

Computer Vision, and Digital Image Processing

Prepared by Dr. Mohamed Mostafa

AI 612

Lecture 8



The background features a series of vertical panels with rounded ends. The first panel shows hands typing on a laptop. The second panel shows a graduation cap icon. The third panel shows a play button icon. The fourth panel shows a target icon. The fifth panel shows an open book icon. The sixth panel shows a magnifying glass icon. The seventh panel shows a group of people icon. The eighth panel shows a bar chart icon. The ninth panel shows a line graph icon. The tenth panel shows a pie chart icon. The eleventh panel shows a scatter plot icon. The twelfth panel shows a line graph icon. The thirteenth panel shows a bar chart icon. The fourteenth panel shows a line graph icon. The fifteenth panel shows a bar chart icon. The sixteenth panel shows a line graph icon. The seventeenth panel shows a bar chart icon. The eighteenth panel shows a line graph icon. The nineteenth panel shows a bar chart icon. The twentieth panel shows a line graph icon. The twenty-first panel shows a bar chart icon. The twenty-second panel shows a line graph icon. The twenty-third panel shows a bar chart icon. The twenty-four panel shows a line graph icon. The twenty-fifth panel shows a bar chart icon. The twenty-sixth panel shows a line graph icon. The twenty-seventh panel shows a bar chart icon. The twenty-eighth panel shows a line graph icon. The twenty-ninth panel shows a bar chart icon. The thirtieth panel shows a line graph icon. The thirty-first panel shows a bar chart icon. The thirty-second panel shows a line graph icon. The thirty-third panel shows a bar chart icon. The thirty-fourth panel shows a line graph icon. The thirty-fifth panel shows a bar chart icon. The thirty-sixth panel shows a line graph icon. The thirty-seventh panel shows a bar chart icon. The thirty-eighth panel shows a line graph icon. The thirty-ninth panel shows a bar chart icon. The fortieth panel shows a line graph icon. The forty-first panel shows a bar chart icon. The forty-second panel shows a line graph icon. The forty-third panel shows a bar chart icon. The forty-fourth panel shows a line graph icon. The forty-fifth panel shows a bar chart icon. The forty-sixth panel shows a line graph icon. The forty-seventh panel shows a bar chart icon. The forty-eighth panel shows a line graph icon. The forty-ninth panel shows a bar chart icon. The fiftieth panel shows a line graph icon. The fifty-first panel shows a bar chart icon. The fifty-second panel shows a line graph icon. The fifty-third panel shows a bar chart icon. The fifty-fourth panel shows a line graph icon. The fifty-fifth panel shows a bar chart icon. The fifty-sixth panel shows a line graph icon. The fifty-seventh panel shows a bar chart icon. The fifty-eighth panel shows a line graph icon. The fifty-ninth panel shows a bar chart icon. The sixtieth panel shows a line graph icon. The sixty-first panel shows a bar chart icon. The sixty-second panel shows a line graph icon. The sixty-third panel shows a bar chart icon. The sixty-fourth panel shows a line graph icon. The sixty-fifth panel shows a bar chart icon. The sixty-sixth panel shows a line graph icon. The sixty-seventh panel shows a bar chart icon. The sixty-eighth panel shows a line graph icon. The sixty-ninth panel shows a bar chart icon. The seventieth panel shows a line graph icon. The seventy-first panel shows a bar chart icon. The seventy-second panel shows a line graph icon. The seventy-third panel shows a bar chart icon. The seventy-fourth panel shows a line graph icon. The seventy-fifth panel shows a bar chart icon. The seventy-sixth panel shows a line graph icon. The seventy-seventh panel shows a bar chart icon. The seventy-eighth panel shows a line graph icon. The seventy-ninth panel shows a bar chart icon. The eightieth panel shows a line graph icon. The eighty-first panel shows a bar chart icon. The eighty-second panel shows a line graph icon. The eighty-third panel shows a bar chart icon. The eighty-fourth panel shows a line graph icon. The eighty-fifth panel shows a bar chart icon. The eighty-sixth panel shows a line graph icon. The eighty-seventh panel shows a bar chart icon. The eighty-eighth panel shows a line graph icon. The eighty-ninth panel shows a bar chart icon. The ninetieth panel shows a line graph icon. The ninety-first panel shows a bar chart icon. The ninety-second panel shows a line graph icon. The ninety-third panel shows a bar chart icon. The ninety-fourth panel shows a line graph icon. The ninety-fifth panel shows a bar chart icon. The ninety-sixth panel shows a line graph icon. The ninety-seventh panel shows a bar chart icon. The ninety-eighth panel shows a line graph icon. The ninety-ninth panel shows a bar chart icon. The hundredth panel shows a line graph icon.

Image Morphology

Lecture 8

AI 612

Prepared by
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Chapter
09

Image Morphology

Resources for this
lecture

Chapter 9:

Gonzalez, Rafael C. *Digital image processing*.
Pearson education, India, 2009.



What is Morphology?

- In nature, morphology refers to the study of the form and structure of organisms.
- It involves the examination and analysis of physical characteristics, such as shape, size, color, and texture, as well as internal structures like organs, tissues, and cells.

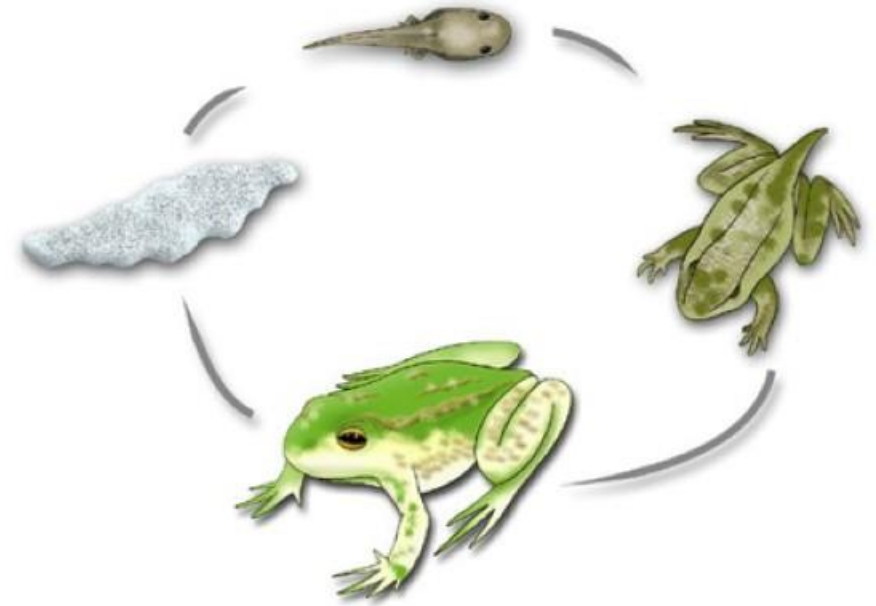
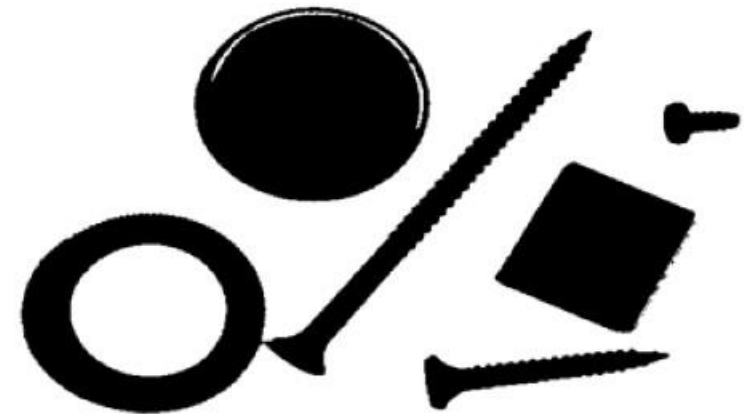
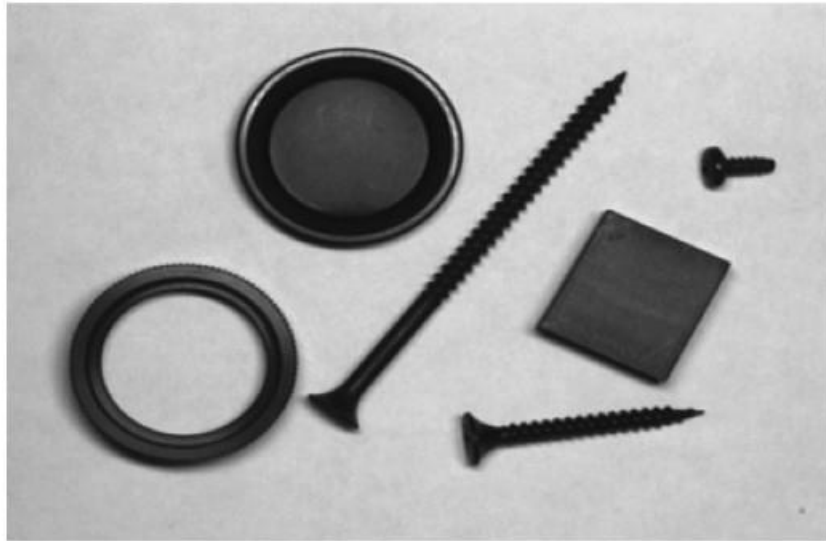


Image Morphology

- We use the same word here in the context of *mathematical morphology* as a tool for extracting image components that are useful in the representation and description of region shape, such as boundaries, skeletons, and the convex hull.
- We are interested also in morphological techniques for pre- or postprocessing, such as morphological filtering, thinning, and pruning.
- The material in the previous lectures begin a transition from methods whose inputs and outputs are images, to methods whose outputs are image attributes, for tasks such as object extraction and description.

Image Morphology

- Morphological operations are typically applied to remove imperfections introduced during segmentation, and so typically operate on bi-level images



Sets (Z^2 and Z^3)

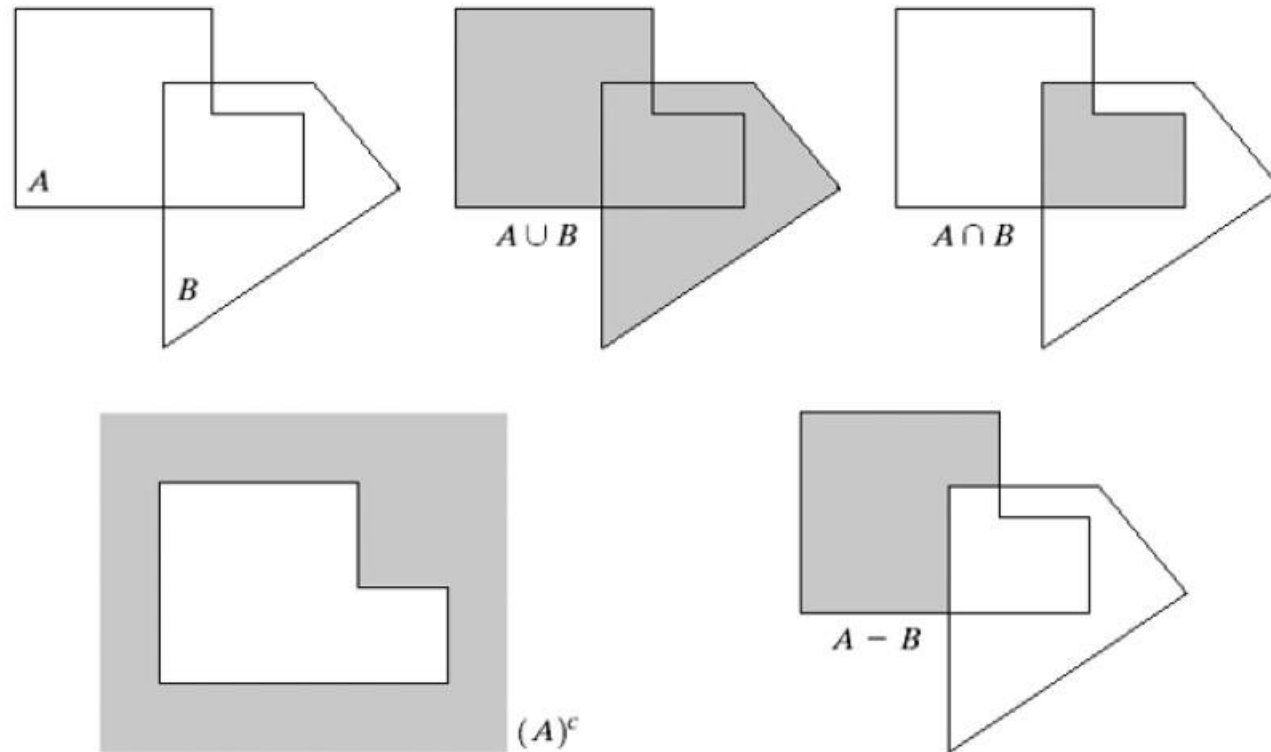
- The morphology is based on the Set Theory.
- In binary images (0 = white, 1 = black), the sets are members of the 2-D integer space Z^2 .

$$F(x,y)$$

- Grayscale digital images can be represented as sets whose components are in Z^3 .

$$F(x,y,i), \text{ where } i \text{ is the "intensity"}$$

Basic Set Theory

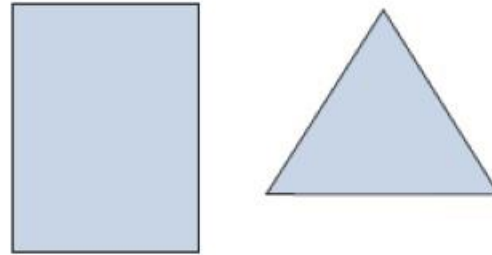


a	b	c
d	e	

FIGURE 9.1

(a) Two sets A and B . (b) The union of A and B . (c) The intersection of A and B . (d) The complement of A . (e) The difference between A and B .

Image Morphology Mathematics



Objects represented
as sets



Structuring element
represented as a set

Reflection and Translation

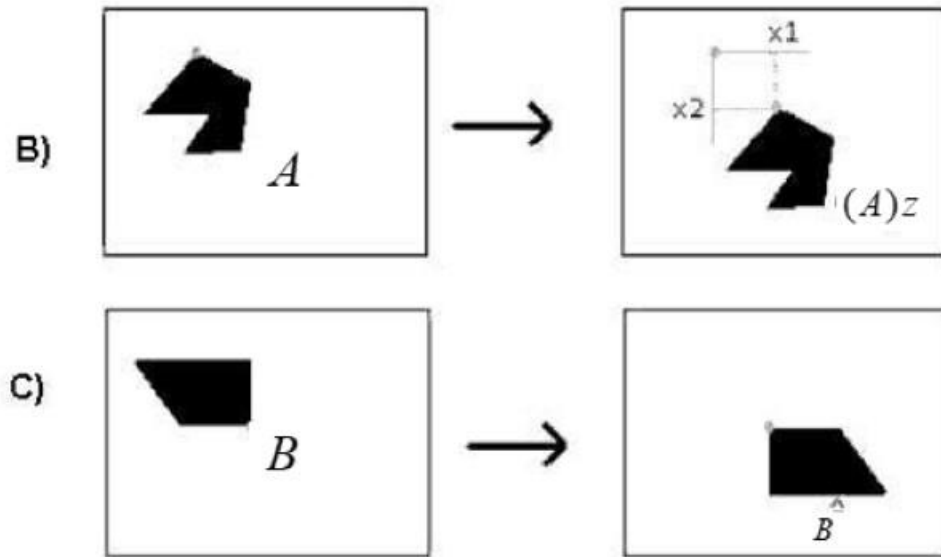


Image Source: [Researchgate](#)

$$(A)_z = \{c \mid c \in a + z, \text{ for } a \in A\}$$

$$\hat{B} = \{w \mid w \in -b, \text{ for } b \in B\}$$

Structuring element (SE)

- a shape mask used in basic morphological operations.
- it can be of any size with an origin.
- Unlike special kernels, SE doesn't required numerical values.

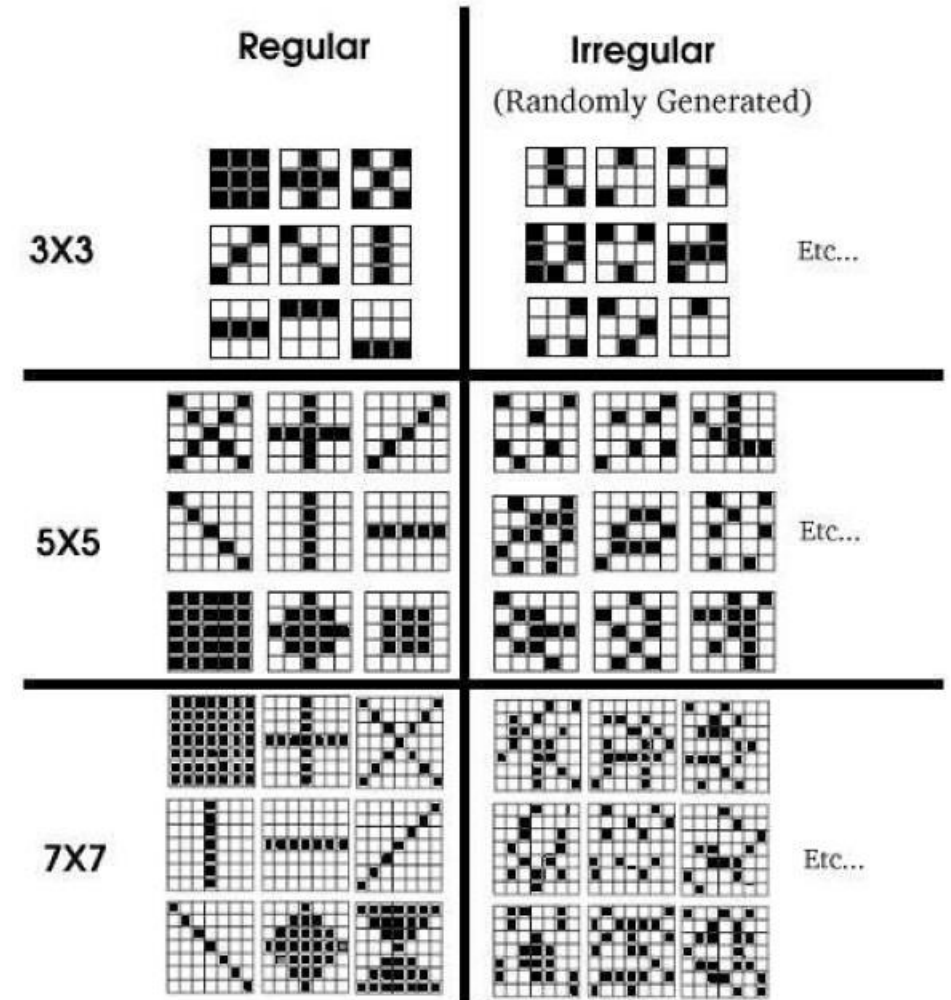
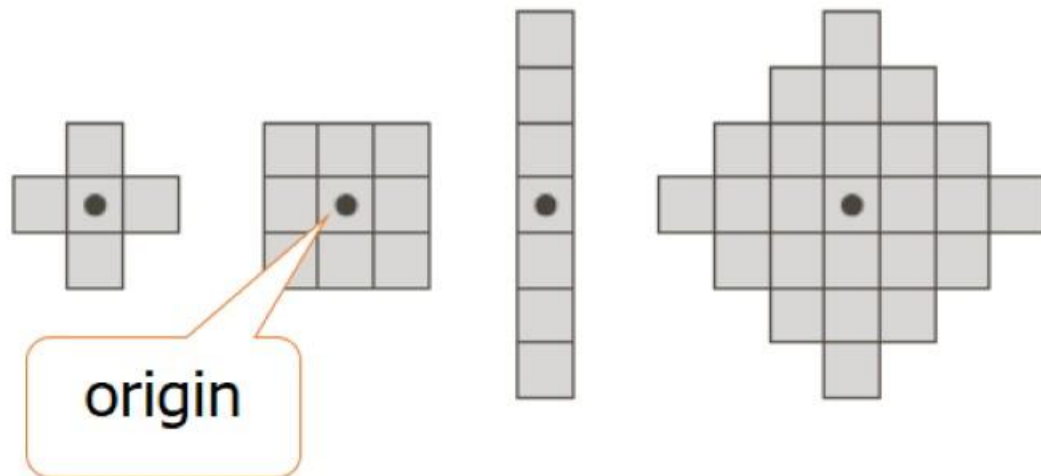


Image Source: [Researchgate](#)

Example

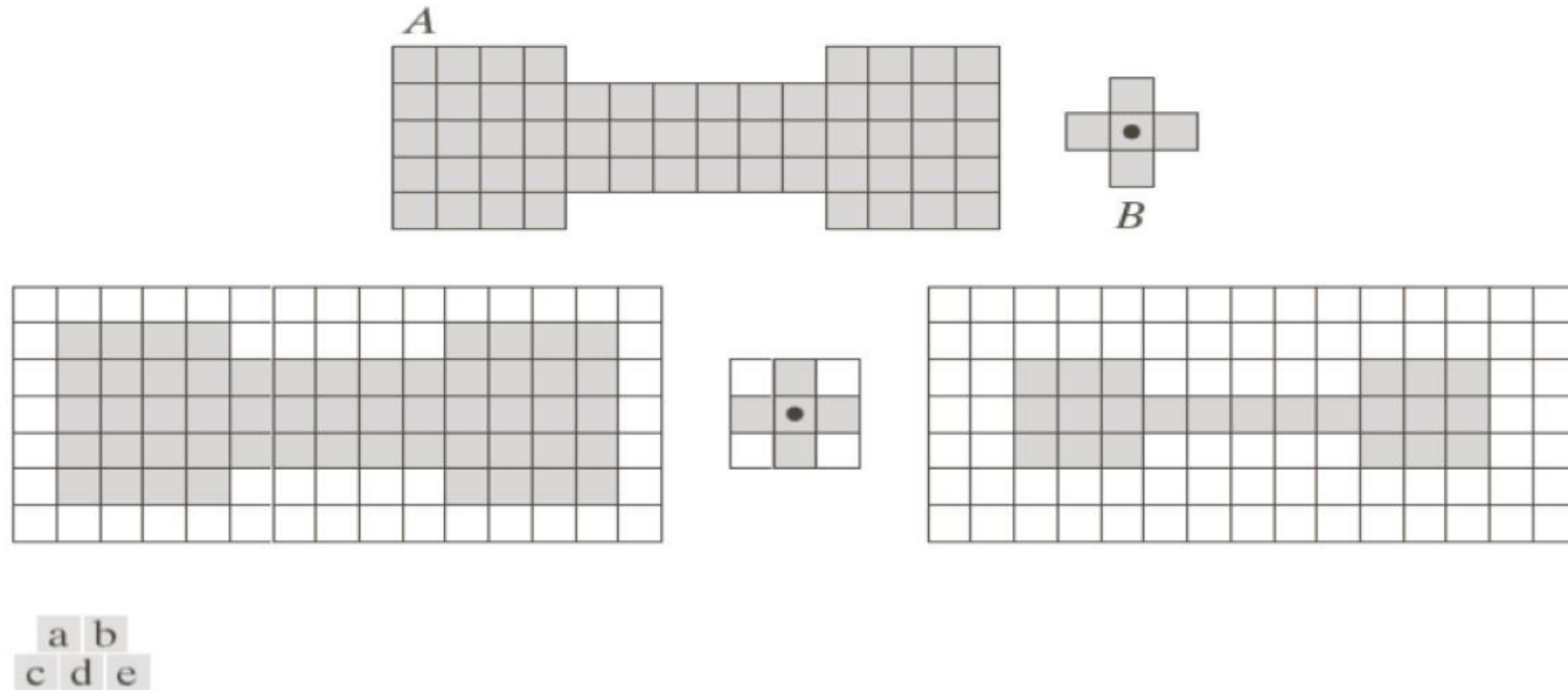


FIGURE 9.3 (a) A set (each shaded square is a member of the set). (b) A structuring element. (c) The set padded with background elements to form a rectangular array and provide a background border. (d) Structuring element as a rectangular array. (e) Set processed by the structuring element.

DILATION

With A and B as sets in Z^2 , the dilation of A by B , denoted $A \oplus B$, is defined as

$$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \emptyset\}$$

The set of all displacements z , the translated $\hat{B}$ and A overlap by at least one element.

$$A \oplus B = \{z \mid [(\hat{B})_z \cap A] \subseteq A\}$$

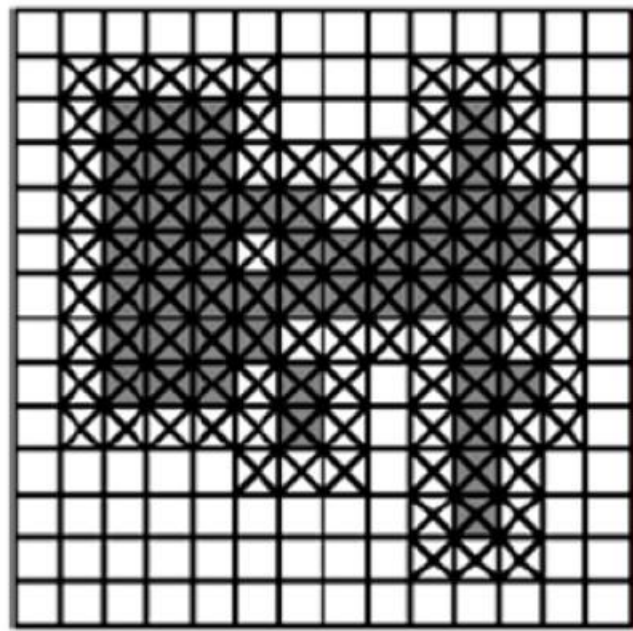
DILATION

- Move the SE B over A, Like the convolution the origin over each pixel
- OR-ing the SE with the pixels in A
- The element is to increase (reflection) by the amount of elements above the origin pixel.

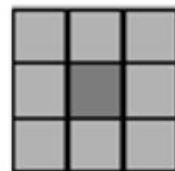
it is used for:

- expanding shapes
- filling small holes and gaps

The number of reflections is calculated by the number of rows and columns around the origin



SE=



Dilation Example

0	0	0	0	0	0
0	0	0	0	0	0
0	0	1	1	0	0
0	0	1	1	0	0
0	0	0	0	0	0
0	0	0	0	0	0

Image

1	1	1
1	1	1
1	1	1

SE

Image

[illegible]

1	1	1					
1	0	1	0	0	0	0	0
1	0	1	0	0	0	0	0
	0	0	1	1	0	0	
	0	0	1	1	0	0	
	0	0	0	0	0	0	
	0	0	0	0	0	0	

Image

[illegible]

Image

[illegible]

Dilation Example

1	0	1	0	1	0	0	0	0	0
1	0	1	0	1	0	0	0	0	0
1	0	1	0	1	1	1	0	0	0
	0	0	1	1	0	0			
	0	0	0	0	0	0			
	0	0	0	0	0	0			

Image

0	0	0	0	0	0
0					

Dilation Example

0	1	0	1	0	1	0	0	0	0
0	1	0	1	0	1	0	0	0	0
0	1	0	1	1	1	1	0	0	0
0	0	1	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

Image

0	0	0	0	0	0
0					

Dilation Example

0	1	0	1	0	1	0	0	0	0
0	1	0	1	0	1	0	0	0	0
0	1	0	1	1	1	1	0	0	0
0	0	1	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

Image

0	0	0	0	0	0
0	1				

Dilation Example

0	0	1	0	1	0	1	0	0
0	0	1	0	1	0	1	0	0
0	0	1	1	1	1	1	0	0
0	0	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

Image

0	0	0	0	0	0
0	1	1			

Dilation Example

0	0	0	1	0	1	0	1	0
0	0	0	1	0	1	0	1	0
0	0	1	1	1	1	0	1	0
0	0	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

Image

0	0	0	0	0	0
0	1	1	1		

Dilation Example

0	0	0	1	1	1
0	0	0	1	1	1
0	0	1	1	1	1
0	0	1	1	0	0
0	0	0	0	0	0
0	0	0	0	0	0

Image

0	0	0	0	0	0
0	1	1	1	1	

Dilation Example

0	0	0	0	1 ⁰	1 ⁰	1
0	0	0	0	1 ⁰	1 ⁰	1
0	0	1	1	1 ⁰	1 ⁰	1
0	0	1	1	0	0	
0	0	0	0	0	0	
0	0	0	0	0	0	

Image

0	0	0	0	0	0
0	1	1	1	1	0

Dilation Example

0	0	0	0	0	0
0	1	0	1	0	0
0	1	0	1	1	0
0	1	0	1	1	0
0	0	0	0	0	0
0	0	0	0	0	0

Image

0	0	0	0	0	0
0	1	1	1	1	0
0	1				

Dilation Example

0	0	0	0	0	0
0	0	1	0	1	0
0	0	1	1	1	0
0	0	1	1	1	0
0	0	0	0	0	0
0	0	0	0	0	0

Image

0	0	0	0	0	0
0	1	1	1	1	0
0	1	1			

Dilation Example

0	0	0	0	0	0
0	0	0	0	0	0
0	0	1	1	0	0
0	0	1	1	0	0
0	0	0	0	0	1
0	0	0	0	0	1
Image				1	1
				1	1

0	0	0	0	0	0
0	1	1	1	1	0
0	1	1	1	1	0
0	1	1	1	1	0
0	1	1	1	1	0
0	0	0	0	0	0

Dilation Example

0	0	0	0	0	0
0	0	0	0	0	0
0	0	1	1	0	0
0	0	1	1	0	0
0	0	0	0	0	0
0	0	0	0	0	0

Image

0	1	0
1	1	1
0	1	0

SE

0	0	0	0	0	0
0	0	1	1	0	0
0	1	1	1	1	0
0	1	1	1	1	0
0	0	1	1	0	0
0	0	0	0	0	0

Dilation Example

0	0	0	0	0	0
0	0	0	0	0	0
0	0	1	1	0	0
0	0	1	1	0	0
0	0	0	0	0	0
0	0	0	0	0	0

Image

1	1	1
1	1	1
1	1	1

SE

1	1	0	0	0	0	0	0
1	1	0	0	0	0	0	0
1	1	0	1	1	0	0	0
		0	0	1	1	0	0
		0	0	0	0	0	0
		0	0	0	0	0	0

Image

[illegible]

1	1	0	0	0	0	0	0
1	1	1	0	0	0	0	0
1	1	1	0	1	1	0	0
		0	0	1	1	0	0
		0	0	0	0	0	0
		0	0	0	0	0	0

Image

[illegible]

1	0	1	0	1	0	0	0	0
1	0	1	0	1	0	0	0	0
1	0	1	0	1	1	1	0	0
0	0	0	1	1	0	0		
0	0	0	0	0	0	0		
0	0	0	0	0	0	0		

Image

[illegible]

Dilation Example

0	0	1	0	1	0	1	0	0
0	0	1	0	1	0	1	0	0
0	0	1	1	1	1	0	0	0
0	0	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

Image

0	0	1	1		

Dilation Example

0	0	0	0 ¹	0 ¹	0 ¹
0	0	0	0 ¹	0 ¹	0 ¹
0	0	1	1 ¹	0 ¹	0 ¹
0	0	1	1	0	0
0	0	0	0	0	0
0	0	0	0	0	0

Image

0	0	1	1	1	1

Dilation Example

			0	0	0	0	0	0
1	1	0	1	0	0	0	0	0
1	1	0	1	0	1	1	0	0
1	1	0	1	0	1	1	0	0
			0	0	0	0	0	0
			0	0	0	0	0	0

Image

0	0	1	1	1	1
0					

Dilation Example

		0	0	0	0	0	0
1	0	1	0	1	0	0	0
1	0	1	0	1	1	1	0
1	0	1	0	1	1	0	0
		0	0	0	0	0	0
		0	0	0	0	0	0

Image

0	0	1	1	1	1
0	0				

Dilation Example

0	0	0	0	0	0
0	1	0	1	0	0
0	1	0	1	1	0
0	1	0	1	1	0
0	0	0	0	0	0
0	0	0	0	0	0

Image

0	0	1	1	1	1
0	0	1			

Dilation Example

0	0	0	0	0	0
0	0	0	01	01	01
0	0	1	11	01	01
0	0	1	11	01	01
0	0	0	0	0	0
0	0	0	0	0	0

Image

0	0	1	1	1	1
0	0	1	1	1	1

Dilation Example

0	0	0	0	0	0
0	0	0	0	0	0
0	0	1	1	0	0
0	0	1	1	0	0
0	0	01	01	01	0
0	0	01	01	01	0
		Image 1	1	1	

0	0	1	1	1	1
0	0	1	1	1	1
0	0	1	1	1	1
0	0	1	1	1	1
0	0	0	0	0	

Image

0	0	1	1	1	1
0	0	1	1	1	1
0	0	1	1	1	1
0	0	1	1	1	1
0	0	0	0	0	0
0	0	0	0	0	0

Dilation Example

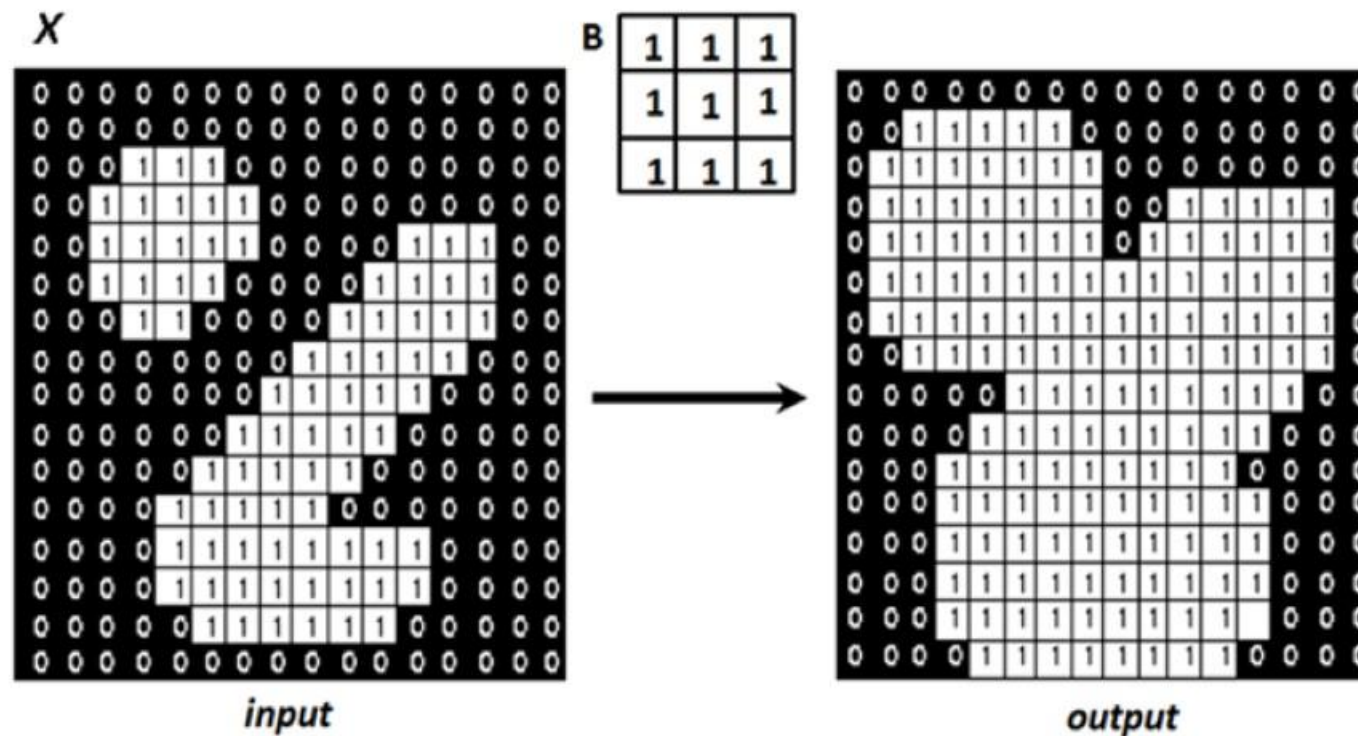
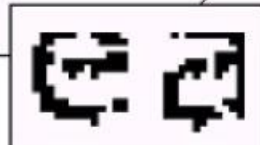


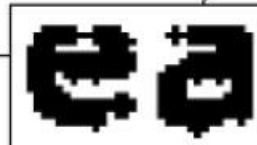
Figure 1. Effect of *dilation* using a 3X3 square structural element B.

Dilation Example

Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.



Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.



0	1	0
1	1	1
0	1	0

a c
b

FIGURE 9.5

(a) Sample text of poor resolution with broken characters (magnified view).
(b) Structuring element.
(c) Dilation of (a) by (b). Broken segments were joined.

Erosion

- Move the SE B over A, Like the convolution the origin over each pixel
- AND-ing the SE with the pixels in A

Erosion used for:

- enlarge the holes
- shrink object

The amount of shrinking is subject to the SE size

$$A \ominus B = \{z \mid (B)_z \subseteq A\}$$

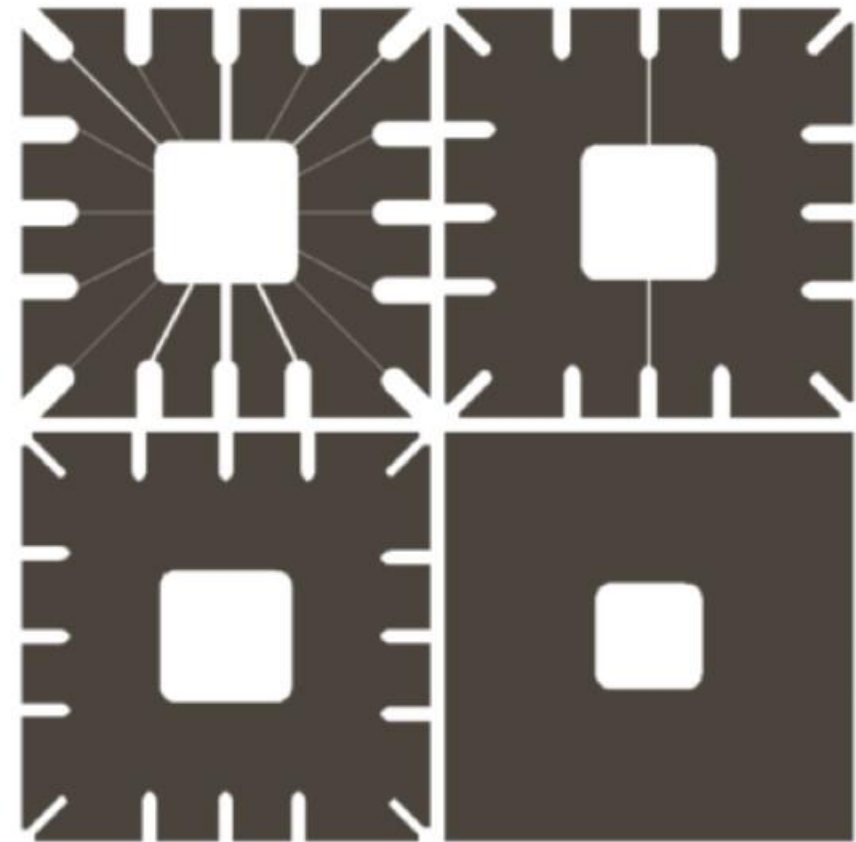


FIGURE 9.5 Using erosion to remove image components. (a) A 486×486 binary image of a wire-bond mask. (b)–(d) Image eroded using square structuring elements of sizes 11×11 , 15×15 , and 45×45 , respectively. The elements of the SEs were all 1s.

Erosion Example

0	0	0	0	0	0
0	0	0	0	0	0
0	0	1	1	0	0
0	0	1	1	0	0
0	0	1	1	0	0
0	0	0	0	0	0

Image

0	1	0
0	1	0
0	1	0

SE

Erosion Example

0	0	0	0	0	0
0	0	0	0	0	0
0	0	1	1	0	0
0	0	1	1	0	0
0	0	1	1	0	0
0	0	0	0	0	0

Image

0	1	0
0	1	0
0	1	0

SE

0	1	0					
0	0	1	0	0	0	0	0
0	0	1	0	0	0	0	0
	0	0	1	1	0	0	
	0	0	1	1	0	0	
	0	0	0	0	0	0	

Image

[illegible]

0	1	0					
0	0	1	0	0	0	0	0
0	0	1	0	0	0	0	0
	0	0	1	1	0	0	
	0	0	1	1	0	0	
	0	0	0	0	0	0	

Image

[illegible]

Erosion Example

0	1	0	0	0	0
0	1	0	0	0	0
0	1	0	1	0	0
0	0	1	1	0	0
0	0	1	1	0	0
0	0	0	0	0	0

Image

0	0	0	0	0	0
0	0	0	0	0	0

Erosion Example

0	0	0	0	0	0
0	0	0	0	0	0
0	0	1	0	0	0
0	0	1	0	0	0
0	0	1	0	0	0
0	0	0	0	0	0

Image

0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	1			

Erosion Example

0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	1	1	0
0	0	0	1	1	0
0	0	0	1	1	0
0	0	0	0	0	0

Image

0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	1	1		

Erosion Example

0	0	0	0	0	0
0	0	0	0	0	0
0	0	1	0	1	0
0	0	1	0	1	0
0	0	1	0	1	0
0	0	0	0	0	0

Image

0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	1	1	0	

Erosion Example

0	0	0	0	0	0
0	0	0	0	0	0
0	0	1	1	0	0
0	0	1	1	0	0
0	0	1	1	0	0
0	0	1	1	0	0
0	0	1	1	0	0

Image

0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	1	1	0	0
0	0	0			

Image

0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	1	1	0	0
0	0	0	0	0	0
0	0	0	0	0	0

Erosion Example

0	0	0	0	0	0
0	0	1	1	0	0
0	0	1	1	0	0
0	1	1	1	1	0
0	1	1	1	1	0
0	0	0	0	0	0

Image

1	1	1
---	---	---

SE

Image

[illegible]

Erosion Example

0	0	0	0	0	0
0	0	11	11	01	0
0	0	1	1	0	0
0	1	1	1	1	0
0	1	1	1	1	0
0	0	0	0	0	0

Image

0	0	0	0	0	0
0	0	0			

Erosion Example

0	0	0	0	0	0
0	0	1	1	0	0
0	0	1	1	0	0
0	1	1	1	1	0
0	1	1	1	1	0
0	0	0	0	0	0

Image

0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0		

Erosion Example

0	0	0	0	0	0
0	0	1	1	0	0
0	0	1	1	0	0
0	1	1	1	1	0
0	1	1	1	1	0
0	0	0	0	0	0

Image

0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	1				

Erosion Example

0	0	0	0	0	0
0	0	1	1	0	0
0	0	1	1	0	0
0	1	1	1	1	0
0	1	1	1	1	0
0	0	0	0	0	0

Image

0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	1	1			

Erosion Example

0	0	0	0	0	0
0	0	1	1	0	0
0	0	1	1	0	0
0	1	1	1	1	0
0	1	1	1	1	0
0	0	0	0	0	0

Image

0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	1	1	0		

Erosion Example

0	0	0	0	0	0
0	0	1	1	0	0
0	0	1	1	0	0
0	1	1	1	1	0
0	1	1	1	1	0
0	0	0	0	0	0

Image

0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	1	1	0	0	0

Erosion Example

0	0	0	0	0	0
0	0	1	1	0	0
0	0	1	1	0	0
0	1	1	1	1	0
0	1	1	1	1	0
0	0	0	0	0	0

Image

0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	1	1	0	0	0
0	1				

Erosion Example

0	0	0	0	0	0
0	0	1	1	0	0
0	0	1	1	0	0
0	1	1	1	1	0
0	1	1	1	1	0
0	0	0	0	0	0

Image

0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	1	1	0	0	
0	1	1			

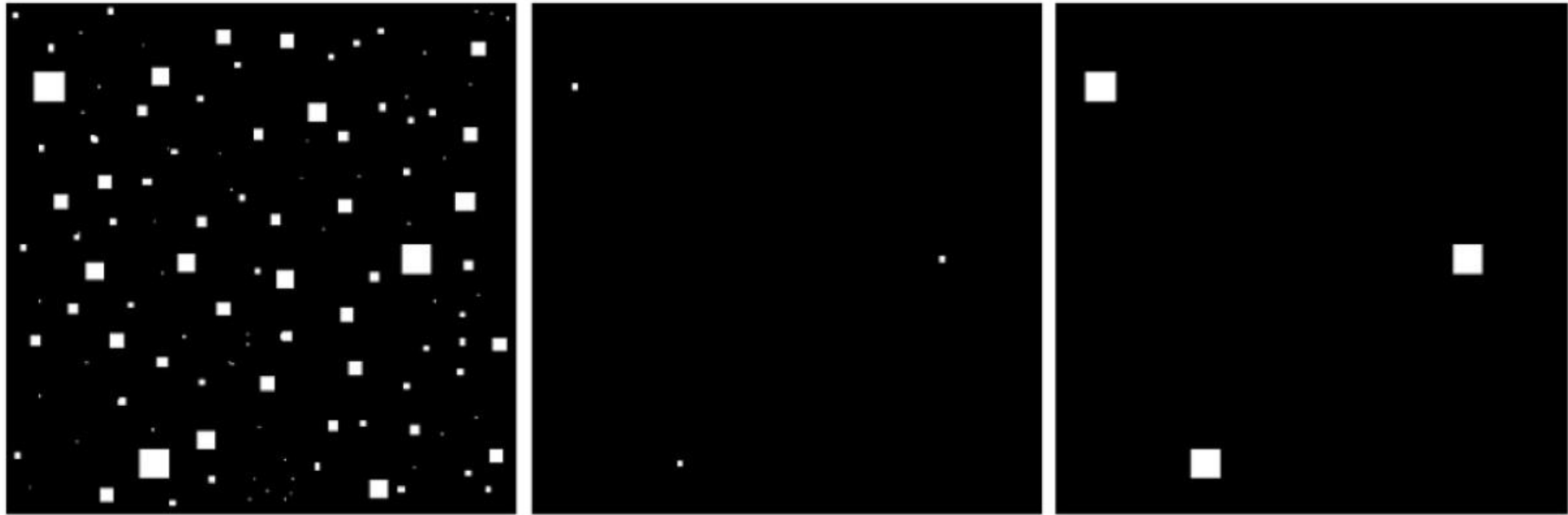
Erosion Example

0	0	0	0	0	0
0	0	1	1	0	0
0	0	1	1	0	0
0	1	1	1	1	0
0	1	1	1	1	0
0	0	0	0	0	1

Image

0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	1	1	0	0	0
0	1	1	0	0	0
0	0	0	0	0	0

Combining erosion and dilation



a b c

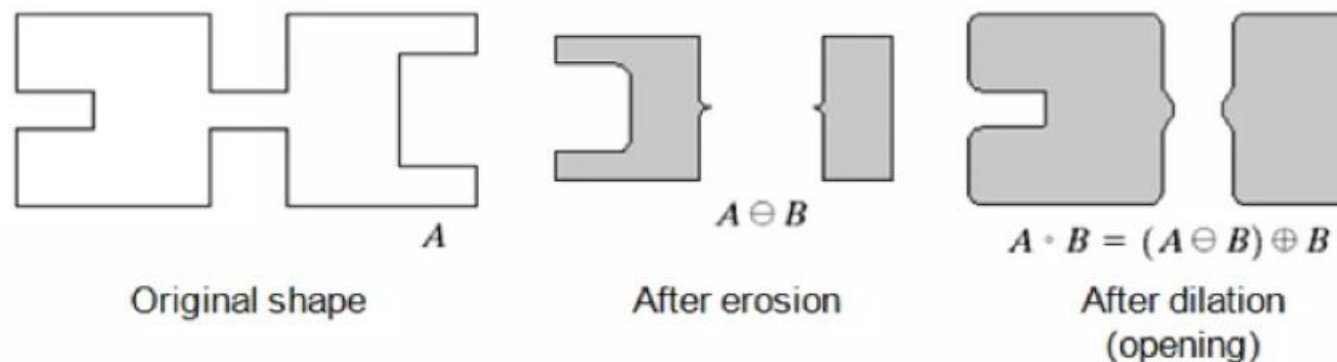
FIGURE 9.7 (a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1's, 13 pixels on the side. (c) Dilation of (b) with the same structuring element.

Opening

Opening generally smoothes the contour of an object, breaks narrow isthmuses, and eliminates thin protrusions

$$A \circ B = (A \ominus B) \oplus B$$

Erosion followed by Dilation, denoted by $\circ$



Opening

Original
Image



Image
After
Opening



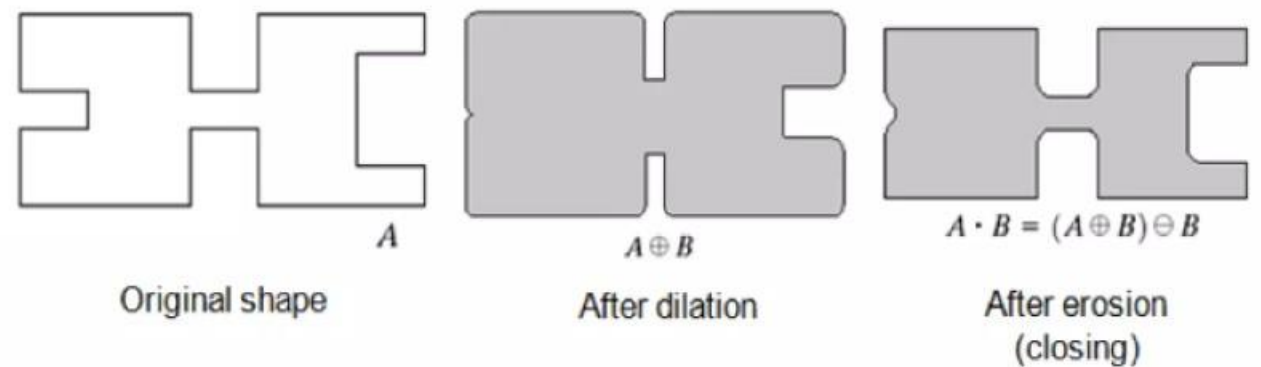
Closing

Closing is used for:

- smooth contour
- fuse narrow breaks and long thin gulfs
- eliminate small holes
- fill gaps in the contour

$$A \bullet B = (A \oplus B) \ominus B$$

Dilation followed by Erosion, denoted •



Closing

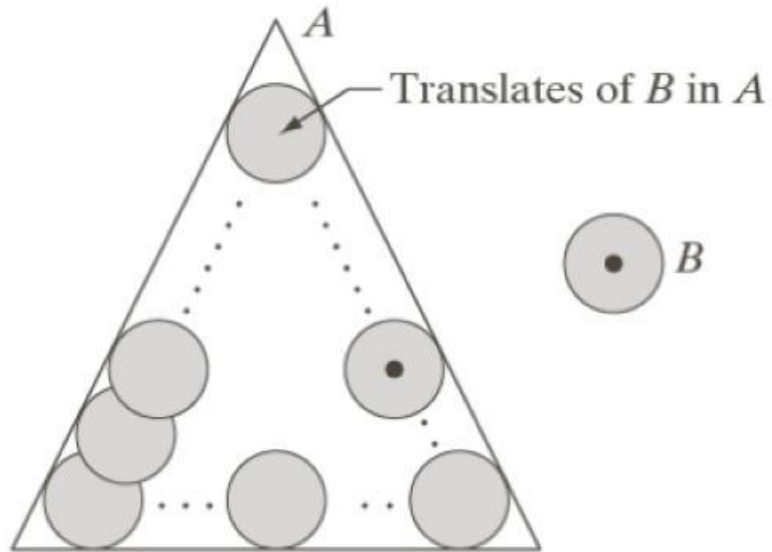
Original
Image



Image
After
Closing



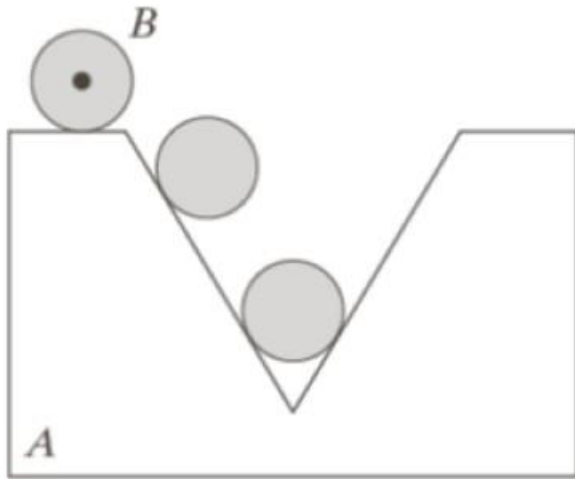
Example: Opening



a b c d

FIGURE 9.8 (a) Structuring element B “rolling” along the inner boundary of A (the dot indicates the origin of B). (b) Structuring element. (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded). We did not shade A in (a) for clarity.

Example: Closing



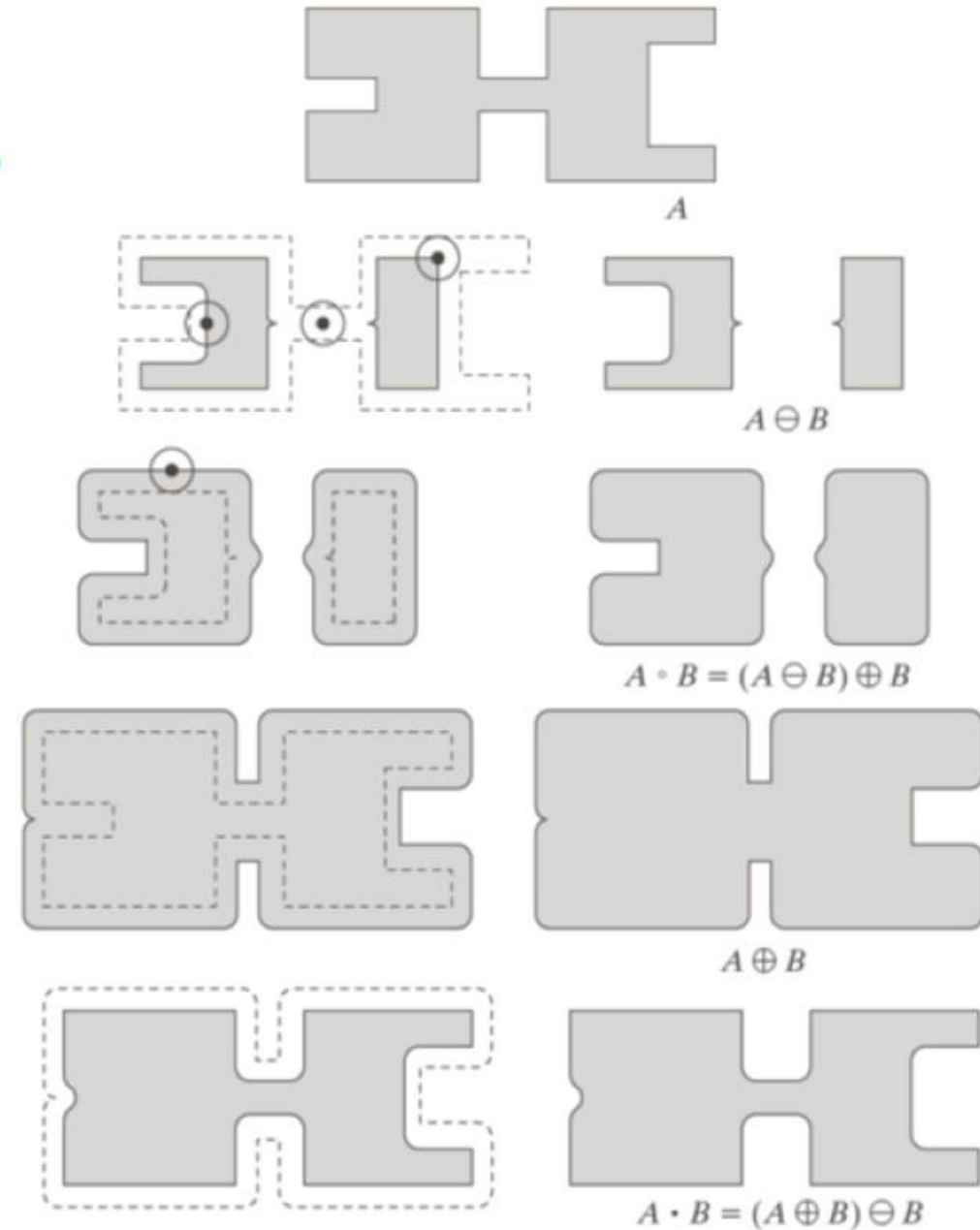
a b c

FIGURE 9.9 (a) Structuring element B “rolling” on the outer boundary of set A . (b) The heavy line is the outer boundary of the closing. (c) Complete closing (shaded). We did not shade A in (a) for clarity.

Opening and Closing

a	
b	c
d	e
f	g
h	i

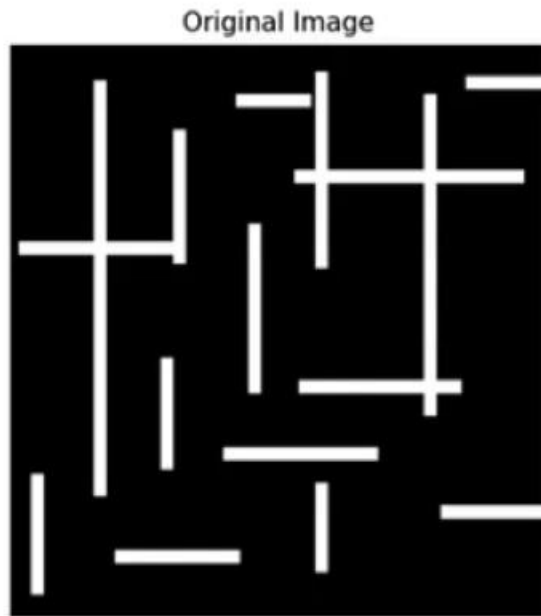
FIGURE 9.10
Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The SE was not shaded here for clarity. The dark dot is the center of the structuring element.





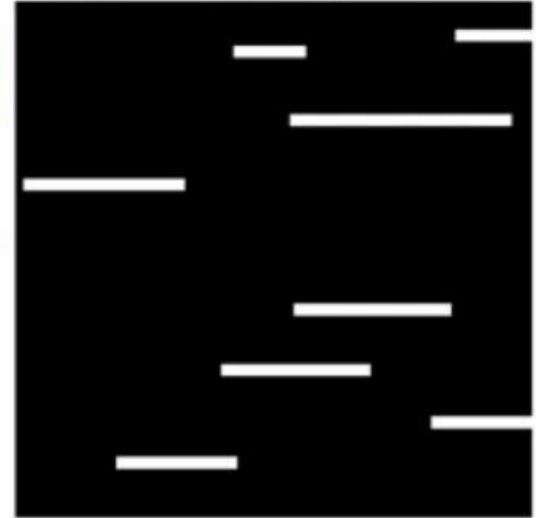
Removing noise by:
Opening followed
by closing

Applications of opening



SE: 3×9

Preprocessed Image (Horizontal Lines)

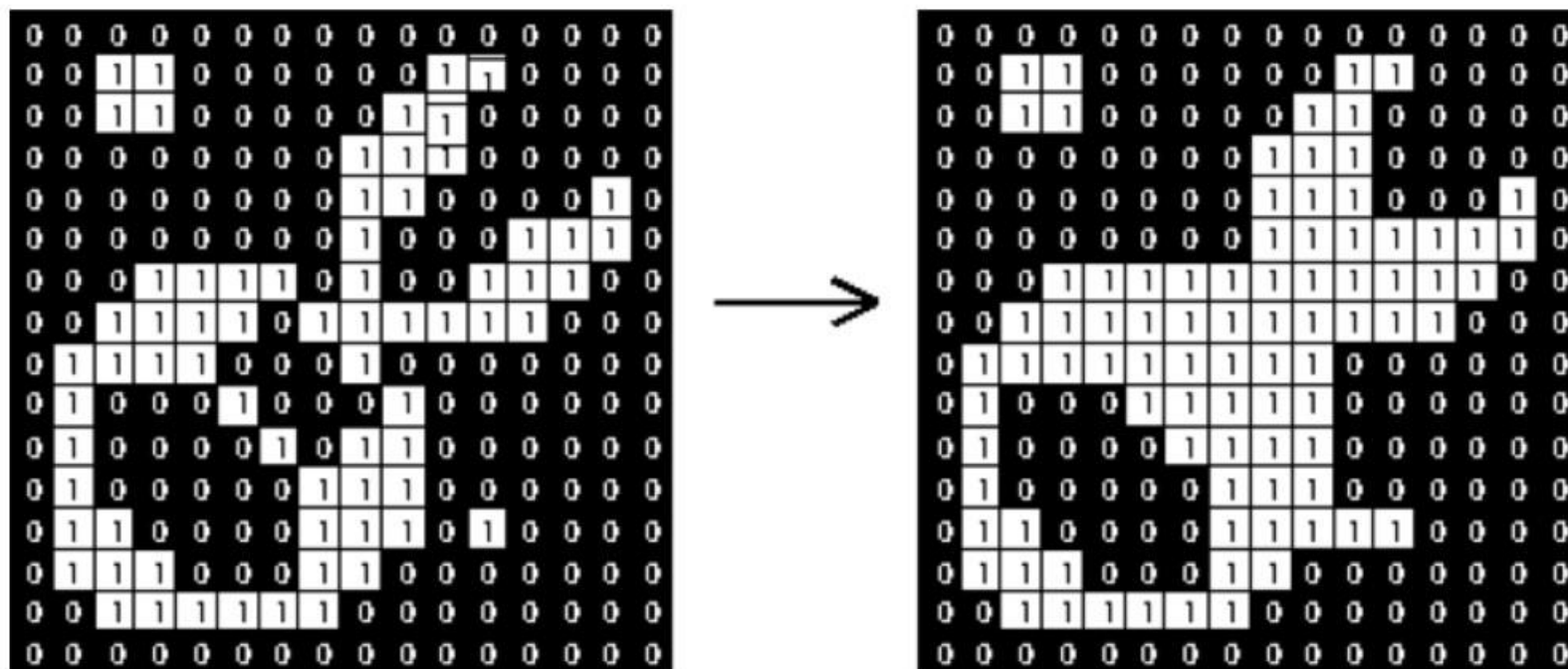


SE: 9×3

Preprocessed Image (Vertical Lines)



Applications of Closing



Effect of closing using a 2×2 square structuring element

- **Extraction of Connected Components**

Central to many automated image analysis applications.

Let A be a set containing one or more connected components, and form an array X_0 (of the same size as the array containing A) whose elements are 0s, except at each location known to correspond to a point in each connected component in A , which is set to 1.

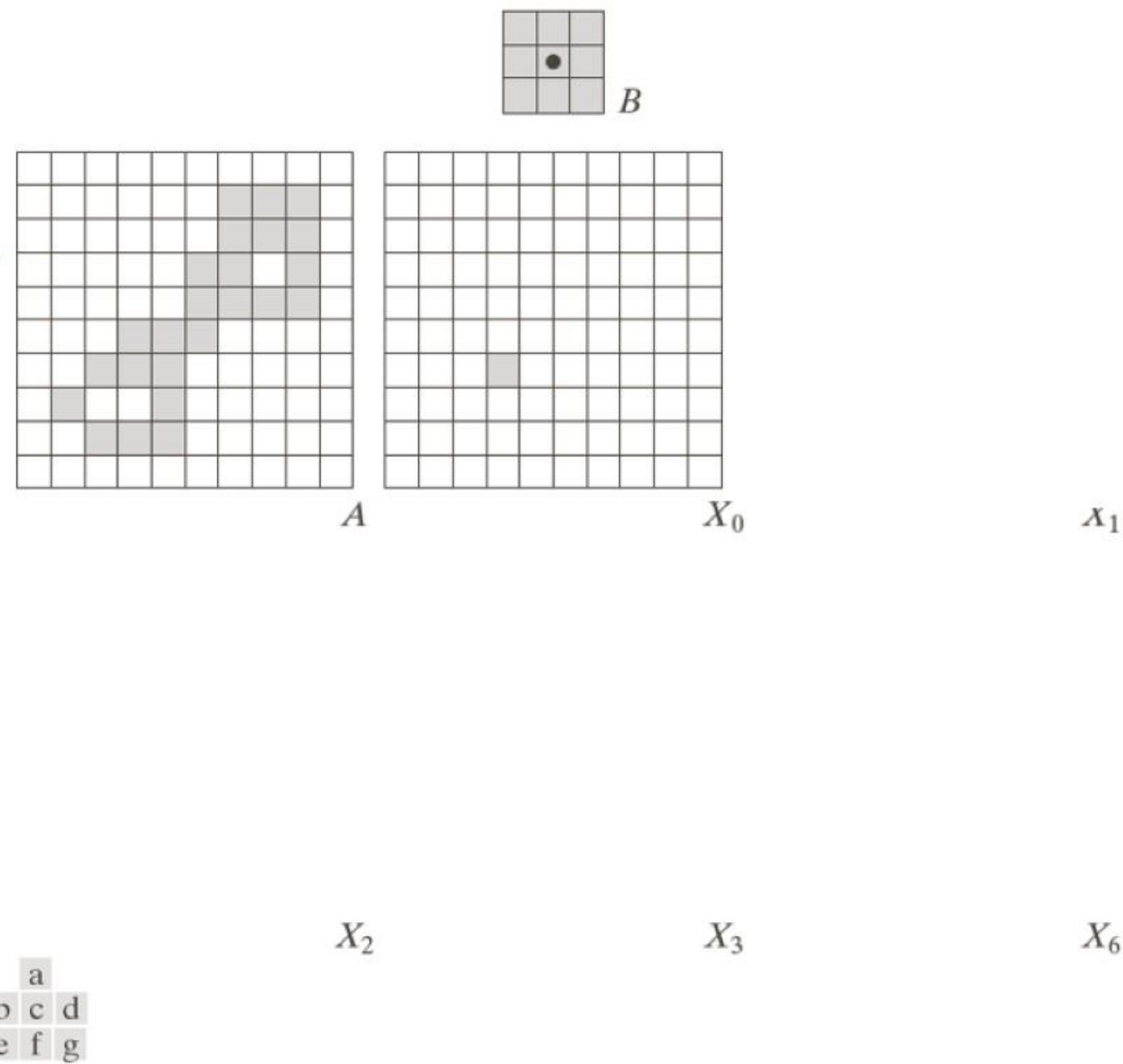


FIGURE 9.17 Extracting connected components. (a) Structuring element. (b) Array containing a set with one connected component. (c) Initial array containing a 1 in the region of the connected component. (d)–(g) Various steps in the iteration of Eq. (9.5-3).

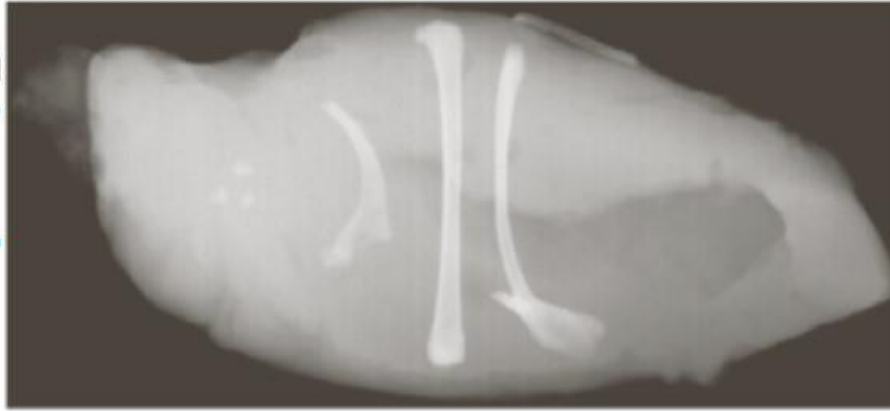
- **Extraction of Connected Components**

Central to many automated image analysis applications.

$$X_k = (X_{k-1} \oplus B) \cap A$$

B : structuring element

until $X_k = X_{k-1}$



Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

a
b
c d

FIGURE 9.18

(a) X-ray image of chicken file with bone fragments.

(b) Thresholded image. (c) Image eroded with a 5×5 structuring element of 1s.

(d) Number of pixels in the connected components of (c).

(Image courtesy of NTB

Elektronische Geraete GmbH, Diepholz, Germany, www.ntbxray.com.)

Some Basic Morphological Algorithms (4)

- **Convex Hull**

Let B^i , $i = 1, 2, 3, 4$, represent the four structuring elements.

The procedure consists of implementing the equation:

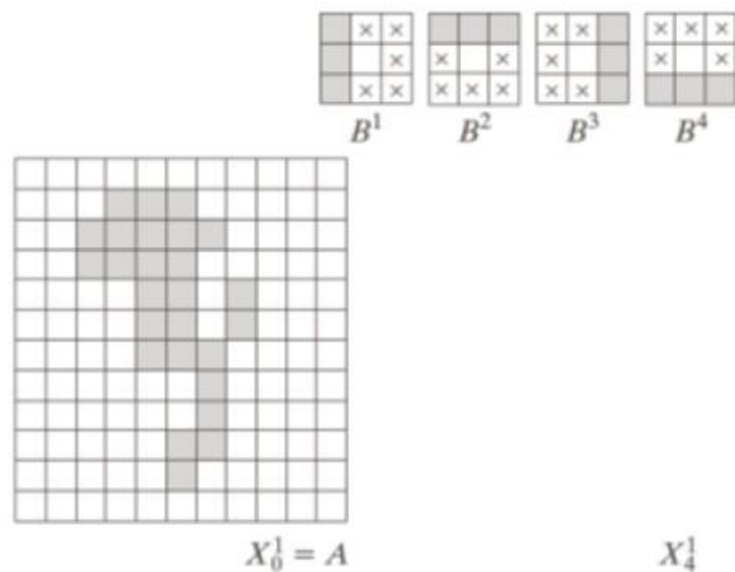
$$X_k^i = (X_{k-1} \circledast B^i) \cup A$$

$$i = 1, 2, 3, 4 \text{ and } k = 1, 2, 3, \dots$$

with $X_0^i = A$.

When the procedure converges, or $X_k^i = X_{k-1}^i$, let $D^i = X_k^i$,
the convex hull of A is

$$C(A) = \bigcup_{i=1}^4 D^i$$



$X_0^1 = A$

X_4^1

X_8^3

X_2^4

B^1
 B^2
 B^3
 B^4

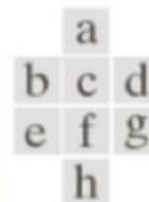


FIGURE 9.19

(a) Structuring elements. (b) Set A . (c)–(f) Results of convergence with the structuring elements shown in (a). (g) Convex hull. (h) Convex hull showing the contribution of each structuring element.

X_2^2

$C(A)$

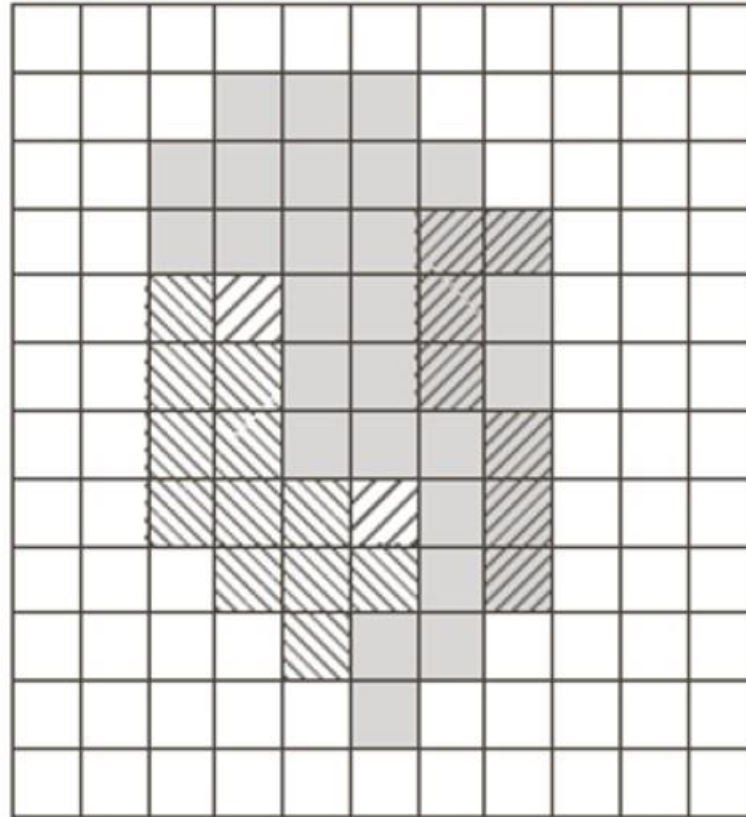
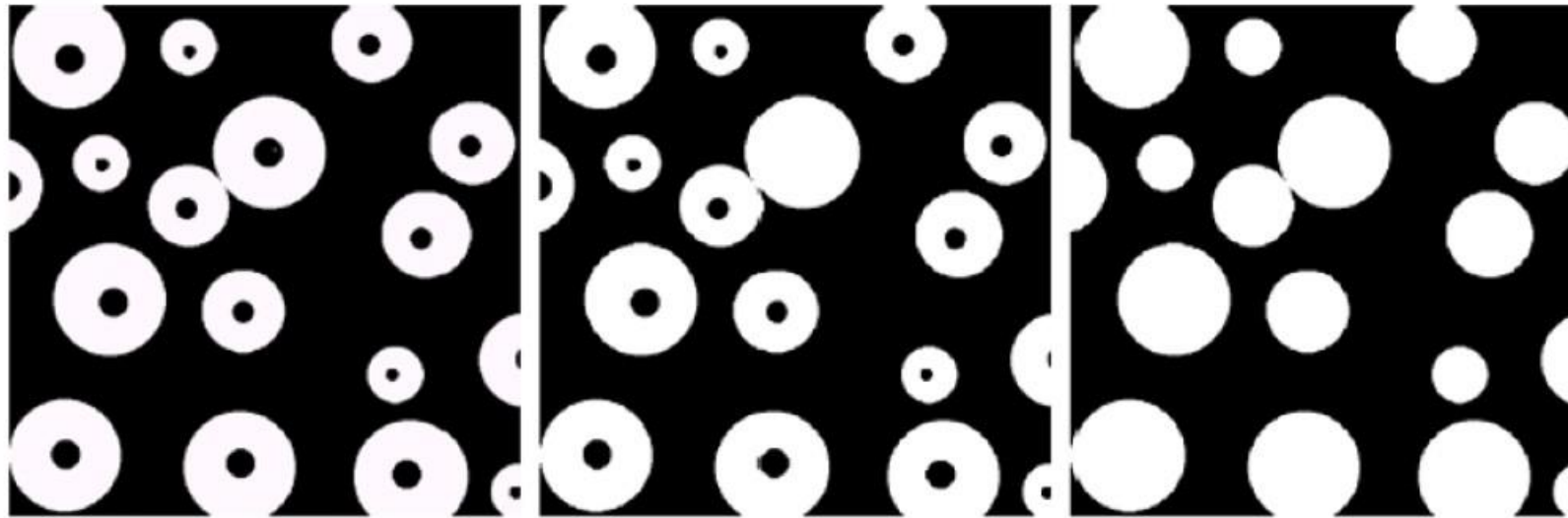


FIGURE 9.20
Result of limiting
growth of the
convex hull
algorithm to the
maximum
dimensions of the
original set of
points along the
vertical and
horizontal
directions.

Some Basic Image Morphology

Region Filling



a b c

FIGURE 9.16 (a) Binary image (the white dot inside one of the regions is the starting point for the region-filling algorithm). (b) Result of filling that region (c) Result of filling all regions.

Some Basic Image Morphology

- Thinning

The thinning of a set A by a structuring element B , defined

$$\begin{aligned} A \otimes B &= A - (A * B) \\ &= A \cap (A * B)^c \end{aligned}$$

It is denoted by the $\otimes$ and it can be defined in terms of hit-or-miss transformation $!$

Some Basic Image Morphology

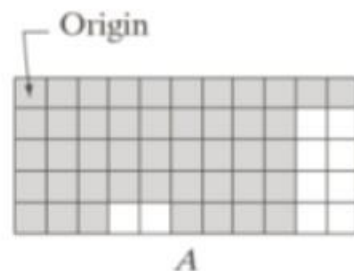
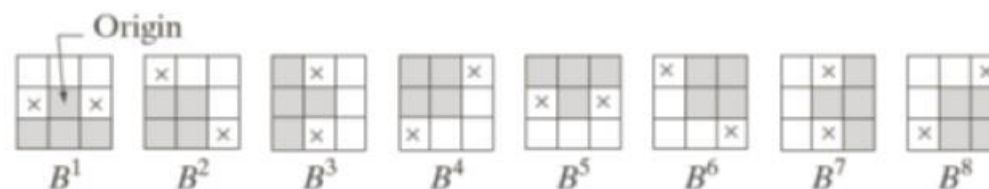
- A more useful expression for thinning A symmetrically is based on a sequence of structuring elements:

$$\{B\} = \{B^1, B^2, B^3, \dots, B^n\}$$

where B^i is a rotated version of B^{i-1}

The thinning of A by a sequence of structuring element $\{B\}$

$$A \otimes \{B\} = (((...((A \otimes B^1) \otimes B^2)...) \otimes B^n)$$



$$\begin{aligned}
 A_1 &= A \otimes B^1 & A_2 &= A_1 \otimes B^2 \\
 A_3 &= A_2 \otimes B^3 & A_4 &= A_3 \otimes B^4 & A_5 &= A_4 \otimes B^5 \\
 A_6 &= A_5 \otimes B^6 & A_8 &= A_6 \otimes B^{7,8} & A_{8,4} &= A_8 \otimes B^{1,2,3,4} \\
 A_{8,5} &= A_{8,4} \otimes B^5 & A_{8,6} &= A_{8,5} \otimes B^6 & A_{8,6} &\text{converted to } m\text{-connectivity.} \\
 && &\text{No more changes after this.} & &
 \end{aligned}$$

a
b c d
e f g
h i j
k l m

FIGURE 9.21 (a) Sequence of rotated structuring elements used for thinning. (b) Set A . (c) Result of thinning with the first element. (d)–(i) Results of thinning with the next seven elements (there was no change between the seventh and eighth elements). (j) Result of using the first four elements again. (l) Result after convergence. (m) Conversion to m -connectivity.

• Thickening:

The thickening is defined by the expression

$$A \odot B = A \cup (A * B)$$

The thickening of A by a sequence of structuring element $\{B\}$

$$A \odot \{B\} = (((...((A \odot B^1) \odot B^2)...) \odot B^n)$$

In practice, the usual procedure is to thin the background of the set and then complement the result.

Some Basic Some Basic Image Morphology

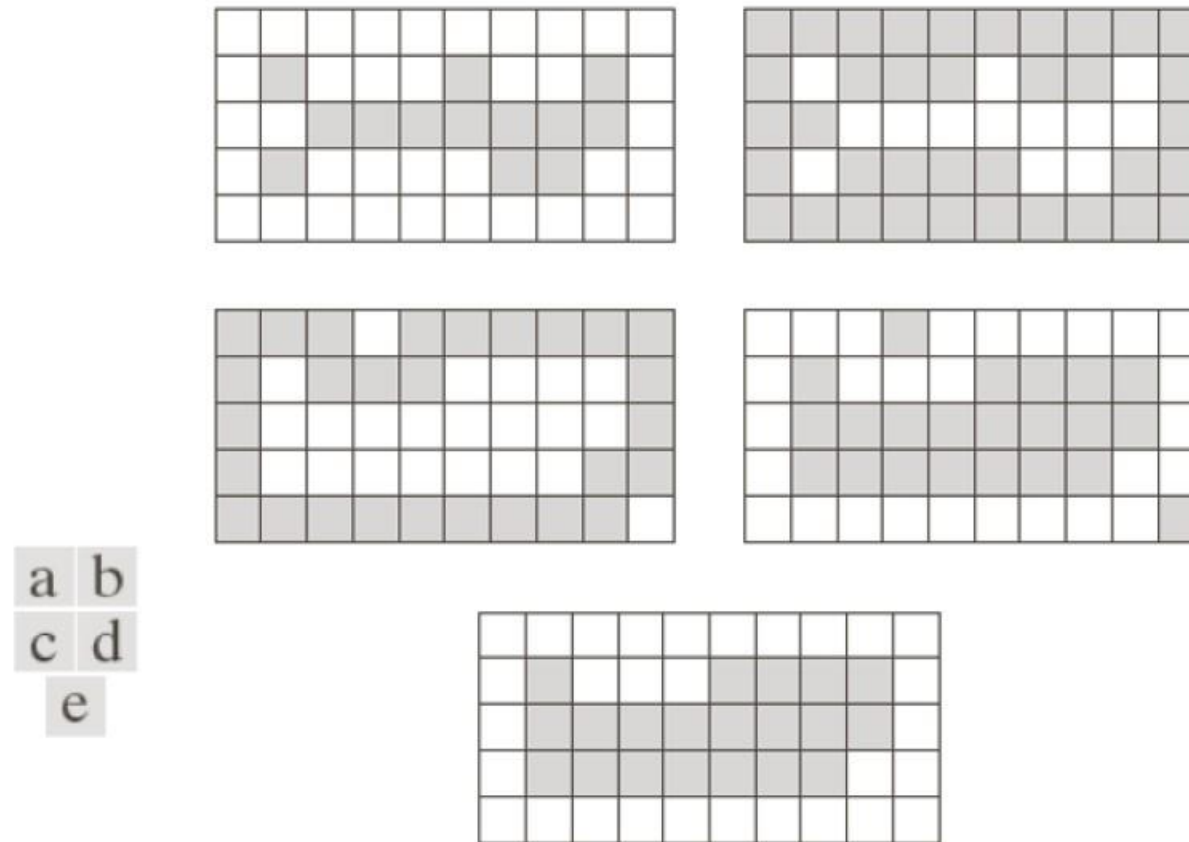
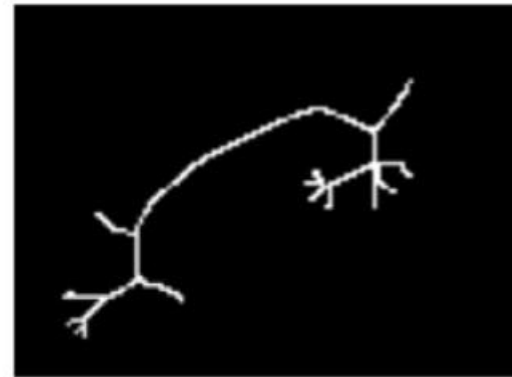
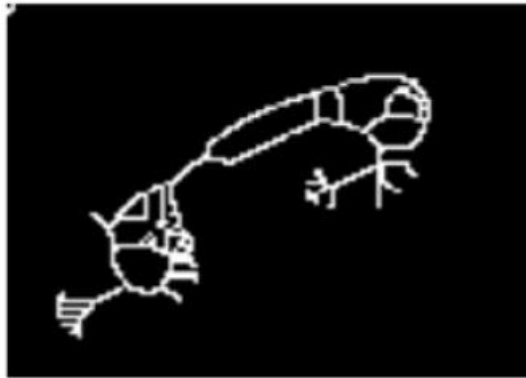


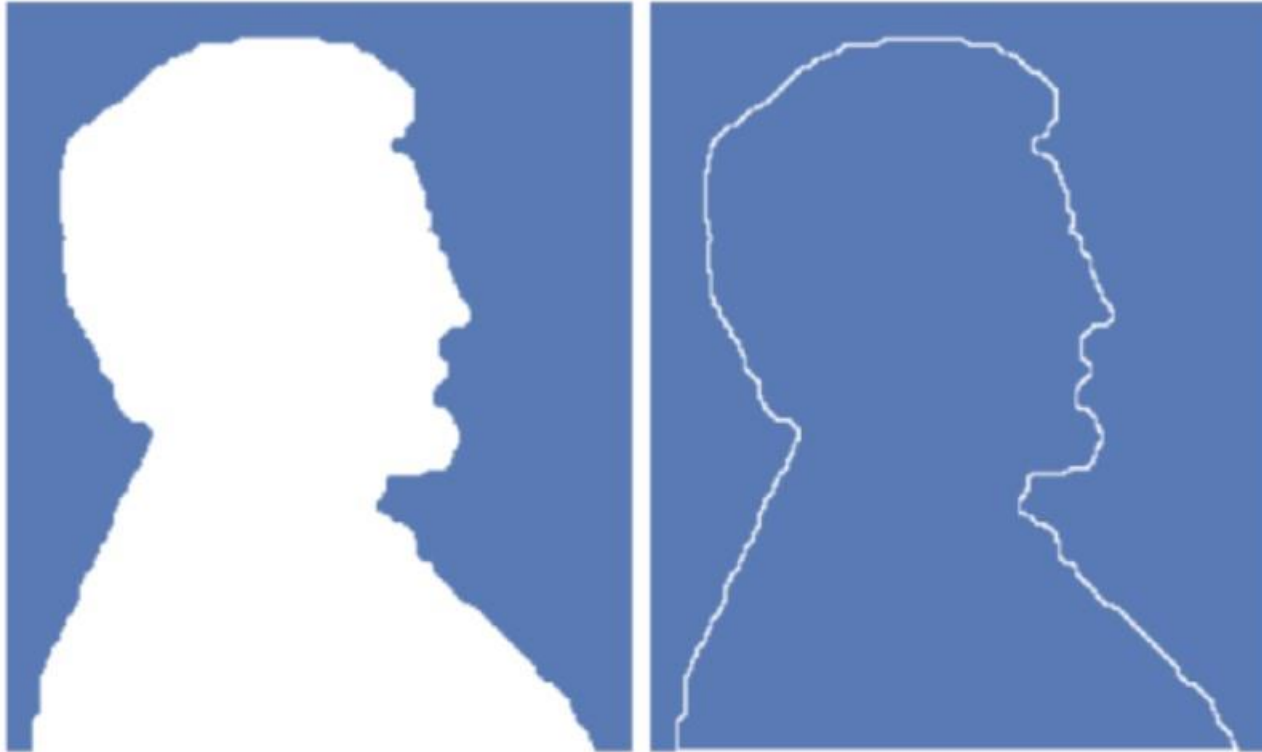
FIGURE 9.22 (a) Set A . (b) Complement of A . (c) Result of thinning the complement of A . (d) Thickened set obtained by complementing (c). (e) Final result, with no disconnected points.

Some Basic Some Basic Image Morphology

Skeletons



Boundary Extraction



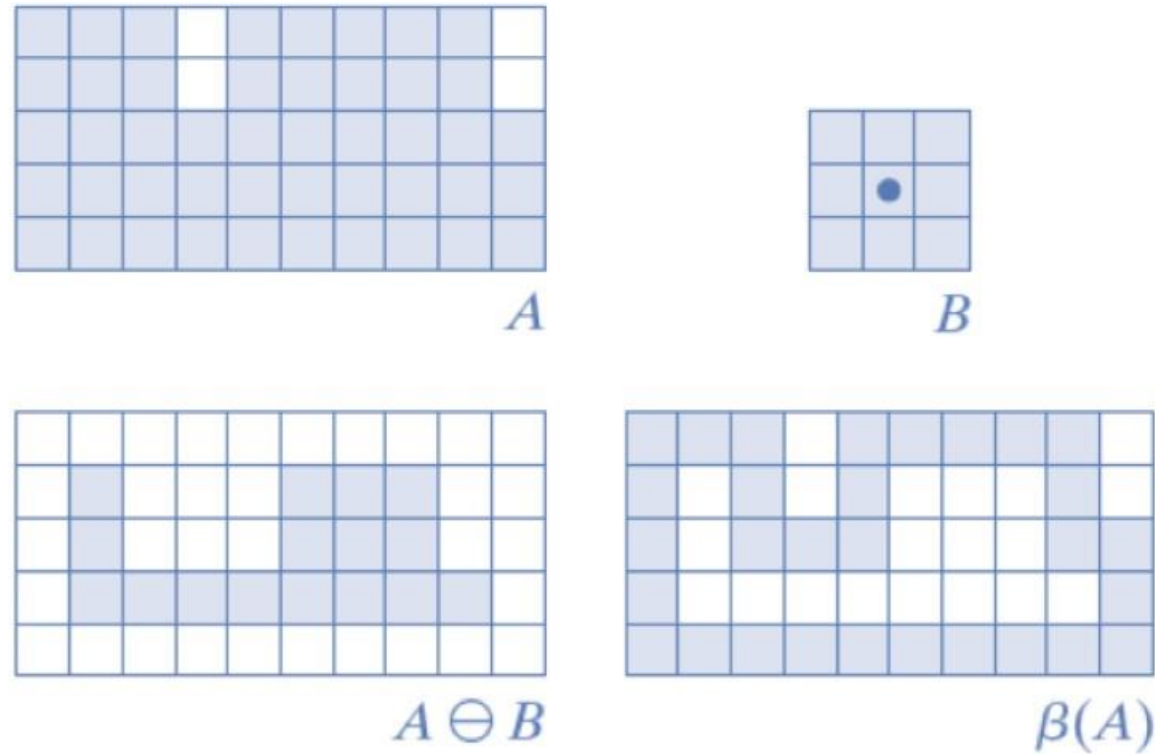
a b

FIGURE 9.14

(a) A simple binary image, with 1s represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).

$$\beta(A) = A - (A \ominus B)$$

Boundary Extraction



a b
c d

FIGURE 9.13 (a) Set A . (b) Structuring element B . (c) A eroded by B . (d) Boundary, given by the set difference between A and its erosion.



Thank You