

Equilibrium and Geometric sequences

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Lecture 06



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MTH 112
Business Mathematics

At the end of this section you should be familiar with:

- Labour demand function
- Market equilibrium for substitute and complementary goods
- Taxes, subsidies and their distribution
- Arithmetic Sequences and serieses

Price Floors

Price floors are used by governments in cases where they believe that the equilibrium price is too low for the producer to receive. Thus, **price floors operate above market equilibrium** and are aimed at protecting producers. Price floors are also known as **minimum prices**, where the price is not allowed to go below the minimum or 'floor' price

For example, the Common Agricultural Policy (CAP) in the European Union and minimum wage laws.

Example:

Given the labour demand and supply functions as

Labour demand function: $w_d = 9 - 0.6L_d$

Labour supply function: $w_s = 2 + 0.4L_s$

analyse the effect on the labour market if the government introduces a minimum wage law of £6 per hour.

Solution:

labour market equilibrium
 $\omega_d = \omega_s$, $L_d = L_s$

$$\rightarrow \omega_d = 9 - 0.6L_d$$
$$\rightarrow \omega_s = 2 + 0.4L_s$$

$$9 - 0.6L_d = 2 + 0.4L_s$$

$$\boxed{L = 7} \quad \text{equilibrium number of workers}$$

$$\omega = 9 - (0.6)(7)$$

$$\boxed{\omega = 4.8} \quad \text{equilibrium wage.}$$



Solution:

labour demanded at $w=6$

$$w_d = 9 - 0.6 L_d$$

$$6 = 9 - 0.6 L_d$$

$$\boxed{L_d = 5}$$

labour supplied at $w=6$ is

$$w_s = 2 + 0.4 L_s$$

$$6 = 2 + 0.4 L_s$$

$$\boxed{L_s = 10}$$



Solution:

Since labour supplied ($L_s = 10$) is greater than labour demanded ($L_d = 5$), there is an excess supply of labour

$$\begin{aligned}XS &= L_s - L_d \\&= 10 - 5 \\&= 5\end{aligned}$$

Solution:

This is also referred to as a surplus,
that is there is unemployment in the labour market.

Chapter 03

Market equilibrium for substitute and complementary goods

Complementary goods are goods that are consumed together (for example, cars and petrol, computer hardware and computer software). One good cannot function without the other.

Substitute goods are consumed instead of each other (for example, coffee versus tea; bus versus train on given routes).

The general demand function is now written as,

$$Q = f(P, P_s, P_c)$$

that is, the quantity demanded of a good is a function of the price of the good itself and the prices of those goods that are substitutes and complements to it.

Note: In this case, P_s refers to the price of substitute goods, not to be confused with P_s which is used to refer to the supply price of a good.

Consider two goods, X and Y. The demand function for good X is written differently depending on whether good Y is a substitute to X or a complement to X.

X and Y are substitute

$$Q_x = a - bP_x + dP_y$$

$$Q_x = (a + dP_y) - bP_x$$

Ex if train fares increase, individuals will reduce their demand for train journeys and increase their demand for bus journey.



Chapter 03

Market equilibrium for substitute and complementary goods

Consider two goods, X and Y. The demand function for good X is written differently depending on whether good Y is a substitute to X or a complement to X.

X and Y are complements

$$Q_x = a - bP_x - dP_y$$

$$Q_x' = (a - dP_y) - bP_x$$

as P_y increases, Q_x decrease

Ex if car prices increase, individuals will reduce their demand for cars and consequently the demand for petrol decreases



Chapter 03

Market equilibrium for substitute and complementary goods

Example:

Find the equilibrium price and quantity for two substitute goods X and Y given their respective demand and supply equations as.

$$Q_{dx} = 82 - 3P_x + P_y \quad (3.11)$$

$$Q_{sx} = -5 + 15P_x \quad (3.12)$$

$$Q_{dy} = 92 + 2P_x - 4P_y \quad (3.13)$$

$$Q_{sy} = -6 + 32P_y \quad (3.14)$$

The equilibrium condition

Solution:

$$Q_{dx} = Q_{sx}$$

$$82 - 3P_x + P_y = -5 + 15P_x$$

$$-18P_x + P_y = -87$$

$$Q_{dy} = Q_{sy}$$

$$92 + 2P_x - 4P_y = -6 + 32P_y$$

$$2P_x - 36P_y = -98$$

Chapter 03

Market equilibrium for substitute and complementary goods

Solution:

$$\begin{array}{r} -18P_x + P_y = -87 \\ + \quad 18P_x - 324P_y = -882 \\ \hline -323P_y = -1969 \\ \hline P_y = 3 \end{array}$$

Chapter 03

Market equilibrium for substitute and complementary goods

Solution:

$$-18P_x + P_y = -87$$

$$-18P_x + 3 = -87$$

$$\cancel{-18}P_x = \cancel{-90} \quad \cancel{18}$$

$$P_x = 5$$

$$P_y = 3$$

Chapter 03

Market equilibrium for substitute and complementary goods

Solution:

Solve for Q_Y by substituting $P_Y=3$, $P_X=5$
into eqns (3.13) or (3.14)

$$\begin{aligned}Q_Y &= -6 + 32 P_Y \\ &= -6 + (32)(3)\end{aligned}$$

$$Q_Y = 90$$

Chapter 03

Market equilibrium for substitute and complementary goods

Solution:

Solve for Q_X by substituting $P_Y = 3$ or $P_X = 5$
into eqns (3.11) or (3.12)

$$Q_X = -5 + 15 P_X$$

$$Q_X = -5 + (15)(5)$$

$$Q_X = 70$$

Chapter 03

Market equilibrium for substitute and complementary goods

Solution:

The equilibrium price and quantities in this two goods market are

$$P_X = 5, \quad Q_X = 70$$

$$P_Y = 3, \quad Q_Y = 90$$

Taxes and subsidies are another example of government intervention in the market.

A tax on a good is known as an indirect tax. Indirect taxes may be either:

- **A fixed amount per unit of output** (excise tax); for example, the tax imposed on petrol and alcohol. This will translate the supply function vertically upwards by the amount of the tax.
- **A percentage of the price of the good**; for example, value added tax. This will change the slope of the supply function. The slope will become steeper since a given percentage tax will be a larger absolute amount the higher the price.

Fixed tax per unit of output

When a tax is imposed on a good, two issues of concern arise:

- How does the imposition of the tax affect the equilibrium price and quantity of the good?
- What is the distribution (incidence) of the tax, that is, what percentage of the tax is paid by consumers and producers respectively?

In these calculations

- The consumer always pays the equilibrium price.
- The supplier receives the equilibrium price minus the tax.

Example

The demand and supply functions for a good are given as

Demand fn
Supply fn

$$P_d = 100 - 0.5 Q_d$$
$$P_s = 10 + 0.5 Q_s$$

(a)

Calculate the equilibrium price and quantity

$$P = £ 55, \quad Q = 90 \text{ units}$$



Example

The demand and supply functions for a good are given as

$$\begin{aligned} \text{Demand function } P_d &= 100 - 0.5 Q_d \\ \text{Supply function } P_s &= 10 + 0.5 Q_s \end{aligned}$$

(b)

Assume that the government imposes a fixed tax of 6 per unit sold.

i) write down the equation of the supply function, adjusted for tax.

ii) find the new equilibrium price and quantity algebraically and graphically

iii) outline the distribution of the tax, that is, calculate the tax paid by the consumer and the producer

Solution: (i) The tax of £6 per unit sold means that the effective price received by the producer is $(P_s - 6)$
The equation of the supply function adjusted for tax is

$$P_s - 6 = 10 + 0.5Q$$
$$\boxed{P_s = 16 + 0.5Q}$$

Solution:

(ii) The new equilibrium price and quantity are calculated by equating the original demand fn. and the supply function adjusted for tax

$$P_d = P_s$$

$$100 - 0.5Q = 16 + 0.5Q$$

$$Q = 84$$

Solution:

Substitute the new equilibrium quantity, $Q = 84$
into either equation in last slide.

$$P = 100 - 0.5 Q$$

$$P = 100 - (0.5)(84)$$

$$P = 58$$

Chapter 03

Taxes, subsidies and their distribution

Solution: The point $(84, 58)$ is shown as point E_1 .

Solution: (iii) The consumer always pays the equilibrium price
£58
an increase of £3 on the original equilibrium price
which was £55
This means that the consumer pays 50% of the tax

Solution:

The Producer receives the new equilibrium price, minus the tax.

$$£58 - £6 = £52$$

$$£55$$

$$55 - 52 = 3$$

This means that the producer pays 50% of the tax.

$$\text{the tax} = £6$$

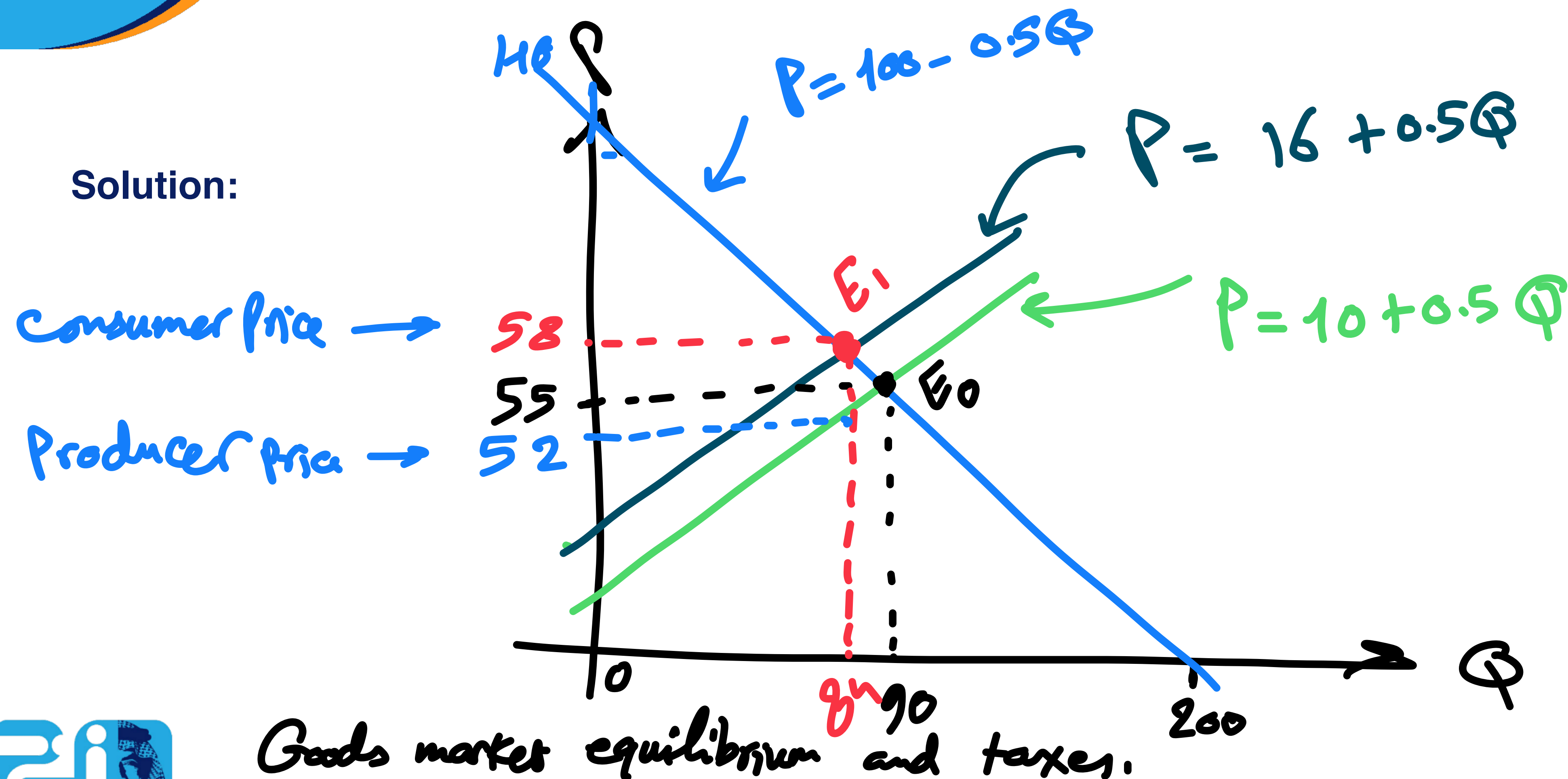
Solution:

The tax is evenly distributed between the consumer and producer.

Chapter 03

Taxes, subsidies and their distribution

Solution:



Example:

The demand and supply functions for a good (£ p per ton of potatoes) are given as

Demand function $P_d = 450 - 2Q_d$

Supply function $P_s = 100 + 5Q_s$

(a) calculate the equilibrium price and quantity

(b) The government provides a subsidy of £70 per unit (ton) solid

(I) write down the equation of the supply function, adjusted for the subsidy

(II) Find the new equilibrium price and quantity algebraically and graphically

(III) Outline the distribution of the subsidy, that is, calculate how much of the subsidy is received by the consumer and the supplier.

Solution:

left for students ...

Chapter 5

Arithmetic & Geometric Sequences and Series



Definitions

A **sequence** is a list of numbers which follow a definite pattern or rule.

Arithmetic sequence

If the rule is that each term, after the first, is obtained by adding a constant, d , to the previous term, then the sequence is called an **arithmetic sequence**, such as 2, 6, 10, 14, 18, 22,... , where $d = 4$.

d is known as the **common difference**.

Geometric sequence

If the rule is that each term, after the first, is obtained by multiplying the previous term by a constant, r , then the sequence is called a **geometric sequence**, such as, 4, 16, 64, 256, 1024, ...
where $r = 4$.

r is known as the **common ratio**.

Definitions

A **series** is the sum of the terms of a sequence.

A series is **finite** if it is the sum of a finite number of terms of a sequence.

A series is **infinite** if it is the sum of an infinite number of terms of a sequence.

Arithmetic series

An **arithmetic series** is the sum of the terms of an arithmetic sequence, such as 2, 6, 10, 14, 18, 22, . . . , with $d=4$. Denoting the first term of an arithmetic sequence with the value a and progressing by adding the value d to each previous term, the arithmetic sequence can be outlined as in Table 5.1.

Each element of a sequence can be identified by reference to its term number;

Find the sum of the first 15 terms of the series: $20 + 18 + 16 + 14 + \dots$

Thank you

