

Probability and Statistics

Prepared by Dr. Alfaisal Hasan

STA 201
Probability and Statistics

Lecture NO 5



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Chapter

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- Chapter 2: Descriptive Statistics
- Chapter 3: Probability
- Chapter 4: Discrete Probability Distributions
- **Chapter 5: Normal Probability Distributions**
- Chapter 6: Confidence Intervals
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Normal Probability Distribution

5.1 Introduction to Normal Distributions and the Standard Normal Distribution

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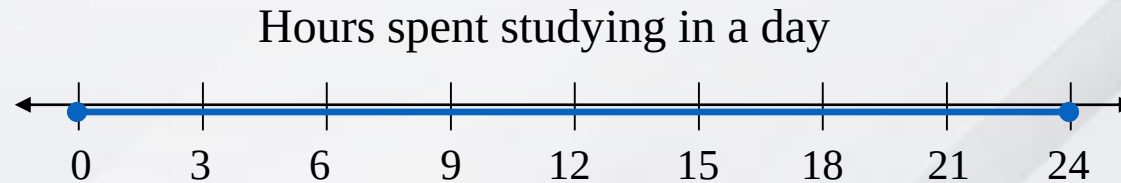
Introduction to Normal Distribution

Continuous random variable

Has an infinite number of possible values that can be represented by an interval on the number line.

متغير عشوائي متصل

لديه عدد لا نهائي من القيم الممكنة التي يمكن تمثيلها بفواصل على خط الأعداد.



The time spent studying can be any number between 0 and 24.

يمكن أن يكون الوقت المستغرق في الدراسة أي رقم بين 0 و 24.

Continuous probability distribution

The probability distribution of a continuous random variable.

التوزيع الاحتمالي للمتغير العشوائي المتصل.

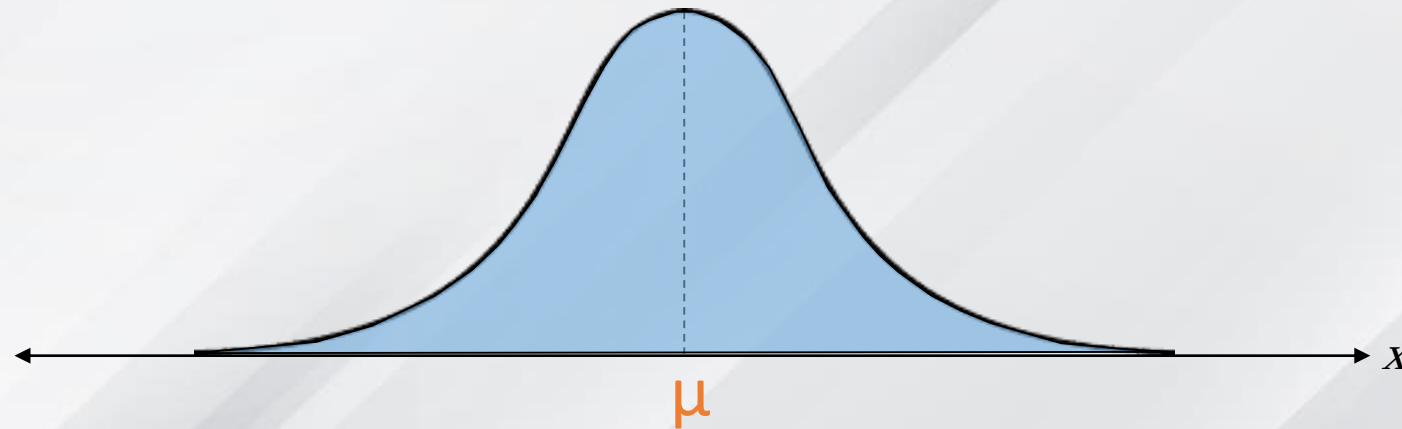
Properties of Normal Distribution

Chapter

خصائص التوزيع الطبيعي

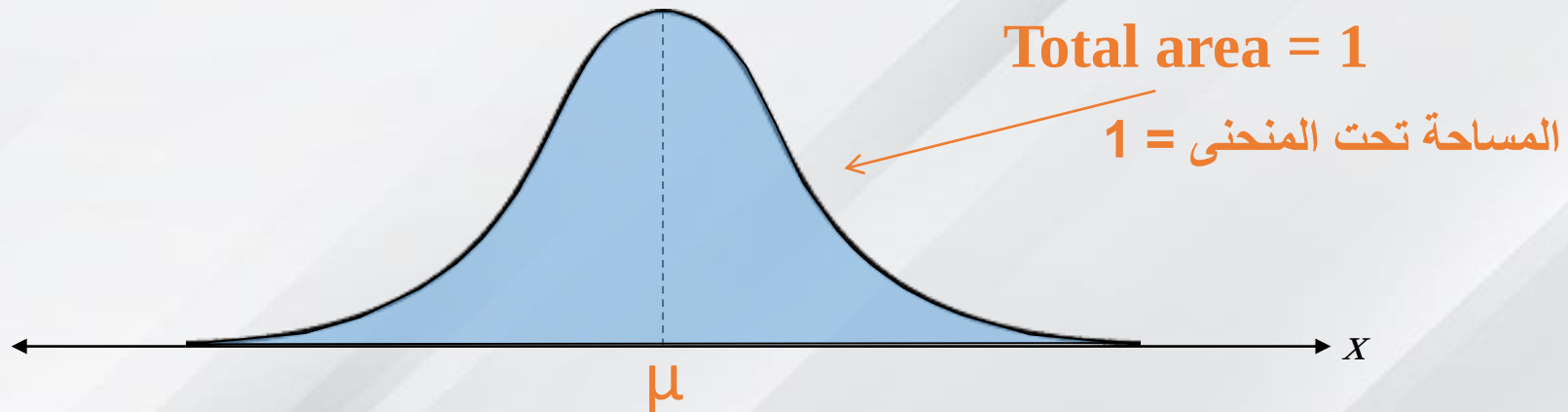
Normal distribution

- A continuous probability distribution for a random variable, x .
التوزيع الاحتمالي المستمر للمتغير العشوائي
- The most important continuous probability distribution in statistics.
أهم التوزيع الاحتمالي المستمر في الإحصاء
- The graph of a normal distribution is called the **normal curve**.
الرسم البياني للتوزيع الطبيعي يسمى المنحنى الطبيعي



Properties of Normal Distribution

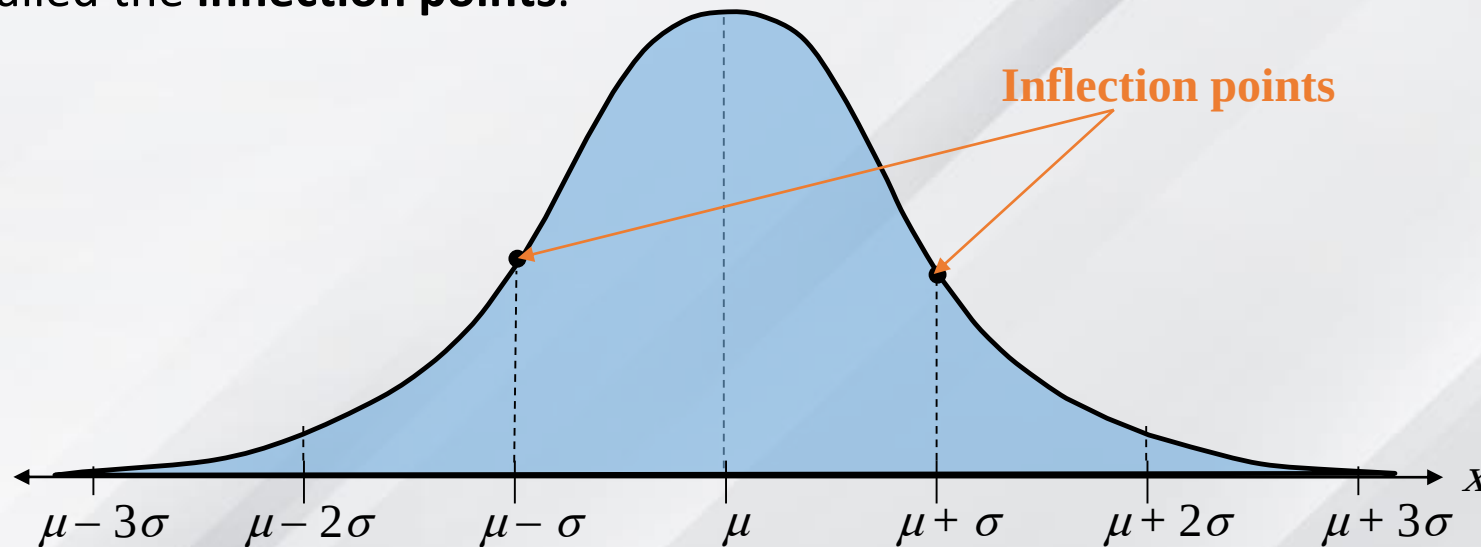
1. The mean, median, and mode are equal.
 2. The normal curve is bell-shaped and symmetric about the mean.
 3. The total area under the curve is equal to one.
 4. The normal curve approaches, but never touches the x -axis as it extends farther and farther away from the mean.
- 1- المتوسط الحسابي والوسيط والمنوال متساوية.
 - 2- المنحنى الطبيعي على شكل جرس ومتماثل حول المتوسط الحسابي.
 - 3- المساحة الكلية تحت المنحنى تساوي واحدًا.
 - 4- يقترب المنحنى الطبيعي، لكنه لا يمس المحور السيني مطلقًا لأنه يمتد أبعد وأبعد عن المتوسط.



Properties of Normal Distribution

5. Between $\mu - \sigma$ and $\mu + \sigma$ (in the center of the curve), the graph curves downward. The graph curves upward to the left of $\mu - \sigma$ and to the right of $\mu + \sigma$. The points at which the curve changes from curving upward to curving downward are called the **inflection points**.

بين $\mu - \sigma$ و $\mu + \sigma$ (في منتصف المنحنى), ينحني المنحنى لأسفل. ينحني المنحنى لأعلى إلى يسار $\mu - \sigma$ وإلى يمين $\mu + \sigma$. تسمى النقاط التي يتغير عندها المنحنى من الانحناء لأعلى إلى الانحناء لأسفل بنقاط الانقلاب.



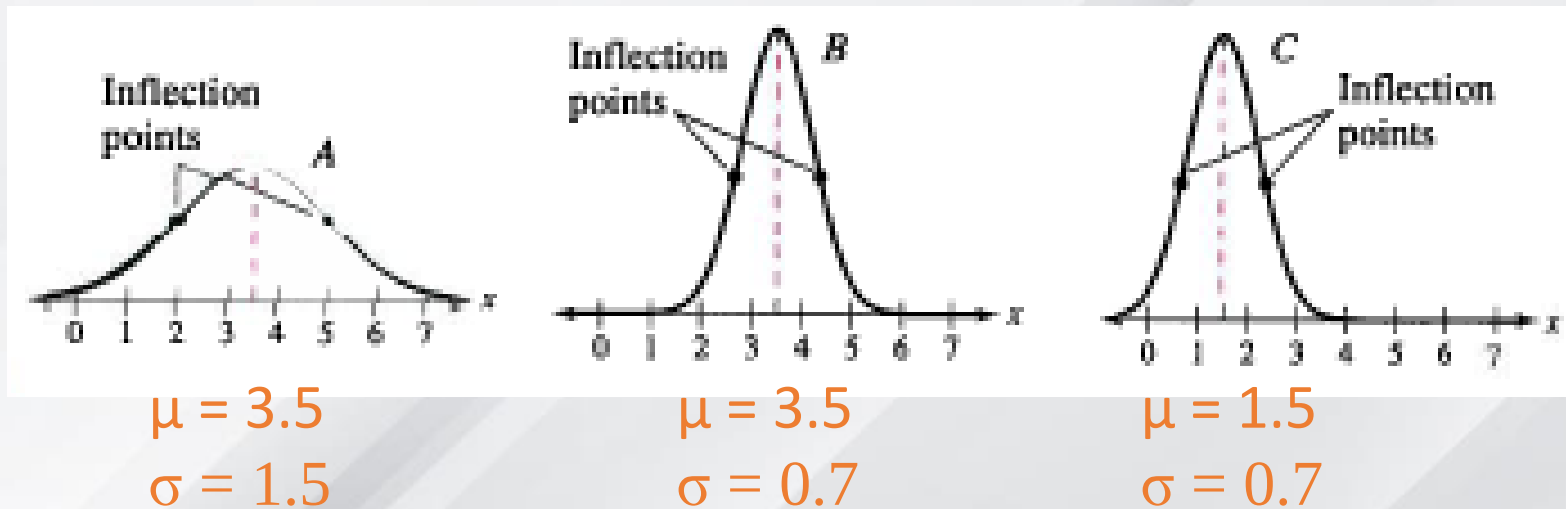
Means and standard deviations

- A normal distribution can have any mean and any positive standard deviation.
- The mean gives the location of the line of symmetry.
- The standard deviation describes the spread of the data.

يمكن أن يكون للتوزيع الطبيعي أي متوسط وأي انحراف معياري موجب.

المتوسط يعطي موقع خط التماثل.

يصف الانحراف المعياري انتشار البيانات.



Example

- Which curve has the greater mean?
- Which curve has the greater standard deviation?

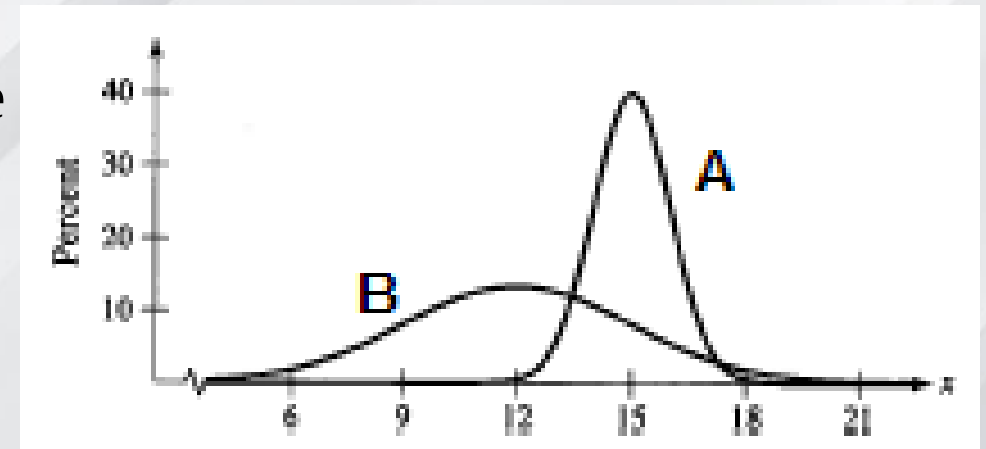
أي منحنى له المتوسط الحسابي الأكبر؟

أي منحنى له الانحراف المعياري الأكبر؟

Solution:

Curve A has the greater mean (The line of symmetry of curve A occurs at $x = 15$. The line of symmetry of curve B occurs at $x = 12$)

Curve B has the greater standard deviation (Curve B is more spread out than curve A.)



Convert from normal to standard normal

التحويل من التوزيع الطبيعي إلى التوزيع الطبيعي القياسي

If x is normal with a mean, μ , and a standard deviation, σ .

Any normal (X) probability can be converted to the standard normal (Z) probability distribution with the following formula

إذا كان X توزيع طبيعي بمتوسط حسابي μ وانحاف معياري σ فإن احتمال التوزيع الطبيعي يمكن تحويله إلى احتمال توزيع طبيعي قياسي عن طريق القانون

$$Z = \frac{X - \mu}{\sigma}$$

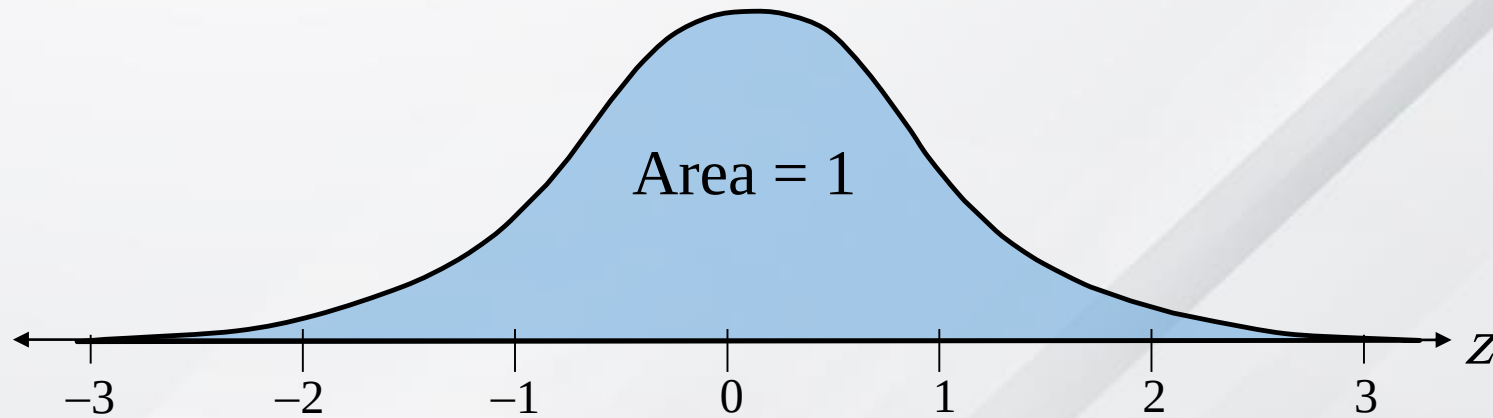
This rule is very important to find the area under the normal curve.

هذه القاعدة مهمة جدًا لإيجاد المساحة تحت المنحنى الطبيعي.

The Standard Normal Distribution

Standard normal distribution

A normal distribution with a mean of 0 and a standard deviation of 1.



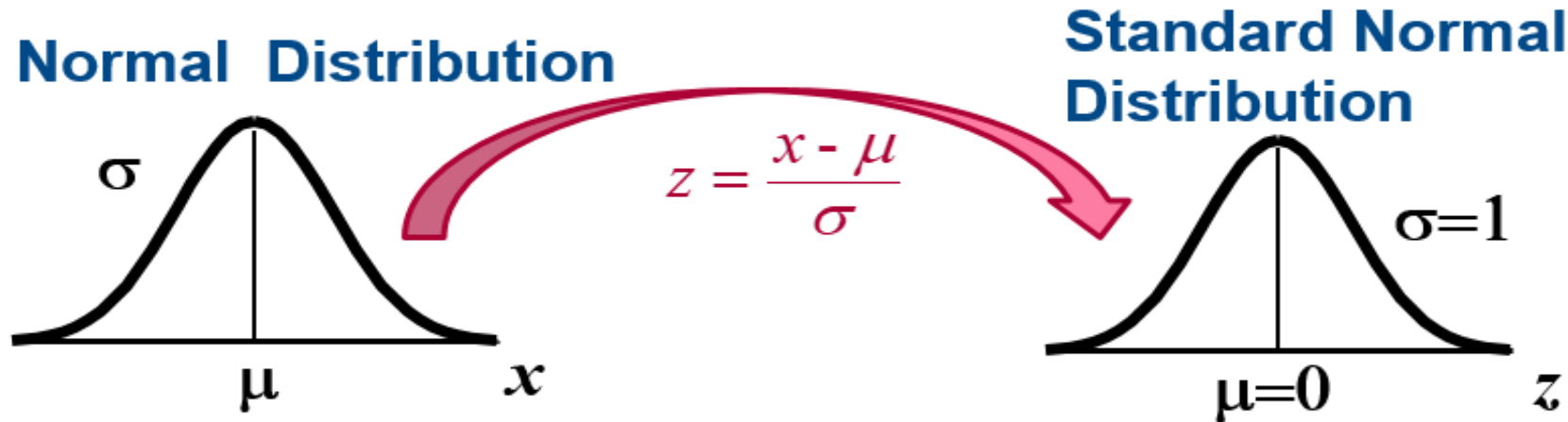
- Any x -value can be transformed into a z -score by using the formula

$$z = \frac{\text{Value} - \text{Mean}}{\text{Standard deviation}} = \frac{x - \mu}{\sigma}$$

The Standard Normal Distribution

Chapter

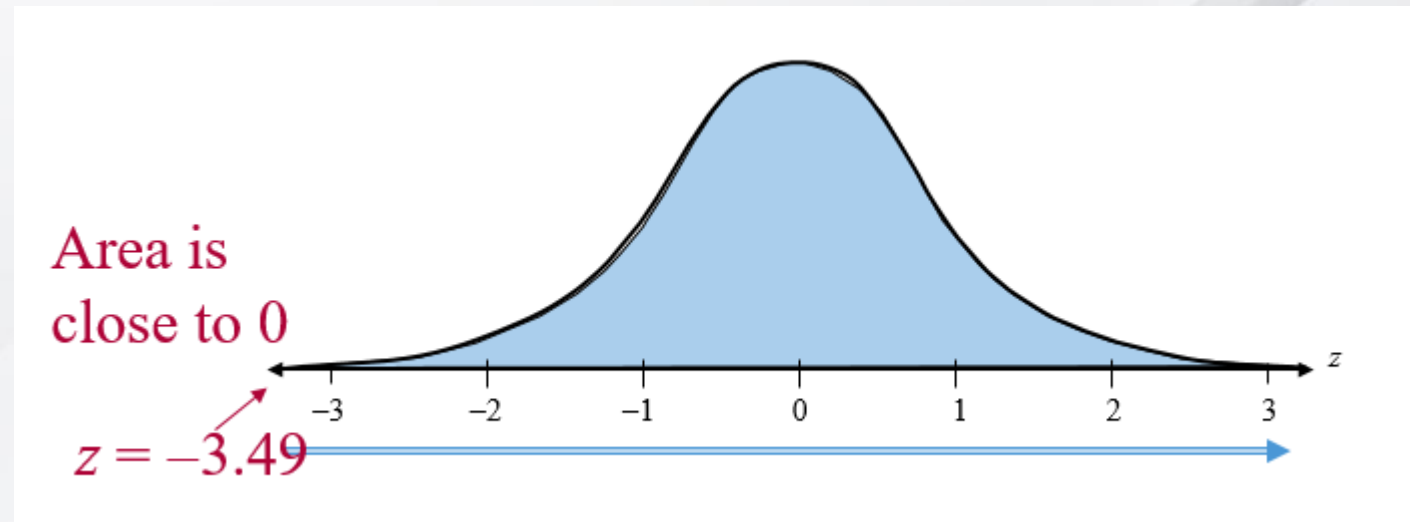
- If each data value of a normally distributed random variable x is transformed into a z -score, the result will be the standard normal distribution.



- Use the Standard Normal Table to find the cumulative area under the standard normal curve.

Properties of the Standard Normal Distribution

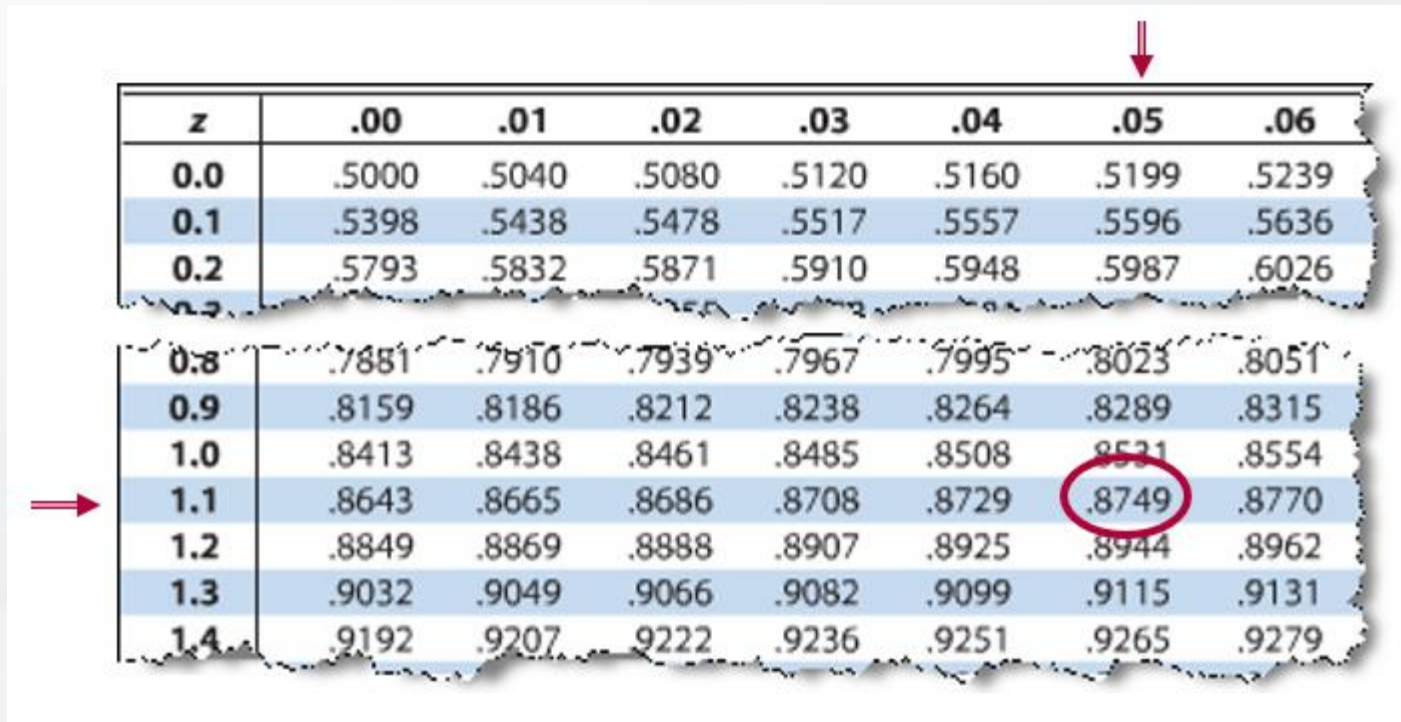
1. The cumulative area is close to 0 for z-scores close to $z = -3.49$.
2. The cumulative area increases as the z-scores increase.



Example: Using The Standard Normal Table

Chapter

- Find the cumulative area that corresponds to a z-score of 1.15.



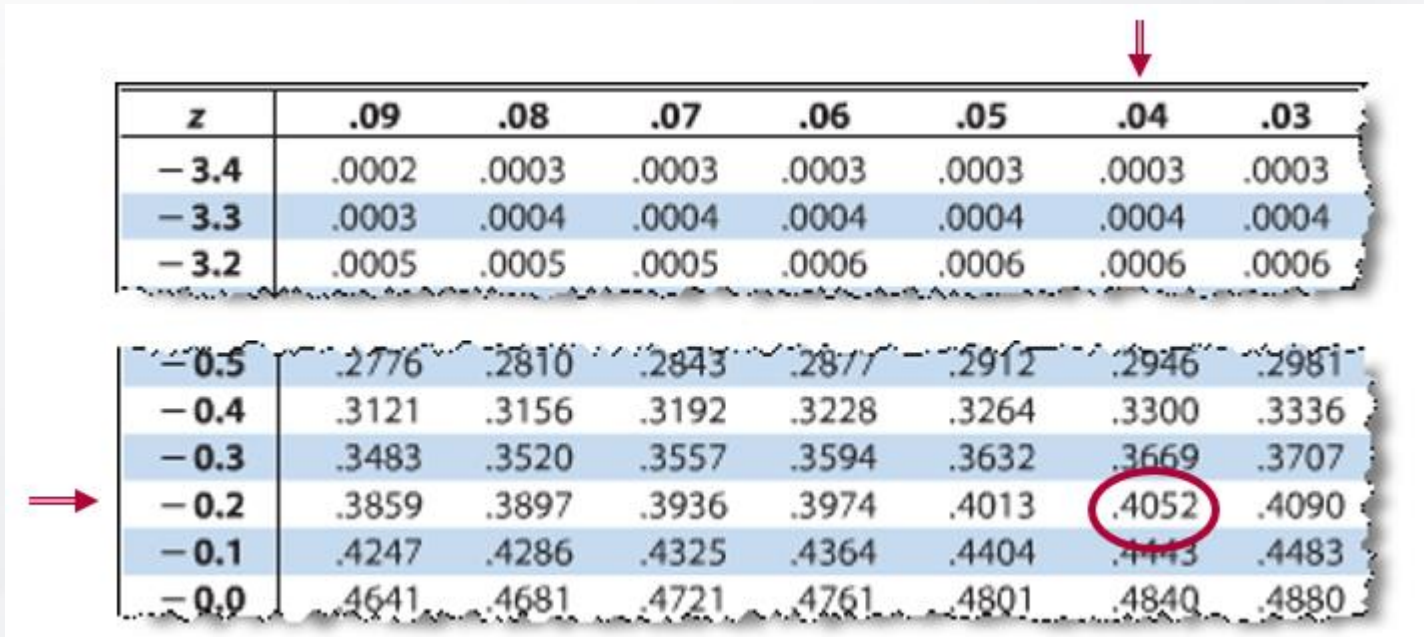
z	.00	.01	.02	.03	.04	.05	.06
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7122
0.6	.7257	.7291	.7324	.7357	.7389	.7421	.7453
0.7	.7580	.7613	.7643	.7673	.7703	.7732	.7761
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279

The area to the left of $z = 1.15$ is 0.8749.

Example: Using The Standard Normal Table

Chapter

- Find the cumulative area that corresponds to a z-score of -0.24.



<i>z</i>	.09	.08	.07	.06	.05	.04	.03
-3.4	.0002	.0003	.0003	.0003	.0003	.0003	.0003
-3.3	.0003	.0004	.0004	.0004	.0004	.0004	.0004
-3.2	.0005	.0005	.0005	.0006	.0006	.0006	.0006
-0.5	.2776	.2810	.2843	.2877	.2912	.2946	.2981
-0.4	.3121	.3156	.3192	.3228	.3264	.3300	.3336
-0.3	.3483	.3520	.3557	.3594	.3632	.3669	.3707
-0.2	.3859	.3897	.3936	.3974	.4013	.4052	.4090
-0.1	.4247	.4286	.4325	.4364	.4404	.4443	.4483
-0.0	.4641	.4681	.4721	.4761	.4801	.4840	.4880

The area to the left of $z = -0.24$ is 0.4052.

The Standard Normal Table

Chapter

<i>z</i>	.09	.08	.07	.06	.05	.04	.03	.02	.01	.00
− 3.4	.0002	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003
− 3.3	.0003	.0004	.0004	.0004	.0004	.0004	.0004	.0005	.0005	.0005
− 3.2	.0005	.0005	.0005	.0006	.0006	.0006	.0006	.0006	.0007	.0007
− 3.1	.0007	.0007	.0008	.0008	.0008	.0008	.0009	.0009	.0009	.0010
− 3.0	.0010	.0010	.0011	.0011	.0011	.0012	.0012	.0013	.0013	.0013
− 2.9	.0014	.0014	.0015	.0015	.0016	.0016	.0017	.0018	.0018	.0019
− 2.8	.0019	.0020	.0021	.0021	.0022	.0023	.0023	.0024	.0025	.0026
− 2.7	.0026	.0027	.0028	.0029	.0030	.0031	.0032	.0033	.0034	.0035
− 2.6	.0036	.0037	.0038	.0039	.0040	.0041	.0043	.0044	.0045	.0047
− 2.5	.0048	.0049	.0051	.0052	.0054	.0055	.0057	.0059	.0060	.0062
− 2.4	.0064	.0066	.0068	.0069	.0071	.0073	.0075	.0078	.0080	.0082
− 2.3	.0084	.0087	.0089	.0091	.0094	.0096	.0099	.0102	.0104	.0107
− 2.2	.0110	.0113	.0116	.0119	.0122	.0125	.0129	.0132	.0136	.0139
− 2.1	.0143	.0146	.0150	.0154	.0158	.0162	.0166	.0170	.0174	.0179
− 2.0	.0183	.0188	.0192	.0197	.0202	.0207	.0212	.0217	.0222	.0228
− 1.9	.0233	.0239	.0244	.0250	.0256	.0262	.0268	.0274	.0281	.0287
− 1.8	.0294	.0301	.0307	.0314	.0322	.0329	.0336	.0344	.0351	.0359
− 1.7	.0367	.0375	.0384	.0392	.0401	.0409	.0418	.0427	.0436	.0446
− 1.6	.0455	.0465	.0475	.0485	.0495	.0505	.0516	.0526	.0537	.0548
− 1.5	.0559	.0571	.0582	.0594	.0606	.0618	.0630	.0643	.0655	.0668
− 1.4	.0681	.0694	.0708	.0721	.0735	.0749	.0764	.0778	.0793	.0808
− 1.3	.0823	.0838	.0853	.0869	.0885	.0901	.0918	.0934	.0951	.0968
− 1.2	.0985	.1003	.1020	.1038	.1056	.1075	.1093	.1112	.1131	.1151
− 1.1	.1170	.1190	.1210	.1230	.1251	.1271	.1292	.1314	.1335	.1357
− 1.0	.1379	.1401	.1423	.1446	.1469	.1492	.1515	.1539	.1562	.1587
− 0.9	.1611	.1635	.1660	.1685	.1711	.1736	.1762	.1788	.1814	.1841
− 0.8	.1867	.1894	.1922	.1949	.1977	.2005	.2033	.2061	.2090	.2119
− 0.7	.2148	.2177	.2206	.2236	.2266	.2296	.2327	.2358	.2389	.2420
− 0.6	.2451	.2483	.2514	.2546	.2578	.2611	.2643	.2676	.2709	.2743
− 0.5	.2776	.2810	.2843	.2877	.2912	.2946	.2981	.3015	.3050	.3085
− 0.4	.3121	.3156	.3192	.3228	.3264	.3300	.3336	.3372	.3409	.3446
− 0.3	.3483	.3520	.3557	.3594	.3632	.3669	.3707	.3745	.3783	.3821
− 0.2	.3859	.3897	.3936	.3974	.4013	.4052	.4090	.4129	.4168	.4207
− 0.1	.4247	.4286	.4325	.4364	.4404	.4443	.4483	.4522	.4562	.4602
− 0.0	.4641	.4681	.4721	.4761	.4801	.4840	.4880	.4920	.4960	.5000

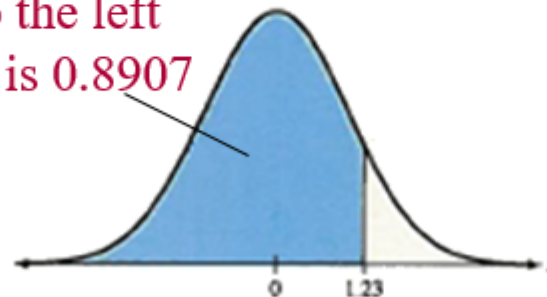
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Finding Areas Under the Standard Normal Curve

Chapter

1. Sketch the standard normal curve and shade the appropriate area under the curve.
2. Find the area by following the directions for each case shown.
 - a. To find the area to the *left* of z , find the area that corresponds to z in the Standard Normal Table.

2. The area to the left of $z = 1.23$ is 0.8907



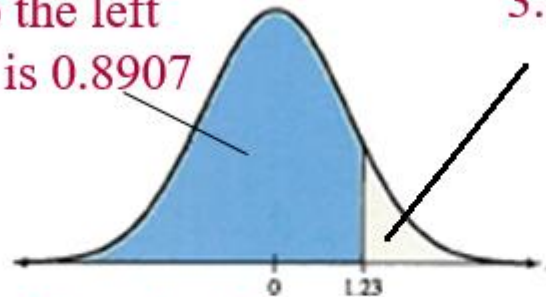
1. Use the table to find the area for the z -score

Finding Areas Under the Standard Normal Curve

To find the area to the *right* of z , use the Standard Normal Table to find the area that corresponds to z . Then subtract the area from 1.

2. The area to the left of $z = 1.23$ is 0.8907

3. Subtract to find the area to the right of $z = 1.23$:
 $1 - 0.8907 = 0.1093$.



1. Use the table to find the area for the z -score

Normal Distributions: Finding Probabilities

Chapter

If X value equal to 600, we have to transform this value to standard value by the rule to be able to use the table:

$$z = \frac{x - \mu}{\sigma} = \frac{600 - 500}{100} = 1$$

Example: Finding Probabilities for Normal Distributions

A survey indicates that people use their computers an average of 2.4 years before upgrading to a new machine. The standard deviation is 0.5 year. A computer owner is selected at random. Find the probability that he or she will use it for fewer than 2 years before upgrading. Assume that the variable x is normally distributed.

تشير إحدى الدراسات إلى أن الأشخاص يستخدمون أجهزة الكمبيوتر الخاصة بهم بمعدل 2.4 سنة قبل الترقية إلى جهاز جديد. الانحراف المعياري هو 0.5 سنة. يتم اختيار مالك الكمبيوتر بشكل عشوائي. أوجد احتمالية استخدامه لمدة أقل من عامين قبل الترقية. افترض أن المتغير X يتم توزيعه بشكل طبيعي.

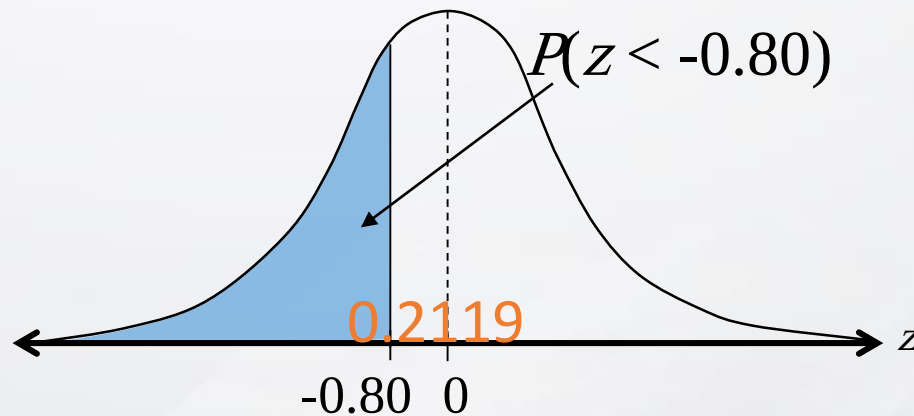
Solution:

We need to find $P(x < 2)$



Example: Finding Probabilities for Normal Distributions

$$z = \frac{x - \mu}{\sigma} = \frac{2 - 2.4}{0.5} = -0.80$$



z	.09	.08	.07	.06	.05	.04	.03	.02	.01	.00
-3.4	.0002	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003
-3.3	.0003	.0004	.0004	.0004	.0004	.0004	.0004	.0005	.0005	.0005
-3.2	.0005	.0005	.0005	.0006	.0006	.0006	.0006	.0006	.0007	.0007
-3.1	.0007	.0007	.0008	.0008	.0008	.0008	.0009	.0009	.0009	.0010
-3.0	.0010	.0010	.0011	.0011	.0011	.0012	.0012	.0013	.0013	.0013
-2.9	.0014	.0014	.0015	.0015	.0016	.0016	.0017	.0018	.0018	.0019
-2.8	.0019	.0020	.0021	.0021	.0022	.0023	.0023	.0024	.0025	.0026
-2.7	.0026	.0027	.0028	.0029	.0030	.0031	.0032	.0033	.0034	.0035
-2.6	.0035	.0036	.0037	.0038	.0039	.0040	.0041	.0042	.0043	.0044
-2.5	.0044	.0045	.0046	.0047	.0048	.0049	.0050	.0051	.0052	.0053
-2.4	.0053	.0054	.0055	.0056	.0057	.0058	.0059	.0060	.0061	.0062
-2.3	.0062	.0063	.0064	.0065	.0066	.0067	.0068	.0069	.0070	.0071
-2.2	.0071	.0072	.0073	.0074	.0075	.0076	.0077	.0078	.0079	.0080
-2.1	.0080	.0081	.0082	.0083	.0084	.0085	.0086	.0087	.0088	.0089
-2.0	.0089	.0090	.0091	.0092	.0093	.0094	.0095	.0096	.0097	.0098
-1.9	.0098	.0099	.0100	.0101	.0102	.0103	.0104	.0105	.0106	.0107
-1.8	.0107	.0108	.0109	.0110	.0111	.0112	.0113	.0114	.0115	.0116
-1.7	.0116	.0117	.0118	.0119	.0120	.0121	.0122	.0123	.0124	.0125
-1.6	.0125	.0126	.0127	.0128	.0129	.0130	.0131	.0132	.0133	.0134
-1.5	.0134	.0135	.0136	.0137	.0138	.0139	.0140	.0141	.0142	.0143
-1.4	.0143	.0144	.0145	.0146	.0147	.0148	.0149	.0150	.0151	.0152
-1.3	.0152	.0153	.0154	.0155	.0156	.0157	.0158	.0159	.0160	.0161
-1.2	.0161	.0162	.0163	.0164	.0165	.0166	.0167	.0168	.0169	.0170
-1.1	.0170	.0171	.0172	.0173	.0174	.0175	.0176	.0177	.0178	.0179
-1.0	.0179	.0180	.0181	.0182	.0183	.0184	.0185	.0186	.0187	.0188
-0.9	.0188	.0189	.0190	.0191	.0192	.0193	.0194	.0195	.0196	.0197
-0.8	.0197	.0198	.0199	.0200	.0201	.0202	.0203	.0204	.0205	.0206
-0.7	.0206	.0207	.0208	.0209	.0210	.0211	.0212	.0213	.0214	.0215
-0.6	.0215	.0216	.0217	.0218	.0219	.0220	.0221	.0222	.0223	.0224
-0.5	.0224	.0225	.0226	.0227	.0228	.0229	.0230	.0231	.0232	.0233
-0.4	.0233	.0234	.0235	.0236	.0237	.0238	.0239	.0240	.0241	.0242
-0.3	.0242	.0243	.0244	.0245	.0246	.0247	.0248	.0249	.0250	.0251
-0.2	.0251	.0252	.0253	.0254	.0255	.0256	.0257	.0258	.0259	.0260
-0.1	.0260	.0261	.0262	.0263	.0264	.0265	.0266	.0267	.0268	.0269
-0.0	.0269	.0270	.0271	.0272	.0273	.0274	.0275	.0276	.0277	.0278

$$P(X < 2) = P(z < -0.80) = 0.2119$$

Example: Finding Probabilities for Normal Distributions

A survey indicates that for each trip to the supermarket, a shopper spends an average of 45 minutes with a standard deviation of 12 minutes in the store. The length of time spent in the store is normally distributed and is represented by the variable x . A shopper enters the store. Find the probability that the shopper will be in the store for between 24 and 54 minutes.

تشير إحدى الدراسات إلى أنه في كل رحلة إلى السوبر ماركت، يقضي المتسوق ما متوسطه 45 دقيقة مع انحراف معياري قدره 12 دقيقة في المتجر. يتم توزيع طول الوقت الذي يقضيه في المتجر بشكل طبيعي ويتم تمثيله بالمتغير X يدخل أحد المتسوقين إلى المتجر. أوجد احتمال بقاء المتسوق في المتجر لمدة تتراوح بين 24 و 54 دقيقة.

Solution:

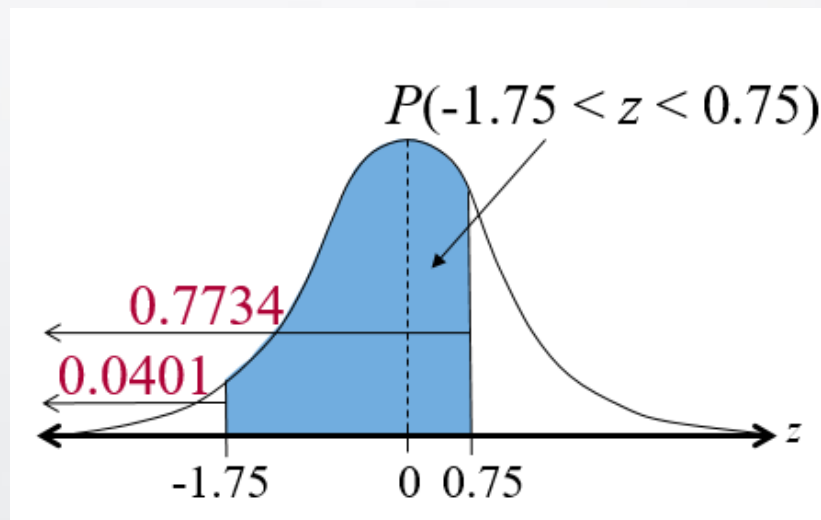
We need to find $P(24 < x < 54)$



Example: Finding Probabilities for Normal Distributions

$$z_1 = \frac{x - \mu}{\sigma} = \frac{24 - 45}{12} = -1.75$$

$$z_2 = \frac{x - \mu}{\sigma} = \frac{54 - 45}{12} = 0.75$$



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7421	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133

z	.09	.08	.07	.06	.05	.04	.03	.02	.01	.00
-3.4	.0002	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003
-3.3	.0003	.0004	.0004	.0004	.0004	.0004	.0004	.0005	.0005	.0005
-1.8	.0294	.0301	.0307	.0314	.0322	.0329	.0336	.0344	.0351	.0359
-1.7	.0367	.0375	.0384	.0392	.0401	.0409	.0418	.0427	.0436	.0446
-1.6	.0455	.0465	.0475	.0485	.0495	.0505	.0516	.0526	.0537	.0548
-1.5	.0559	.0571	.0582	.0594	.0606	.0618	.0630	.0643	.0655	.0668
-1.4	.0681	.0694	.0708	.0721	.0735	.0749	.0764	.0778	.0793	.0808
-1.3	.0823	.0838	.0853	.0869	.0885	.0901	.0918	.0934	.0951	.0968
-1.2	.0985	.1003	.1020	.1038	.1056	.1075	.1093	.1112	.1131	.1151
-1.1	.1170	.1190	.1210	.1230	.1251	.1271	.1292	.1314	.1335	.1357
-1.0	.1379	.1401	.1423	.1446	.1469	.1492	.1515	.1539	.1562	.1587
-0.9	.1611	.1635	.1660	.1685	.1711	.1736	.1762	.1788	.1814	.1841
-0.8	.1867	.1894	.1922	.1949	.1977	.2005	.2033	.2061	.2090	.2119

$$P(24 < x < 54) = P(-1.75 < z < 0.75) = 0.7734 - 0.0401 = 0.7333$$

Example: Finding Probabilities for Normal Distributions

Find the probability that the shopper will be in the store more than 39 minutes. (Recall $\mu = 45$ minutes and $\sigma = 12$ minutes)

أوجد احتمال تواجد المتسوق في المتجر لمدة تزيد عن 39 دقيقة.

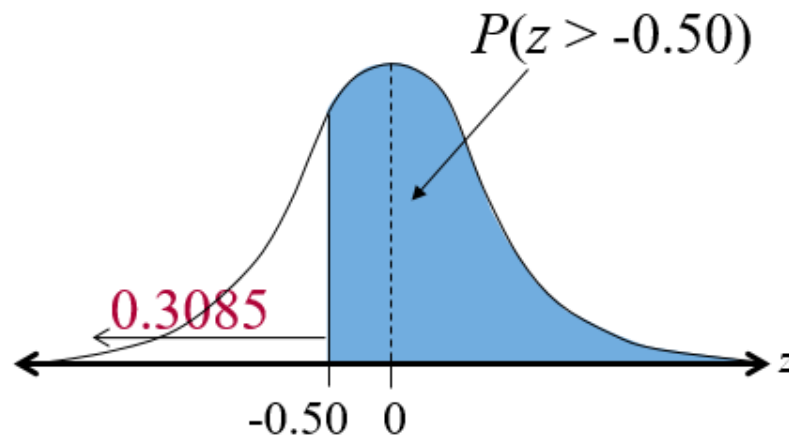
Solution:

We need to get $P(x > 39)$



Example: Finding Probabilities for Normal Distributions

$$z = \frac{x - \mu}{\sigma} = \frac{39 - 45}{12} = -0.50$$



z	.09	.08	.07	.06	.05	.04	.03	.02	.01	.00
-3.4	.0002	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003
-3.3	.0003	.0004	.0004	.0004	.0004	.0004	.0004	.0005	.0005	.0005
-3.2	.0005	.0005	.0005	.0006	.0006	.0006	.0006	.0006	.0007	.0007
-0.5	.3776	.3810	.3842	.3877	.3912	.3946	.3981	.3915	.3858	.3085
-0.4	.3121	.3156	.3192	.3228	.3264	.3300	.3336	.3372	.3409	.3446
-0.3	.3483	.3520	.3557	.3594	.3632	.3669	.3707	.3745	.3783	.3821
-0.2	.3859	.3897	.3936	.3974	.4013	.4052	.4090	.4129	.4168	.4207
-0.1	.4247	.4286	.4325	.4364	.4404	.4443	.4483	.4522	.4562	.4602
-0.0	.4641	.4681	.4721	.4761	.4801	.4840	.4880	.4920	.4960	.5000

$$P(x > 39) = P(z > -0.50) = 1 - 0.3085 = 0.6915$$

5.3 Normal Distributions: Finding Values

Chapter

- Find a z-score given the area under the normal curve
- Transform a z-score to an x -value
- Find a specific data value of a normal distribution given the probability

أوجد درجة Z بمعلومية المساحة الواقعة أسفل المنحنى الطبيعي

حول قيمة Z إلى قيمة X

أوجد قيمة بيانات محددة للتوزيع الطبيعي بمعلومية الاحتمال

Example: Finding a z-Score Given an Area

Chapter 1

Find the z-score that corresponds to a cumulative area of 0.3632.

- Locate 0.3632 in the body of the Standard Normal Table.

z	.09	.08	.07	.06	.05	.04	.03
-3.4	.0002	.0003	.0003	.0003	.0003	.0003	.0003
-3.3	.0003	.0004	.0004	.0004	.0004	.0004	.0004
-3.2	.0005	.0005	.0005	.0006	.0006	.0006	.0006
-3.1	.0006	.0006	.0006	.0007	.0007	.0007	.0007
-3.0	.0008	.0008	.0008	.0008	.0008	.0008	.0008
-2.9	.0009	.0009	.0009	.0009	.0009	.0009	.0009
-2.8	.0010	.0010	.0010	.0010	.0010	.0010	.0010
-2.7	.0011	.0011	.0011	.0011	.0011	.0011	.0011
-2.6	.0012	.0012	.0012	.0012	.0012	.0012	.0012
-2.5	.0013	.0013	.0013	.0013	.0013	.0013	.0013
-2.4	.0014	.0014	.0014	.0014	.0014	.0014	.0014
-2.3	.0015	.0015	.0015	.0015	.0015	.0015	.0015
-2.2	.0016	.0016	.0016	.0016	.0016	.0016	.0016
-2.1	.0017	.0017	.0017	.0017	.0017	.0017	.0017
-2.0	.0018	.0018	.0018	.0018	.0018	.0018	.0018
-1.9	.0019	.0019	.0019	.0019	.0019	.0019	.0019
-1.8	.0020	.0020	.0020	.0020	.0020	.0020	.0020
-1.7	.0021	.0021	.0021	.0021	.0021	.0021	.0021
-1.6	.0022	.0022	.0022	.0022	.0022	.0022	.0022
-1.5	.0023	.0023	.0023	.0023	.0023	.0023	.0023
-1.4	.0024	.0024	.0024	.0024	.0024	.0024	.0024
-1.3	.0025	.0025	.0025	.0025	.0025	.0025	.0025
-1.2	.0026	.0026	.0026	.0026	.0026	.0026	.0026
-1.1	.0027	.0027	.0027	.0027	.0027	.0027	.0027
-1.0	.0028	.0028	.0028	.0028	.0028	.0028	.0028
-0.9	.0029	.0029	.0029	.0029	.0029	.0029	.0029
-0.8	.0030	.0030	.0030	.0030	.0030	.0030	.0030
-0.7	.0031	.0031	.0031	.0031	.0031	.0031	.0031
-0.6	.0032	.0032	.0032	.0032	.0032	.0032	.0032
-0.5	.0033	.0033	.0033	.0033	.0033	.0033	.0033
-0.4	.0034	.0034	.0034	.0034	.0034	.0034	.0034
-0.3	.0035	.0035	.0035	.0035	.0035	.0035	.0035
-0.2	.0036	.0036	.0036	.0036	.0036	.0036	.0036
-0.1	.0037	.0037	.0037	.0037	.0037	.0037	.0037
-0.0	.0038	.0038	.0038	.0038	.0038	.0038	.0038

The z-score is -0.35.

- The values at the beginning of the corresponding row and at the top of the column give the z-score.

Transforming a z-Score to an x-Score

Chapter

To transform a standard z -score to a data value x in a given population, use the formula

للتحويل من z إلى x نستخدم القانون التالي

$$x = \mu + z\sigma$$

Example: Finding an x-Value

Chapter

The speeds of vehicles along a stretch of highway are normally distributed, with a mean of 67 miles per hour and a standard deviation of 4 miles per hour. Find the speeds x corresponding to z -scores of 1.96, -2.33, and 0

يتم توزيع سرعات المركبات على امتداد الطريق السريع بشكل طبيعي بمتوسط 67 ميلاً في الساعة وانحراف معياري قدره 4 أميال في الساعة.

Solution: Use the formula $x = \mu + z\sigma$

- $z = 1.96$: $x = 67 + 1.96(4) = 74.84$ miles per hour
- $z = -2.33$: $x = 67 + (-2.33)(4) = 57.68$ miles per hour
- $z = 0$: $x = 67 + 0(4) = 67$ miles per hour



Sampling Distributions and the Central Limit Theorem

Find sampling distributions and verify their properties

Interpret the Central Limit Theorem

Apply the Central Limit Theorem to find the probability of a sample mean

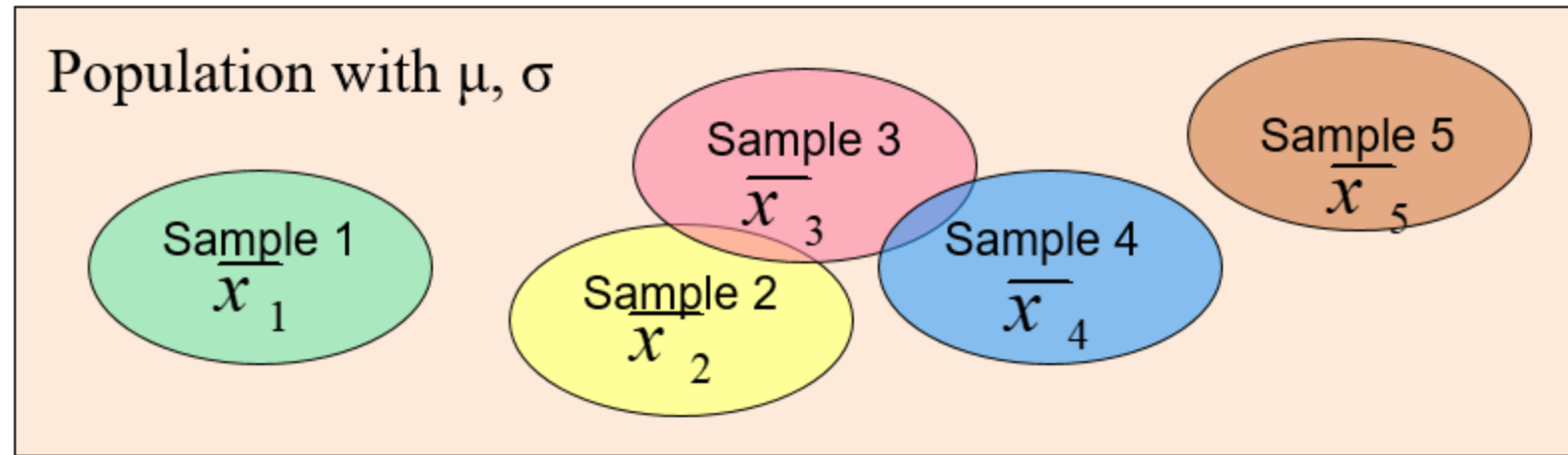
البحث عن توزيعات العينات والتحقق من خصائصها

تفسير نظرية الحد المركزي

طبّق نظرية الحد المركزي لإيجاد احتمال متوسط العينة

Sampling distribution of sample mean

Chapter



The sampling distribution consists of the values of the sample means, $\bar{x}_1, \bar{x}_2, \bar{x}_3, \bar{x}_4, \bar{x}_5, \dots$

Properties of Sampling distribution of sample mean

1. The mean of the sample means, $\mu_{\bar{x}}$, is equal to the population mean μ .

$$\mu_{\bar{x}} = \mu$$

2. The standard deviation of the sample means, $\sigma_{\bar{x}}$, is equal to the population standard deviation, σ divided by the square root of the sample size, n .

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

- Called the **standard error of the mean**.

المتوسط الحسابي للعينه هو نفس المتوسط الحسابي

للمجتمع

الانحراف المعياري للعينه يساوي الانحراف المعياري

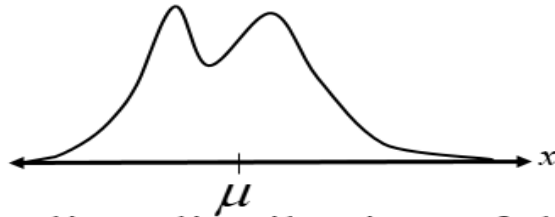
للمجتمع مقسوما على الجذر التربيعي لحجم العينة

ويعرف الخطأ القياسي للمتوسط الحسابي

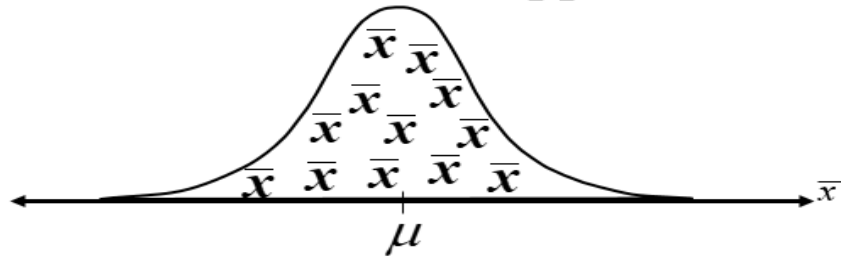
Central Limit Theorem

Chapter

1. If samples of size $n \geq 30$, are drawn from any population with mean $= \mu$ and standard deviation $= \sigma$,



then the sampling distribution of the sample means approximates a normal distribution. The greater the sample size, the better the approximation.



عندما يكون حجم العينة أكبر من أو يساوي 30

ويتم تمثيلها من أي مجتمع بمتوسط حسابي وانحراف معياري

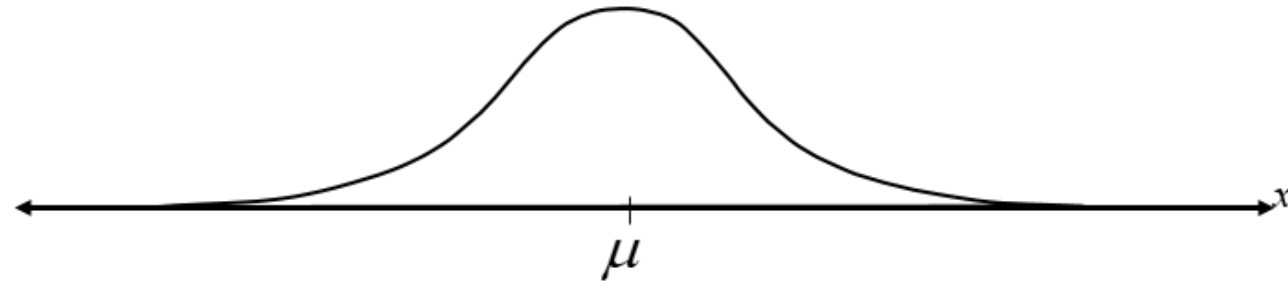
يمكن تقريب العينة إلى توزيع طبيعي وكلما زاد حجم العينة

كان التقريب أفضل

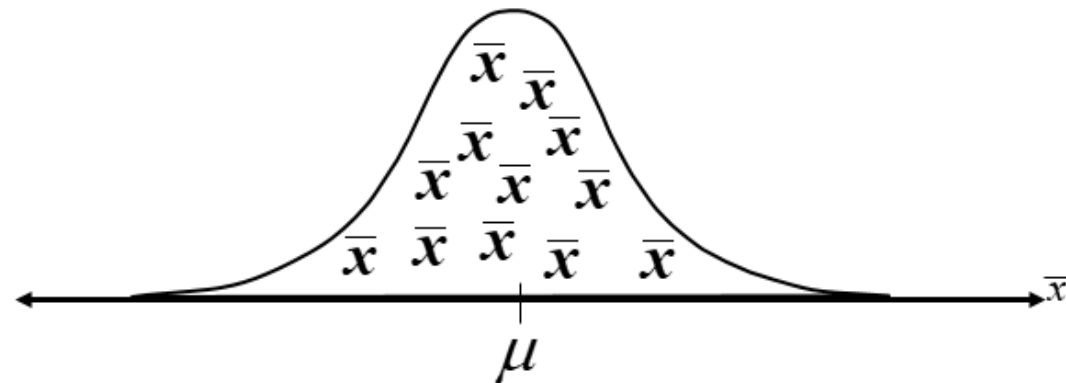
Central Limit Theorem

Chapter

2. If the population itself is normally distributed,



the sampling distribution of the sample means is normally distribution for **any** sample size n .



Central Limit Theorem

Chapter

- In either case, the sampling distribution of sample means has a mean equal to the population mean.

$$\mu_{\bar{x}} = \mu$$

- The sampling distribution of sample means has a variance equal to $1/n$ times the variance of the population and a standard deviation equal to the population standard deviation divided by the square root of n .

$$\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n} \quad \text{Variance}$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \quad \text{Standard deviation (standard error of the mean)}$$

Example: Probabilities for x and $\bar{x}$

A bank auditor claims that credit card balances are normally distributed, with a mean of \$2870 and a standard deviation of \$900.

1. What is the probability that a randomly selected credit card holder has a credit card balance less than \$2500?



Solution:

You are asked to find the probability associated with a certain value of the random variable x .

في أحد البنوك يمكن اعتبار حسابات بطاقات الائتمان

تتبع توزيع طبيعي بمتوسط حسابي 2870 دولار

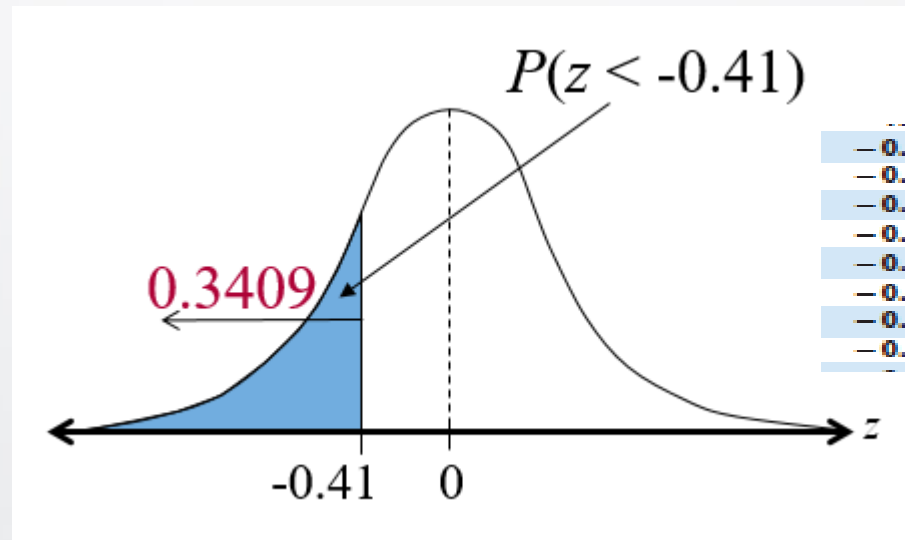
وانحراف معياري 900 دولار.

1- أوجد احتمال البطاقة المسحوبة بطريقة عشوائية بها

رصيد أقل من 2500 دولار؟

Solution: Probabilities for x and $\bar{x}$

$$z = \frac{x - \mu}{\sigma} = \frac{2500 - 2870}{900} \approx -0.41$$



-0.9	.1611	.1635	.1660	.1685	.1711	.1736	.1762	.1788	.1814	.1841
-0.8	.1867	.1894	.1922	.1949	.1977	.2005	.2033	.2061	.2090	.2119
-0.7	.2148	.2177	.2206	.2236	.2266	.2296	.2327	.2358	.2389	.2420
-0.6	.2451	.2483	.2514	.2546	.2578	.2611	.2643	.2676	.2709	.2743
-0.5	.2776	.2810	.2843	.2877	.2912	.2946	.2981	.3015	.3050	.3085
-0.4	.3121	.3156	.3192	.3228	.3264	.3300	.3336	.3372	.3409	.3446
-0.3	.3483	.3520	.3557	.3594	.3632	.3669	.3707	.3745	.3783	.3821
-0.2	.3859	.3897	.3936	.3974	.4013	.4052	.4090	.4129	.4168	.4207

- $P(x < 2500) = P(z < -0.41) = 0.3409$

Example: Probabilities for x and $\bar{x}$

2. You randomly select 25 credit card holders. What is the probability that their mean credit card balance is less than \$2500?



Solution:

You are asked to find the probability associated with a sample mean $\bar{x}$.

$$\mu = 2870 \qquad \frac{\sigma}{\sqrt{n}} = \frac{900}{\sqrt{25}} = 180$$

2- تم اختيار 25 بطاقة بطريقة عشوائية، أوجد احتمال

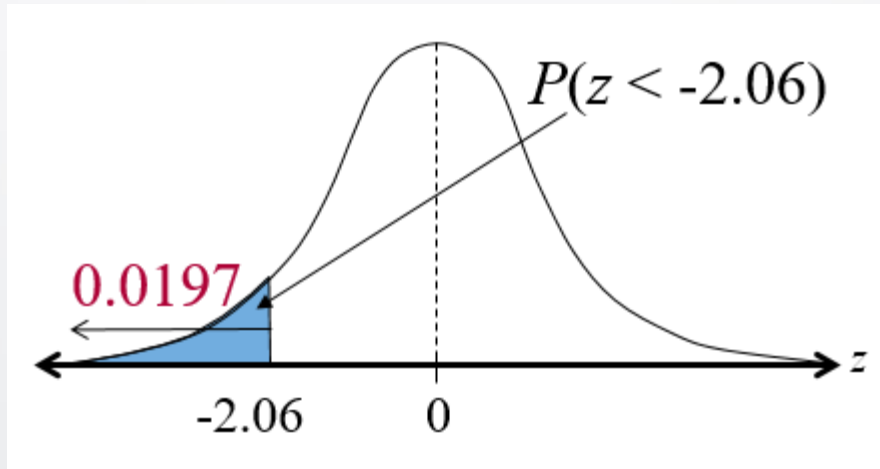
أن يكون رصيد البطاقات المسحوبة بطريقة عشوائية

أقل من 2500 دولار؟

Example: Probabilities for x and $\bar{x}$

Chapter

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{2500 - 2870}{\frac{900}{\sqrt{25}}} \approx -2.06$$



z	.09	.08	.07	.06	.05
-3.4	.0002	.0003	.0003	.0003	.0003
-3.3	.0003	.0004	.0004	.0004	.0004
-3.2	.0005	.0005	.0005	.0006	.0006
-3.1	.0007	.0007	.0008	.0008	.0008
-3.0	.0010	.0010	.0011	.0011	.0011
-2.9	.0014	.0014	.0015	.0015	.0016
-2.8	.0019	.0020	.0021	.0021	.0022
-2.7	.0026	.0027	.0028	.0029	.0030
-2.6	.0036	.0037	.0038	.0039	.0040
-2.5	.0048	.0049	.0051	.0052	.0054
-2.4	.0064	.0066	.0068	.0069	.0071
-2.3	.0084	.0087	.0089	.0091	.0094
-2.2	.0110	.0113	.0116	.0119	.0122
-2.1	.0143	.0146	.0150	.0154	.0158
-2.0	.0183	.0188	.0192	.0197	.0202
-1.9	.0233	.0239	.0244	.0250	.0256
-1.8	.0294	.0301	.0307	.0314	.0322

$$P(x < 2500) = P(z < -2.06) = 0.0197$$

5.5 Normal Approximations to Binomial

Chapter
Distributions

Determine when the normal distribution can approximate the binomial distribution

حدد متى يمكن أن يقترب التوزيع الطبيعي من التوزيع ذي الحدين

Use the normal distribution to approximate binomial probabilities

استخدم التوزيع الطبيعي لتقريب الاحتمالات ذات الحدين

Normal Approximation to a Binomial

Chapter

- The normal distribution is used to approximate the binomial distribution when it would be impractical to use the binomial distribution to find a probability.

Normal Approximation to a Binomial Distribution

- If $np \geq 5$ and $nq \geq 5$, then the binomial random variable x is approximately normally distributed with
 - mean $\mu = np$
 - standard deviation $\sigma = \sqrt{npq}$

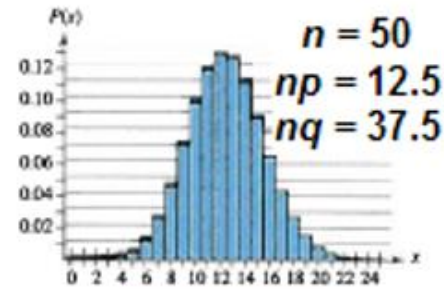
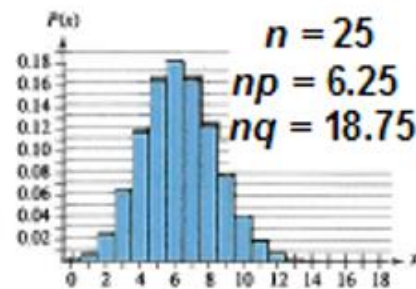
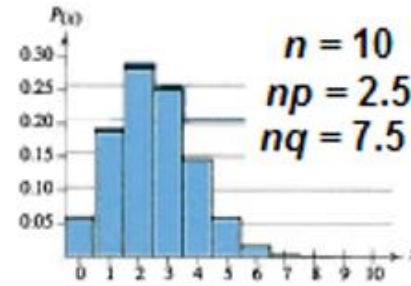
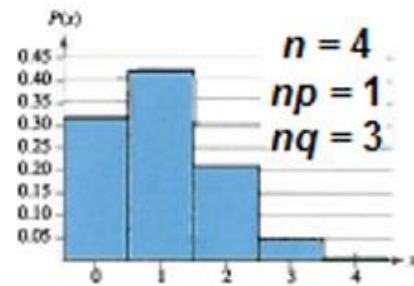
توزيع ذي الحدين يمكن تقريبه إلى توزيع طبيعي

واعتباره توزيعا طبيعيا بتحقيق شرطين

Normal Approximation to a Binomial

Chapter

- Binomial distribution: $p = 0.25$



- As n increases the histogram approaches a normal curve.

Normal Approximation to a Binomial

Chapter

- The normal distribution is used to approximate the binomial distribution when it would be impractical to use the binomial distribution to find a probability.

Normal Approximation to a Binomial Distribution

- If $np \geq 5$ and $nq \geq 5$, then the binomial random variable x is approximately normally distributed with
 - mean $\mu = np$
 - standard deviation $\sigma = \sqrt{npq}$

Approximating the Binomial

Chapter

Decide whether you can use the normal distribution to approximate x , the number of people who reply yes. If you can, find the mean and standard deviation.

1. Fifty-one percent of adults in the U.S. whose New Year's resolution was to exercise more achieved their resolution. You randomly select 65 adults in the U.S. whose resolution was to exercise more and ask each if he or she achieved that resolution.

حدد ما إذا كان يمكن اعتبار توزيع طبيعي أم لا؟

51 % من الشباب في الولايات المتحدة أجابوا نعم

تم اختيار 65 شخص بطريقة عشوائية

Solution: Approximating the Binomial

Chapter

- You can use the normal approximation

$$n = 65, \quad p = 0.51, \quad q = 0.49$$

$$np = (65)(0.51) = 33.15 \geq 5$$

$$nq = (65)(0.49) = 31.85 \geq 5$$

- Mean: $\mu = np = 33.15$
- Standard Deviation: $\sigma = \sqrt{npq} = \sqrt{65 \cdot 0.51 \cdot 0.49} \approx 4.03$

Chapter Summary

- Found probabilities for normally distributed
- Found a z -score given the area under the normal curve
- Transformed a z -score to an x -value
- Interpreted the Central Limit Theorem
- Used the normal distribution to approximate binomial probabilities

تم العثور على احتمالات التوزيع الطبيعي
 تم العثور على درجة z بالنظر إلى المنطقة الواقعة
 تحت المنحنى الطبيعي
 حول قيمة z إلى قيمة x
 تفسير نظرية الحد المركزي
 استخدم التوزيع الطبيعي لتقريب الاحتمالات ذي الحدين

Un Solved Examples

Chapter

In Normal Distribution If $X = 700$, $\mu = 600$ and $\sigma = 100$ then $Z = ?$

في التوزيع الطبيعي إذا كان $X = 700$, $\mu = 600$ and $\sigma = 100$ فإن قيمة $Z = ?$

Un Solved Examples

Chapter

In Normal Distribution If $X = 24$, $\mu = 45$ and $\sigma = 12$ then $Z = -1.75$

في التوزيع الطبيعي إذا كان $X = 24$, $\mu = 45$ and $\sigma = 12$ فإن قيمة $Z = -1.75$

Un Solved Examples

Chapter

In Normal Distribution If $Z = -2.33$, $\mu = 67$ and $\sigma = 4$ then $X = 10$

في التوزيع الطبيعي إذا كان $Z = -2.33$, $\mu = 67$ and $\sigma = 4$ فإن قيمة $X = 10$

Un Solved Examples

Chapter

If in Binomial distribution $n = 4$, $p = 0.25$, $q = 0.75$ then we can approximate it to normaly disputation

إذا كان في توزيع ذي الحدين $n = 4$, $p = 0.25$, $q = 0.75$ فإنه يمكن تقريبه إلى توزيع طبيعي



Thank You