

Mathematics 3 – MAT132

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Lecture 5 – Calculus for Fourier Series



Lecture Objective

- Revision for some integration Rules
- Integration of Basic Trigonometric Functions
- Integration for other functions with Trigonometric Functions
- definite Integration
- Extra Practice

Basic Rules in Integration

Revising some main rules in the integration

Rules of Integration

- Basic Integration

Properties:

$$1. \int k dx = kx + C \quad k \text{ is a constant}$$

$$2. \int x^n dx = \frac{x^{n+1}}{n+1} + C \quad n \neq -1$$

$$3. \int x^{-1} dx = \int \frac{1}{x} dx = \int \frac{dx}{x} = \ln x + C \quad \text{for } x > 0$$

$$4. \int e^x dx = e^x + C$$

$$5. \int kf(x) dx = k \int f(x) dx \quad k \text{ is a constant}$$

$$6. \int (f(x) \pm g(x)) dx = \int f(x) dx \pm \int g(x) dx$$

Example

Example 1 - Finding an Indefinite Integral

Find $\int 5dx$.

Solution: $\int 5dx = 5x + C$

Example

Example - Indefinite Integral of a Constant Times a Function

Find $\int 7x dx$

Solution: $\int 7x dx = \frac{7x^2}{2} + C$

Example

Example 5 - Finding Indefinite Integrals

a. $\int \frac{1}{\sqrt{t}} dx$

$$= \int t^{-1/2} dx = \frac{t^{1/2}}{1/2} + C = 2\sqrt{t} + C$$

b. $\int \frac{1}{6x^3} dx$

$$= \frac{1}{6} \left(\frac{x^{-3+1}}{-3+1} \right) + C = -\frac{1}{12x^2} + C$$

Basic Rules in Integration

Important Notes for Trigonometric Functions

Important Notes

Derivative Rule	Antiderivative Rule
$\frac{d}{dx} \sin x = \cos x$	$\int \cos x \, dx = \sin x + C$
$\frac{d}{dx} \cos x = -\sin x$	$\int \sin x \, dx = -\cos x + C$
$\frac{d}{dx} \tan x = \sec^2 x$	$\int \sec^2 x \, dx = \tan x + C$
$\frac{d}{dx} \cot x = -\csc^2 x$	$\int \csc^2 x \, dx = -\cot x + C$
$\frac{d}{dx} \sec x = \sec x \tan x$	$\int (\sec x \tan x) \, dx = \sec x + C$
$\frac{d}{dx} \csc x = -\csc x \cot x$	$\int (\csc x \cot x) \, dx = -\csc x + C$

Important Notes

Integral Rule	General Rule
$\int \cos x \, dx = \sin x + C$	$\int \cos(ax + b) \, dx = \frac{1}{a} \sin(ax + b) + C$
$\int \sin x \, dx = -\cos x + C$	$\int \sin(ax + b) \, dx = -\frac{1}{a} \cos(ax + b) + C$

Extra Practice

Extra Practice for different ideas in Integration !!!!!

Extra Practice – Example 1

Integrate the following function:

$$\int (3 \sin x - 4 \sec^2 x) dx$$

Example 1 Solution

$$\begin{aligned} & \int (3 \sin x - 4 \sec^2 x) dx \\ &= \int 3 \sin x \, dx - \int 4 \sec^2 x \, dx \\ &= 3 \int \sin x \, dx - 4 \int \sec^2 x \, dx \\ &= -3 \cos x - 4 \tan x + C \end{aligned}$$

Extra Practice – Example 2

Integrate the following function:

$$\int \cos(2x - 6) dx$$

Example 2 Solution

$$\int \cos(2x - 6) dx$$
$$= \frac{1}{2} \sin(2x - 6) + C.$$

Extra Practice – Example 3

Integrate the following function:

$$\int \sin x \cos^2 x \, dx$$

Example 3 Solution

$$\int \sin x \cos^2 x \, dx$$
$$= -\frac{\cos^3 x}{3} + C$$

Extra Practice – Example 4

Integrate the following function:

$$\int \sin 3x \, dx$$

Example 4 Solution

$$\begin{aligned}\int \sin 3x \, dx \\ = -\frac{1}{3} \cos 3x + C\end{aligned}$$

Extra Practice – Example 5

Integrate the following function:

$$\int 5 \sec 4x \tan 4x \, dx$$

Example 5 Solution

$$\int 5 \sec 4x \tan 4x \, dx$$

$$= \frac{5}{4} \sec 4x + C$$

Extra Practice – Example 6

Integrate the following function:

$$\int \tan x \, dx$$

Example 6 Solution

$$\begin{aligned}\int \tan x \, dx &= \int \frac{\sin x}{\cos x} \, dx \\ &= -\ln |\cos x| + C.\end{aligned}$$

Extra Practice – Example 7

Integrate the following function:

$$\int \tan 5x \, dx$$

Example 7 Solution

$$\begin{aligned}\int \tan 5x \, dx \\ = -\frac{1}{5} \ln |\cos 5x| + C\end{aligned}$$

Extra Practice – Example 8

Integrate the following function:

$$\int (\sin x + \cos x)^2 dx$$

Example 8 Solution

$$\begin{aligned}\int (\sin x + \cos x)^2 dx &= \int (\sin^2 x + 2 \sin x \cos x + \cos^2 x) dx \\&= \int ((\sin^2 x + \cos^2 x) + 2 \sin x \cos x) dx \\&= \int (1 + 2 \sin x \cos x) dx \\&= \int 1 dx + \int 2 \sin x \cos x dx \\&= x + 2 \int \sin x \cos x dx .\end{aligned}$$

Example 8 Solution

$$\begin{aligned} &= \int (1 + 2 \sin x \cos x) dx \\ &= \int 1 dx + \int 2 \sin x \cos x dx \\ &= x + 2 \int \sin x \cos x dx . \\ &= x + 2 \int \sin x \cos x dx \\ &= x + \sin^2 x + C \end{aligned}$$

Extra Practice – Example 9

Integrate the following function:

$$3 \int \frac{1 + \cos 2(5x)}{2} dx$$

Example 9 Solution

$$\begin{aligned} 3 \int \frac{1 + \cos 2(5x)}{2} dx &= (3/2) \int (1 + \cos 10x) dx \\ &= (3/2) \left(\int 1 dx + \int \cos 10x dx \right) \\ &= (3/2) \left(x + \int \cos 10x dx \right) . \end{aligned}$$

Example 9 Solution

$$= (3/2) \left(x + \int \cos 10x \, dx \right)$$

$$= (3/2) \left(x + \int \cos 10x \, dx \right)$$

$$= (3/2)x + (3/20) \sin 10x + C .$$

Extra Practice – Example 10

Integrate the following function:

$$\text{Find } \int_{-1}^3 (3x^2 - x + 6) dx.$$

Example 10 Solution

Find $\int_{-1}^3 (3x^2 - x + 6)dx$.

Solution:

$$\begin{aligned}\int_{-1}^3 (3x^2 - x + 6)dx &= \left[x^3 - \frac{x^2}{2} + 6x \right]_{-1}^3 \\ &= \left[3^3 - \frac{3^2}{2} + 6(3) \right] - \left[(-1)^3 - \frac{(-1)^2}{2} + 6(-1) \right] \\ &= 48\end{aligned}$$

Extra Practice – Example 11

Integrate the following function:

$$\int_3^4 \frac{3}{(x+3)^2} dx$$

Example 11 Solution

$$\begin{aligned}\int_3^4 \frac{3}{(x+3)^2} dx &= 3 \int_3^4 (x+3)^{-2} dx \\&= 3 \cdot \frac{(x+3)^{-1}}{-1} \Big|_3^4 \\&= -3 \left[\frac{1}{4+3} - \frac{1}{3+3} \right] \\&= -3 \left(\frac{1}{7} - \frac{1}{6} \right) \\&= \frac{1}{14}\end{aligned}$$

Extra Practice – Example 12

Integrate the following function:

$$\int_{\frac{1}{2}}^{\frac{3}{2}} (x^2 + x + 1) dx$$

Example 12 Solution

$$\int_{1/2}^{3/2} (x^2 + x + 1) dx$$

$$= \left(\frac{x^3}{3} + \frac{x^2}{2} + x \right) \bigg|_{1/2}^{3/2}$$

$$= \frac{15}{4} - \frac{2}{3} = \frac{37}{12}$$

Extra Practice – Example 13

Integrate the following function:

$$\int_{-2}^{-1} (3w^2 - w - 1) dw$$

Example 13 Solution

$$\int_{-2}^{-1} (3w^2 - w - 1) dw$$

$$= \left(w^3 - \frac{w^2}{2} - w \right) \Big|_{-2}^{-1}$$

$$= -\frac{1}{2} - (-8) = \frac{15}{2}$$

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