

## Initial Basic Feasible Solution (North-West Corner)

|               |               |               |    |    |   |                                |
|---------------|---------------|---------------|----|----|---|--------------------------------|
|               |               | 8             | 6  | 10 | 9 | <i>Supply</i>                  |
|               | <b>35</b>     |               |    |    |   | <del>35</del> 0                |
|               | 9             | 12            | 13 | 7  |   | <del>50</del> <del>40</del> 20 |
|               | <b>10</b>     | <b>20</b>     |    |    |   |                                |
|               | 14            | 9             | 16 | 5  |   | 40                             |
| <i>Demand</i> | <del>45</del> | <del>20</del> | 30 | 30 |   |                                |
|               | <del>10</del> | 0             |    |    |   |                                |
|               | 0             |               |    |    |   |                                |

# Transportation Applications

## Initial Basic Feasible Solution (North-West Corner)

|               |                  |                 |    |                  |                 |
|---------------|------------------|-----------------|----|------------------|-----------------|
|               | 8                | 6               | 10 | 9                | <i>Supply</i>   |
| <b>35</b>     |                  |                 |    |                  | <del>35</del> 0 |
| 9             | 12               | 13              | 7  | <del>50</del> 40 | 20              |
| <b>10</b>     | <b>20</b>        |                 |    |                  |                 |
| 14            | 9                | 16              | 5  | 40               |                 |
| <i>Demand</i> | <del>45</del> 10 | <del>20</del> 0 | 30 | 30               |                 |
|               | 0                |                 |    |                  |                 |

# Transportation Applications

## Initial Basic Feasible Solution (North-West Corner)

|               |               |    |               |    |               |    |    |   |                                             |
|---------------|---------------|----|---------------|----|---------------|----|----|---|---------------------------------------------|
|               |               | 8  |               | 6  |               | 10 |    | 9 | <i>Supply</i>                               |
|               | <b>35</b>     |    |               |    |               |    |    |   | <del>35</del> 0                             |
|               |               | 9  |               | 12 |               | 13 |    | 7 | <del>50</del> <del>40</del> <del>20</del> 0 |
|               | <b>10</b>     |    | <b>20</b>     |    | <b>20</b>     |    |    |   |                                             |
|               |               | 14 |               | 9  |               | 16 |    | 5 | 40                                          |
| <i>Demand</i> | <del>45</del> |    | <del>20</del> |    | <del>30</del> |    | 30 |   |                                             |
|               | <del>10</del> |    | 0             |    | 10            |    |    |   |                                             |
|               | 0             |    |               |    |               |    |    |   |                                             |

# Transportation Applications

## Initial Basic Feasible Solution (North-West Corner)

|               |               |    |               |    |               |    |    |   |                                             |
|---------------|---------------|----|---------------|----|---------------|----|----|---|---------------------------------------------|
|               |               | 8  |               | 6  |               | 10 |    | 9 | <i>Supply</i>                               |
|               | <b>35</b>     |    |               |    |               |    |    |   | <del>35</del> 0                             |
|               |               | 9  |               | 12 |               | 13 |    | 7 | <del>50</del> <del>40</del> <del>20</del> 0 |
|               | <b>10</b>     |    | <b>20</b>     |    | <b>20</b>     |    |    |   |                                             |
|               |               | 14 |               | 9  |               | 16 |    | 5 | 40                                          |
| <i>Demand</i> | <del>45</del> |    | <del>20</del> |    | <del>30</del> |    | 30 |   |                                             |
|               | <del>10</del> |    | 0             |    | 10            |    |    |   |                                             |
|               | 0             |    |               |    |               |    |    |   |                                             |

# Transportation Applications

## Initial Basic Feasible Solution (North-West Corner)

|               |               |    |               |    |               |    |    |   |               |               |               |   |  |
|---------------|---------------|----|---------------|----|---------------|----|----|---|---------------|---------------|---------------|---|--|
|               |               | 8  |               | 6  |               | 10 |    | 9 | <i>Supply</i> |               |               |   |  |
|               | <b>35</b>     |    |               |    |               |    |    |   | <del>35</del> | 0             |               |   |  |
|               |               | 9  |               | 12 |               | 13 |    | 7 | <del>50</del> | <del>40</del> | <del>20</del> | 0 |  |
|               | <b>10</b>     |    | <b>20</b>     |    | <b>20</b>     |    |    |   |               |               |               |   |  |
|               |               | 14 |               | 9  |               | 16 |    | 5 | <del>40</del> | 30            |               |   |  |
|               |               |    |               |    | <b>10</b>     |    |    |   |               |               |               |   |  |
| <i>Demand</i> | <del>45</del> |    | <del>20</del> |    | <del>30</del> |    | 30 |   |               |               |               |   |  |
|               | <del>10</del> |    | 0             |    | <del>10</del> |    |    |   |               |               |               |   |  |
|               | 0             |    |               |    | 0             |    |    |   |               |               |               |   |  |

# Transportation Applications

## Initial Basic Feasible Solution (North-West Corner)

|               |               |    |               |    |               |    |    |   |               |               |               |   |  |
|---------------|---------------|----|---------------|----|---------------|----|----|---|---------------|---------------|---------------|---|--|
|               |               | 8  |               | 6  |               | 10 |    | 9 | <i>Supply</i> |               |               |   |  |
|               | <b>35</b>     |    |               |    |               |    |    |   | <del>35</del> | 0             |               |   |  |
|               |               | 9  |               | 12 |               | 13 |    | 7 | <del>50</del> | <del>40</del> | <del>20</del> | 0 |  |
|               | <b>10</b>     |    | <b>20</b>     |    | <b>20</b>     |    |    |   |               |               |               |   |  |
|               |               | 14 |               | 9  |               | 16 |    | 5 | <del>40</del> | 30            |               |   |  |
|               |               |    |               |    | <b>10</b>     |    |    |   |               |               |               |   |  |
| <i>Demand</i> | <del>45</del> |    | <del>20</del> |    | <del>30</del> |    | 30 |   |               |               |               |   |  |
|               | <del>10</del> |    | 0             |    | <del>10</del> |    |    |   |               |               |               |   |  |
|               | 0             |    |               |    | 0             |    |    |   |               |               |               |   |  |

# Transportation Applications

## Initial Basic Feasible Solution (North-West Corner)

|               |               |               |               |               |           |    |           |   |               |                               |
|---------------|---------------|---------------|---------------|---------------|-----------|----|-----------|---|---------------|-------------------------------|
|               |               | 8             |               | 6             |           | 10 |           | 9 | <i>Supply</i> |                               |
|               | <b>35</b>     |               |               |               |           |    |           |   | <del>35</del> | 0                             |
|               |               | 9             |               | 12            |           | 13 |           | 7 | <del>50</del> | <del>40</del> <del>20</del> 0 |
|               | <b>10</b>     |               | <b>20</b>     |               | <b>20</b> |    |           |   |               |                               |
|               |               | 14            |               | 9             |           | 16 |           | 5 | <del>40</del> | <del>30</del> 0               |
|               |               |               |               |               | <b>10</b> |    | <b>30</b> |   |               |                               |
| <i>Demand</i> | <del>45</del> | <del>20</del> | <del>30</del> | <del>30</del> |           |    |           |   |               |                               |
|               | <del>10</del> | 0             | <del>10</del> | 0             |           |    |           |   |               |                               |
|               | 0             |               |               |               | 0         |    |           |   |               |                               |

# Transportation Applications

## Initial Basic Feasible Solution (North-West Corner)

|               |               |               |               |               |           |    |           |   |               |                               |
|---------------|---------------|---------------|---------------|---------------|-----------|----|-----------|---|---------------|-------------------------------|
|               |               | 8             |               | 6             |           | 10 |           | 9 | <i>Supply</i> |                               |
|               | <b>35</b>     |               |               |               |           |    |           |   | <del>35</del> | 0                             |
|               |               | 9             |               | 12            |           | 13 |           | 7 | <del>50</del> | <del>40</del> <del>20</del> 0 |
|               | <b>10</b>     |               | <b>20</b>     |               | <b>20</b> |    |           |   |               |                               |
|               |               | 14            |               | 9             |           | 16 |           | 5 | <del>40</del> | <del>30</del> 0               |
|               |               |               |               |               | <b>10</b> |    | <b>30</b> |   |               |                               |
| <i>Demand</i> | <del>45</del> | <del>20</del> | <del>30</del> | <del>30</del> |           |    |           |   |               |                               |
|               | <del>10</del> | 0             | <del>10</del> | 0             |           |    |           |   |               |                               |
|               | 0             |               |               |               | 0         |    |           |   |               |                               |

# Transportation Applications

## Initial Basic Feasible Solution (North-West Corner)

|           |           |           |           |   |               |
|-----------|-----------|-----------|-----------|---|---------------|
|           | 8         | 6         | 10        | 9 | <i>Supply</i> |
| <b>35</b> |           |           |           |   | 35            |
|           | 9         | 12        | 13        | 7 | 50            |
| <b>10</b> | <b>20</b> | <b>20</b> |           |   |               |
|           | 14        | 9         | 16        | 5 | 40            |
|           |           | <b>10</b> | <b>30</b> |   |               |
| 45        | 20        | 30        | 30        |   |               |

**cell filled** =  $(m+n-1) = 4+3-1 = 6$   
The result is True

**Total Cost** =  $(35*8) + (10*9) + (20*12) + (20*13) + (10*16) + (30*5) = 1180 \$$

## 2- Least cost Method:

# Transportation Applications

## Initial Basic Feasible Solution (Least-Cost Method)

|  |    |    |    |    |                     |
|--|----|----|----|----|---------------------|
|  | 2  | 10 | 20 | 11 | <i>Supply</i><br>15 |
|  | 12 | 7  | 9  | 20 | 25                  |
|  | 4  | 14 | 16 | 18 | 10                  |

**Supply = 50**

*Demand*      5                  15                  15                  15

**Demand = 50**

**Supply = Demand = 50**



**Balanced Transportation**

# Transportation Applications

## Initial Basic Feasible Solution (Least-Cost Method)

|               |              |    |    |    |                  |
|---------------|--------------|----|----|----|------------------|
|               | 2            | 10 | 20 | 11 | <i>Supply</i>    |
| <b>5</b>      |              |    |    |    | <del>15</del> 10 |
|               | 12           | 7  | 9  | 20 | 25               |
|               | 4            | 14 | 16 | 18 | 10               |
| <i>Demand</i> | <del>5</del> | 15 | 15 | 15 |                  |
|               | 0            |    |    |    |                  |

# Transportation Applications

## Initial Basic Feasible Solution (Least-Cost Method)

|               |                |    |    |    |    |                  |
|---------------|----------------|----|----|----|----|------------------|
|               |                | 2  | 10 | 20 | 11 | <i>Supply</i>    |
|               | <b>5</b>       |    |    |    |    | <del>15</del> 10 |
|               |                | 12 | 7  | 9  | 20 | 25               |
|               |                | 4  | 14 | 16 | 18 | 10               |
| <i>Demand</i> | <del>5</del> 0 | 15 | 15 | 15 |    |                  |

# Transportation Applications

## Initial Basic Feasible Solution (Least-Cost Method)

|               |              |               |           |    |    |               |
|---------------|--------------|---------------|-----------|----|----|---------------|
|               |              | 2             | 10        | 20 | 11 |               |
|               | <b>5</b>     |               |           |    |    | <i>Supply</i> |
|               |              | 12            | <b>15</b> | 7  | 9  | 15            |
|               |              |               |           |    | 20 | <i>10</i>     |
|               |              |               |           |    |    | <i>25</i>     |
|               |              | 4             | 14        | 16 | 18 | <i>10</i>     |
| <i>Demand</i> | <del>5</del> | <del>15</del> | 15        | 15 |    |               |
|               | 0            | 0             |           |    |    |               |

# Transportation Applications

## Initial Basic Feasible Solution (Least-Cost Method)

|               |              |    |               |           |   |    |   |    |               |    |
|---------------|--------------|----|---------------|-----------|---|----|---|----|---------------|----|
|               |              | 2  |               | 10        |   | 20 |   | 11 | <i>Supply</i> |    |
|               | <b>5</b>     |    |               |           |   |    |   |    | <del>15</del> | 10 |
|               |              | 12 |               | <b>15</b> | 7 |    | 9 |    | <del>25</del> | 10 |
|               |              | 4  |               | 14        |   | 16 |   | 18 | 10            |    |
| <i>Demand</i> | <del>5</del> |    | <del>15</del> |           |   | 15 |   | 15 |               |    |
|               | 0            |    | 0             |           |   |    |   |    |               |    |

# Transportation Applications

## Initial Basic Feasible Solution (Least-Cost Method)

|               |              |    |               |    |               |    |    |    |  |  |
|---------------|--------------|----|---------------|----|---------------|----|----|----|--|--|
|               |              | 2  |               | 10 |               | 20 |    | 11 |  |  |
|               | <b>5</b>     |    |               |    |               |    |    |    |  |  |
|               |              | 12 |               | 7  |               | 9  |    | 20 |  |  |
|               |              |    | <b>15</b>     |    | <b>10</b>     |    |    |    |  |  |
|               |              | 4  |               | 14 |               | 16 |    | 18 |  |  |
| <i>Demand</i> | <del>5</del> |    | <del>15</del> |    | <del>15</del> |    | 15 |    |  |  |
|               | 0            |    | 0             |    | 5             |    |    |    |  |  |

*Supply*

~~15~~    10

~~25~~    ~~10~~    0

10

# Transportation Applications

## Initial Basic Feasible Solution (Least-Cost Method)

|               |              |    |               |    |               |    |    |    |               |                 |
|---------------|--------------|----|---------------|----|---------------|----|----|----|---------------|-----------------|
|               |              | 2  |               | 10 |               | 20 |    | 11 |               | <i>Supply</i>   |
|               | <b>5</b>     |    |               |    |               |    |    |    | <del>15</del> | 10              |
|               |              | 12 |               | 7  |               | 9  |    | 20 | <del>25</del> | <del>10</del> 0 |
|               |              |    | <b>15</b>     |    | <b>10</b>     |    |    |    |               |                 |
|               |              | 4  |               | 14 |               | 16 |    | 18 | 10            |                 |
| <i>Demand</i> | <del>5</del> |    | <del>15</del> |    | <del>15</del> |    | 15 |    |               |                 |
|               | 0            |    | 0             |    | 5             |    |    |    |               |                 |

# Transportation Applications

## Initial Basic Feasible Solution (Least-Cost Method)

|               |              |    |               |    |               |    |               |           |               |  |
|---------------|--------------|----|---------------|----|---------------|----|---------------|-----------|---------------|--|
|               |              | 2  |               | 10 |               | 20 |               | 11        |               |  |
|               | <b>5</b>     |    |               |    |               |    |               | <b>10</b> |               |  |
|               |              | 12 |               | 7  |               | 9  |               |           | 20            |  |
|               |              |    | <b>15</b>     |    | <b>10</b>     |    |               |           |               |  |
|               |              | 4  |               | 14 |               | 16 |               |           | 18            |  |
| <i>Demand</i> | <del>5</del> |    | <del>15</del> |    | <del>15</del> |    | <del>15</del> |           | <del>15</del> |  |
|               | 0            |    | 0             |    | 5             |    | 5             |           |               |  |

*Supply*

~~15~~   ~~10~~   0

~~25~~   ~~10~~   0

10

# Transportation Applications

## Initial Basic Feasible Solution (Least-Cost Method)

|        |              |    |               |    |               |    |               |           |               |               |   |
|--------|--------------|----|---------------|----|---------------|----|---------------|-----------|---------------|---------------|---|
|        |              | 2  |               | 10 |               | 20 |               | 11        | Supply        |               |   |
|        | <b>5</b>     |    |               |    |               |    |               | <b>10</b> | <del>15</del> | <del>10</del> | 0 |
|        |              | 12 |               | 7  |               | 10 | 9             |           | <del>25</del> | <del>10</del> | 0 |
|        |              |    | <b>15</b>     |    |               |    |               |           |               |               |   |
|        |              | 4  |               | 14 |               | 16 |               | 18        | 10            |               |   |
| Demand | <del>5</del> |    | <del>15</del> |    | <del>15</del> |    | <del>15</del> |           |               |               |   |
|        | 0            |    | 0             |    | 5             |    | 5             |           |               |               |   |

# Transportation Applications

## Initial Basic Feasible Solution (Least-Cost Method)

|               |              |    |               |    |               |    |               |           |               |               |               |   |
|---------------|--------------|----|---------------|----|---------------|----|---------------|-----------|---------------|---------------|---------------|---|
|               |              | 2  |               | 10 |               | 20 |               | 11        |               | <i>Supply</i> |               |   |
|               | <b>5</b>     |    |               |    |               |    |               | <b>10</b> |               | <del>15</del> | <del>10</del> | 0 |
|               |              | 12 |               | 7  |               | 10 | 9             |           | 20            | <del>25</del> | <del>10</del> | 0 |
|               |              |    | <b>15</b>     |    |               |    |               |           |               | <del>10</del> |               |   |
|               |              | 4  |               | 14 | <b>5</b>      | 16 |               |           | 18            | <del>10</del> | 5             |   |
| <i>Demand</i> | <del>5</del> |    | <del>15</del> |    | <del>15</del> |    | <del>15</del> |           | <del>15</del> |               |               |   |
|               | 0            |    | 0             |    | <del>5</del>  |    | 5             |           |               |               |               |   |
|               |              |    |               |    | 0             |    |               |           |               |               |               |   |

# Transportation Applications

## Initial Basic Feasible Solution (Least-Cost Method)

|               |              |    |               |    |               |          |               |           |               |               |               |   |
|---------------|--------------|----|---------------|----|---------------|----------|---------------|-----------|---------------|---------------|---------------|---|
|               |              | 2  |               | 10 |               | 20       |               | 11        |               | <i>Supply</i> |               |   |
|               | <b>5</b>     |    |               |    |               |          |               | <b>10</b> |               | <del>15</del> | <del>10</del> | 0 |
|               |              | 12 |               | 7  |               | 10       | 9             |           | 20            | <del>25</del> | <del>10</del> | 0 |
|               |              |    | <b>15</b>     |    |               |          |               |           |               | <del>10</del> |               |   |
|               |              | 4  |               | 14 |               | <b>5</b> | 16            |           | 18            | <del>10</del> | 5             |   |
| <i>Demand</i> | <del>5</del> |    | <del>15</del> |    | <del>15</del> |          | <del>15</del> |           | <del>15</del> |               |               |   |
|               | 0            |    | 0             |    | <del>5</del>  |          | 5             |           |               |               |               |   |
|               |              |    |               |    | 0             |          |               |           |               |               |               |   |

# Transportation Applications

## Initial Basic Feasible Solution (Least-Cost Method)

|               |              |    |               |    |               |    |               |           |               |               |               |   |
|---------------|--------------|----|---------------|----|---------------|----|---------------|-----------|---------------|---------------|---------------|---|
|               |              | 2  |               | 10 |               | 20 |               | 11        |               | <i>Supply</i> |               |   |
|               | <b>5</b>     |    |               |    |               |    |               | <b>10</b> |               | <del>15</del> | <del>10</del> | 0 |
|               |              | 12 |               | 7  |               | 10 | 9             |           | 20            | <del>25</del> | <del>10</del> | 0 |
|               |              |    | <b>15</b>     |    |               |    |               |           |               |               |               |   |
|               |              | 4  |               | 14 | <b>5</b>      | 16 |               | <b>5</b>  | 18            | <del>10</del> | <del>5</del>  | 0 |
| <i>Demand</i> | <del>5</del> |    | <del>15</del> |    | <del>15</del> |    | <del>15</del> |           | <del>15</del> |               |               |   |
|               | 0            |    | 0             |    | <del>5</del>  |    | <del>5</del>  |           |               |               |               |   |
|               |              |    |               |    | 0             |    | 0             |           |               |               |               |   |

# Transportation Applications

## Initial Basic Feasible Solution (Least-Cost Method)

|               |           |    |           |           |               |
|---------------|-----------|----|-----------|-----------|---------------|
|               | 2         | 10 | 20        | 11        | <i>Supply</i> |
| <b>5</b>      |           |    |           | <b>10</b> | 15            |
|               | 12        | 7  | 9         | 20        | 25            |
|               | <b>15</b> |    | <b>10</b> |           |               |
|               | 4         | 14 | 5         | 18        | 10            |
|               |           |    |           |           |               |
| <i>Demand</i> | 5         | 15 | 15        | 15        |               |

**cell filled** =  $(m+n-1) = 4+3-1 = 6$   
The result is True

**Total Cost =  $(5*2) + (15*7) + (10*9) + (10*11) + (5*16) + (5*18) = 485$  \$**

## 3- Vogel approximation Method(VAM)

VAM is improved version of the least cost method that generally, but not always, produces better starting

- Initial Basic Feasible Solution (VAM)

|  |   |    |    |                      |
|--|---|----|----|----------------------|
|  | 6 | 8  | 10 | <i>Supply</i><br>150 |
|  | 7 | 11 | 11 | 175                  |
|  | 4 | 5  | 12 | 275                  |

*Demand*      200          100          300

**Demand = 600**

**Supply = 600**

**Supply = Demand = 600**



**Balanced Transportation**

- Initial Basic Feasible Solution (VAM)

|  |   |  |    |  |    |
|--|---|--|----|--|----|
|  | 6 |  | 8  |  | 10 |
|  | 7 |  | 11 |  | 11 |
|  | 4 |  | 5  |  | 12 |

*Supply*  
150

175

275

*Demand*      200          100          300

**penalty**          2                  3                  1

**penalty**

2

4

1

- Initial Basic Feasible Solution (VAM)

|            |   |  |    |  |    |
|------------|---|--|----|--|----|
|            | 6 |  | 8  |  | 10 |
| <b>175</b> | 7 |  | 11 |  | 11 |
|            | 4 |  | 5  |  | 12 |

|                |     |                |
|----------------|-----|----------------|
|                |     | <b>penalty</b> |
| Supply         | 150 | 2              |
| <del>175</del> | 0   | 4              |
| 275            |     | 1              |

|                |                |     |     |
|----------------|----------------|-----|-----|
| Demand         | <del>200</del> | 100 | 300 |
|                | 25             |     |     |
| <b>penalty</b> | 2              | 3   | 1   |

- Initial Basic Feasible Solution (VAM)

|            |                      |     |     |                  |   |
|------------|----------------------|-----|-----|------------------|---|
|            | 6                    | 8   | 10  | Supply<br>150    | 2 |
| <b>175</b> | 7                    | 11  | 11  | <del>175</del> 0 | 0 |
|            | 4                    | 5   | 12  | 275              | 1 |
| Demand     | <del>200</del><br>25 | 100 | 300 |                  |   |
| penalty    | 2                    | 3   | 2   |                  |   |



- |            |   |            |    |                    |   |
|------------|---|------------|----|--------------------|---|
|            | 6 | 8          | 10 | Supply<br>150      | 2 |
| <b>175</b> | 7 | 11         | 11 | <del>175</del> 0   |   |
|            | 4 | <b>100</b> | 5  | <del>275</del> 175 | 1 |
- 
- |    |                |                |     |
|----|----------------|----------------|-----|
| nd | <del>200</del> | <del>100</del> | 300 |
|    | 25             | 0              |     |
| ty | 2              | 3              | 2   |



- |            |   |            |    |  |    |                    |
|------------|---|------------|----|--|----|--------------------|
|            | 6 |            | 8  |  | 10 | <i>Supply</i>      |
| <b>175</b> | 7 |            | 11 |  | 11 | <del>175</del> 0   |
|            | 4 | <b>100</b> | 5  |  | 12 | <del>275</del> 175 |

25

2

2

4

8

- Initial Basic Feasible Solution (VAM)

|         |                |                |     |  |    |                |                |     |
|---------|----------------|----------------|-----|--|----|----------------|----------------|-----|
|         | 6              |                | 8   |  | 10 | Supply         | 150            |     |
| 175     | 7              |                | 11  |  | 11 | <del>175</del> | 0              | 4   |
| 25      | 4              | 100            | 5   |  | 12 | <del>275</del> | <del>175</del> | 150 |
| Demand  | <del>200</del> | <del>100</del> | 300 |  |    |                |                |     |
|         | <del>25</del>  | 0              |     |  |    |                |                |     |
|         | 0              |                |     |  |    |                |                |     |
| penalty | 2              |                | 2   |  |    |                |                |     |

- Initial Basic Feasible Solution (VAM)

|               |                |                |     |    |
|---------------|----------------|----------------|-----|----|
|               | 6              | 8              | 10  |    |
| <b>175</b>    | 7              | 11             | 11  |    |
| <b>25</b>     | 4              | <b>100</b>     | 5   | 12 |
| <i>Demand</i> | <del>200</del> | <del>100</del> | 300 |    |

Supply 150  
 175 0  
 275 175 150  
 0  
 2  
 penalty

- Initial Basic Feasible Solution (VAM)

|         |               |     |     |                |    |
|---------|---------------|-----|-----|----------------|----|
|         | 6             | 8   | 10  | Supply         |    |
| 175     | 7             | 11  | 11  | <del>175</del> | 0  |
| 25      | 4             | 100 | 5   | 150            | 12 |
| Demand  | 200           | 100 | 300 |                |    |
|         | <del>25</del> | 0   | 150 |                |    |
|         | 0             |     |     |                |    |
| penalty |               |     |     |                |    |

- Initial Basic Feasible Solution (VAM)

|     |   |     |    |     |
|-----|---|-----|----|-----|
|     | 6 | 8   | 10 |     |
| 175 | 7 | 11  | 11 |     |
| 25  | 4 | 100 | 5  | 150 |

*Demand*      ~~200~~      ~~100~~      300

*Supply*      150

penalty

10

~~175~~    0

~~275~~    ~~175~~    ~~150~~    0

~~25~~      0      150

0

penalty

- Initial Basic Feasible Solution (VAM)

|               |                |                |                |            |                                 |                |
|---------------|----------------|----------------|----------------|------------|---------------------------------|----------------|
|               | 6              | 8              | <b>150</b>     | 10         | <i>Supply</i><br><del>150</del> | 0              |
| <b>175</b>    | 7              | 11             |                | 11         | <del>175</del>                  | 0              |
| <b>25</b>     | 4              | <b>100</b>     | 5              | <b>150</b> | <del>275</del>                  | <del>175</del> |
| <i>Demand</i> | <del>200</del> | <del>100</del> | <del>300</del> |            | <del>25</del>                   | 0              |
|               | 0              |                | 0              |            |                                 |                |

- Initial Basic Feasible Solution (VAM)

|            |   |            |    |            |    |                      |
|------------|---|------------|----|------------|----|----------------------|
|            | 6 |            | 8  | <b>150</b> | 10 | <i>Supply</i><br>150 |
| <b>175</b> | 7 |            | 11 |            | 11 | 175                  |
| <b>25</b>  | 4 | <b>100</b> | 5  | <b>150</b> | 12 | 275                  |

*Demand*      200                  100                  300

**cell filled** =  $(m+n-1) = 3+3-1 = 5$   
The result is True

Total Cost =  $(175*7) + (25*4) + (100*5) + (150*10) + (150*12) = 5125$  \$



Thank You