

Simultaneous Eqns. & Equilibrium

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Lecture 05



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MTH 112
Business Mathematics

At the end of this section you should be familiar with:

- How to solve two simultaneous equations in two unknowns using: (a) Algebra
(b) Graphical methods
- Determining when two equations in two unknowns have: (a) A unique solution
(b) No solution
(c) Infinitely many solutions
- Solving three simultaneous equations in three unknowns.
- Equilibrium of Goods

Reminder:

A solution of an equation in an unknown, say x , is simply the value for which the left-hand side (LHS) of the equation is equal to the right-hand side (RHS).

For example, consider the equation $x + 4 = 6$

The solution of this equation is $x = 2$ $x=2$ is the only value of x for which the LHS= RHS.

The statement ' $x = 2$ satisfies the equation' is another way of saying that $x = 2$ is a solution.

Therefore, the solution of a set of simultaneous equations is a set of values for the variables which satisfy all the equations.

Two equations in two unknowns

Example: Given the simultaneous equations

$$x + 3y = 4$$

$$-x + 2y = 6$$

- (a) Solve for x and y **algebraically**.
- (b) Solve for x and y **graphically**.

Two equations in two unknowns

Solution: (a)Method:

Eliminate x from the system of equations by adding equations (1) and (2).

The two equations reduce to a single equation in which the only unknown is y .

Solve for y , then substitute the value of y into either of the original equations and solve for x :

Two equations in two unknowns

Solution: (a)

Step 1:

$$\begin{array}{rcl} + & x + 3y = 4 & (1) \\ & -x + 2y = 6 & (2) \\ \hline \end{array}$$

$$0 + 5y = 10 \quad (\text{adding (1) and (2)})$$

$$y = \frac{10}{5} = 2$$

solving for y

$$y = 2$$



Two equations in two unknowns

Solution: (a)

Step 2:

Solve for x by substituting $y = 2$ into either equation (1) or equation (2):

$$-x + (2)(2) = 6 \quad (\text{Substituting } y = 2 \text{ into equation (2)})$$

$$-x = 6 - 4$$

$$x = -2$$

Two equations in two unknowns

Solution: (a)

Step 3:

Check the solution $x = -2$, $y = 2$ by substituting these values into equations (1) and (2) and confirm that both equations balance.

- Substitute $x = -2$ and $y = 2$ into equation (1):

$$x + 3y = 4$$

$$(-2) + 3(2) = 4$$

(substituting in $x = -2$ and $y = 2$)

$$-2 + 6 = 4 \rightarrow 4 = 4 \quad \text{so equation (1) balances and } x = -2, y = 2 \text{ is a solution}$$

Two equations in two unknowns

Solution: (a)

Step 3:

- Substitute $x = -2$ and $y = 2$ into equation (2):

$$-x + 3y = 6$$

$$-(-2) + 2(2) = 6 \quad (\text{substituting in } x = -2 \text{ and } y = 2)$$

$$-2 + 4 = 6 \rightarrow \underline{6 = 6} \quad \text{so equation (2) balances and } x = -2, y = 2 \text{ is a solution.}$$

Since the point $(-2, 2)$ satisfies equations (1) and (2), then this point is at the point of intersection of the lines represented by equations (1) and (2) as shown in (b).

Two equations in two unknowns

Solution: (b)

The two lines are plotted in Figure 3.1. The point of intersection is the solution. The coordinates of this point are $x = -2$ and $y = 2$. In this case, it is a **unique solution**, that is, the lines intersect at only one point. This point is on the first line so it satisfies equation (1) and also on the second line, so it satisfies equation (2).

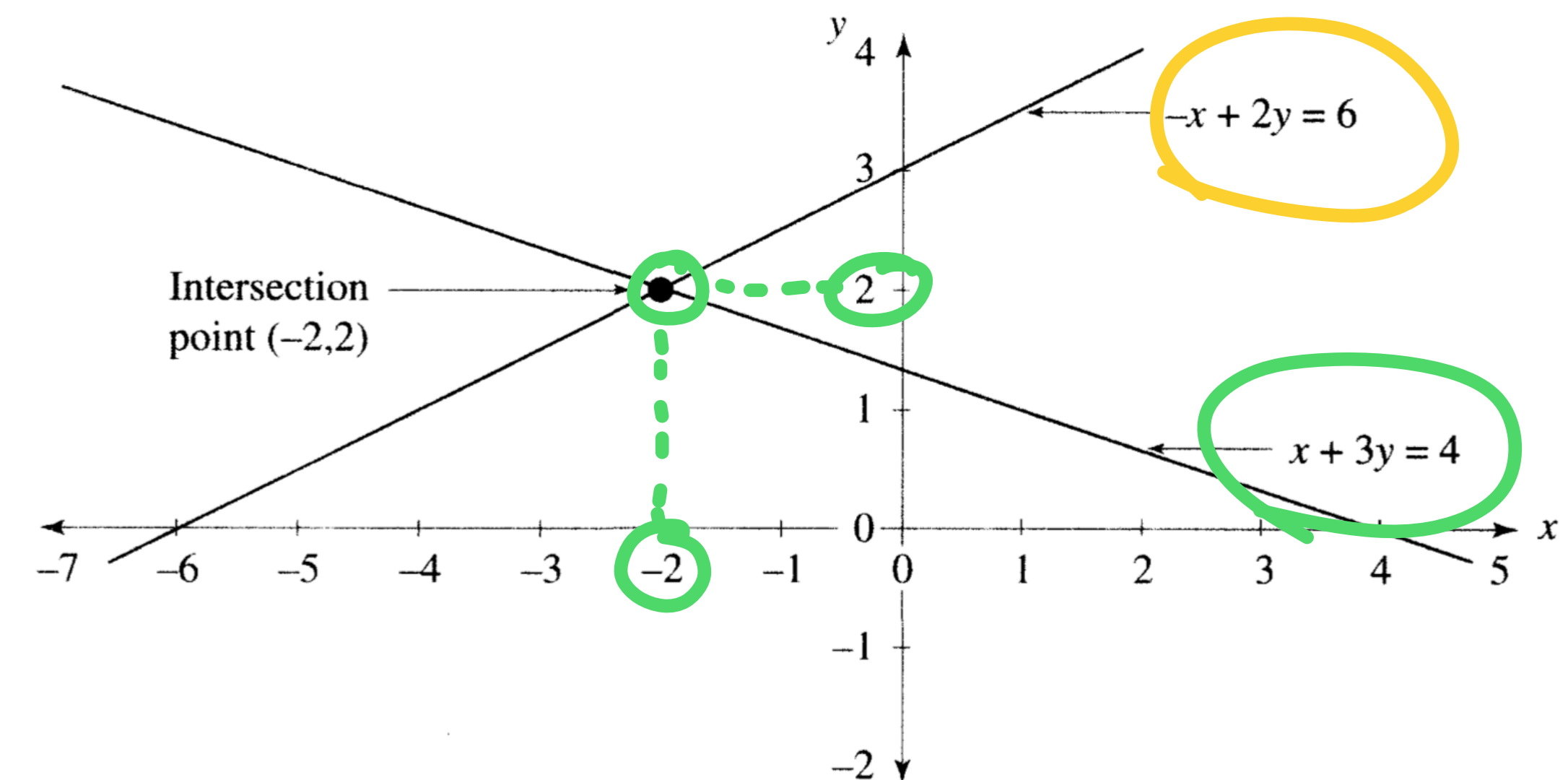


Figure 3.1 Unique solution

A set of simultaneous equations may have:

- **Unique solution:** This occurs when a set of equations has one set of values which satisfy all equations.
- **No solution:** This occurs when a set of equations has no set of values which satisfy all equations.
- **Infinitely many solutions:** A set of equations has infinitely many solutions when there is an infinite number of sets of values that satisfy all equations.

Example:

Given the simultaneous equations

$$y = 1 + x$$

$$y = 2 + x$$

(a) Solve for x and y algebraically.

(b) Solve for x and y graphically.

Solution:

$$\begin{array}{r} x = 1 + 2 \\ + \quad -y = -2 + x \\ \hline 0 = -1 \end{array}$$

$0 = -1$ is not possible.
Therefore, there is no solution.

Solution:

$$y = 1 + x$$
$$y = 2 + x$$

How can y be equal to $(1+x)$ and $(2+x)$ at the same time?!

Solution:

Note: A false statement (contradiction)

like $0 = -1$

indicates a set of equations with no solution

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Simultaneous equations -No solution Example

①

Solution:

⑥

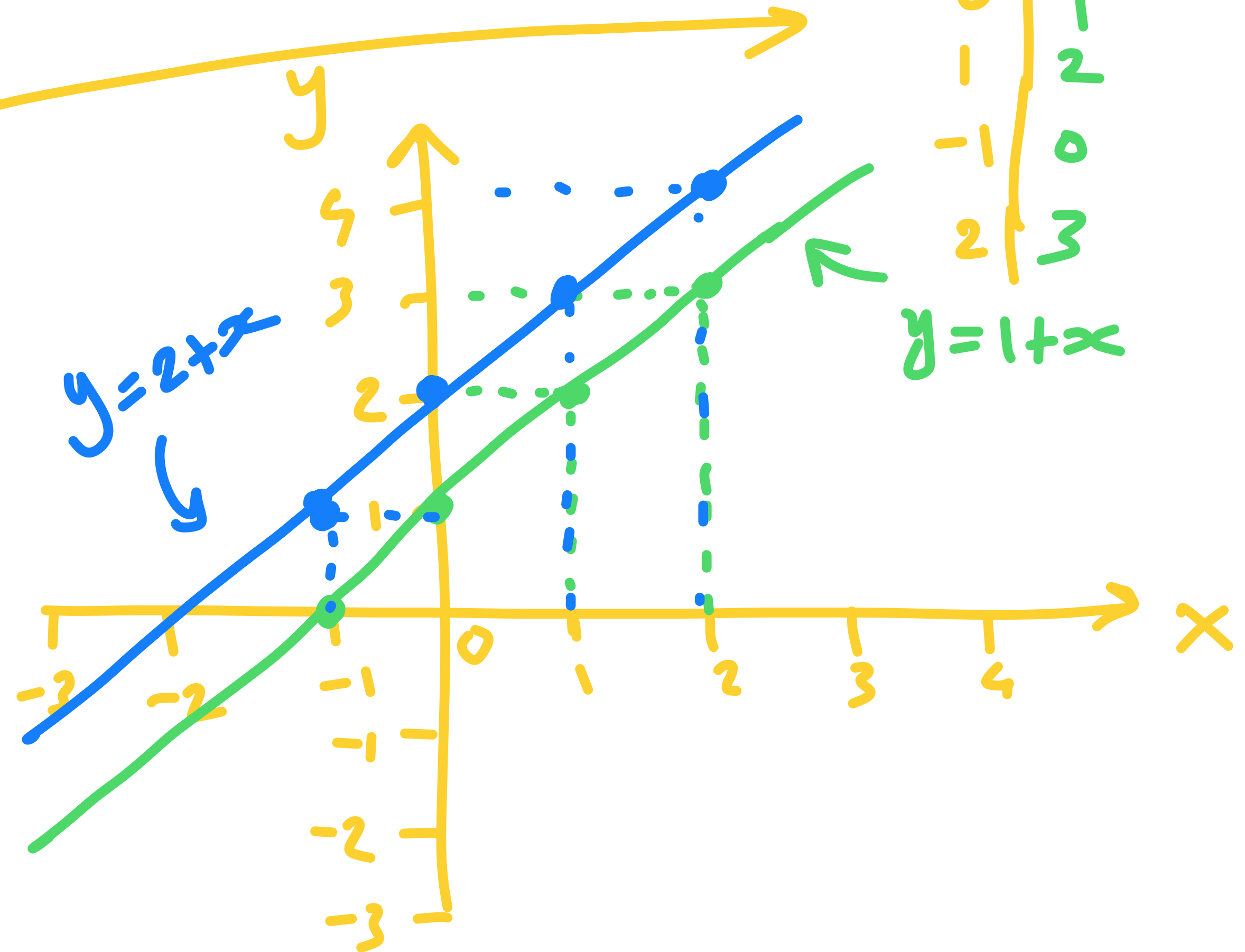
$$y = 1 + x$$

$$y = 2 + x$$

②

x	y
0	2
1	3
2	4
3	5

x	y
0	1
1	2
2	3
3	4



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Resource Book: T. Bradley and P. Patton, *Essential Mathematics for Economics and Business*, 2nd Ed. John Willey & Sons, Ltd, 2002

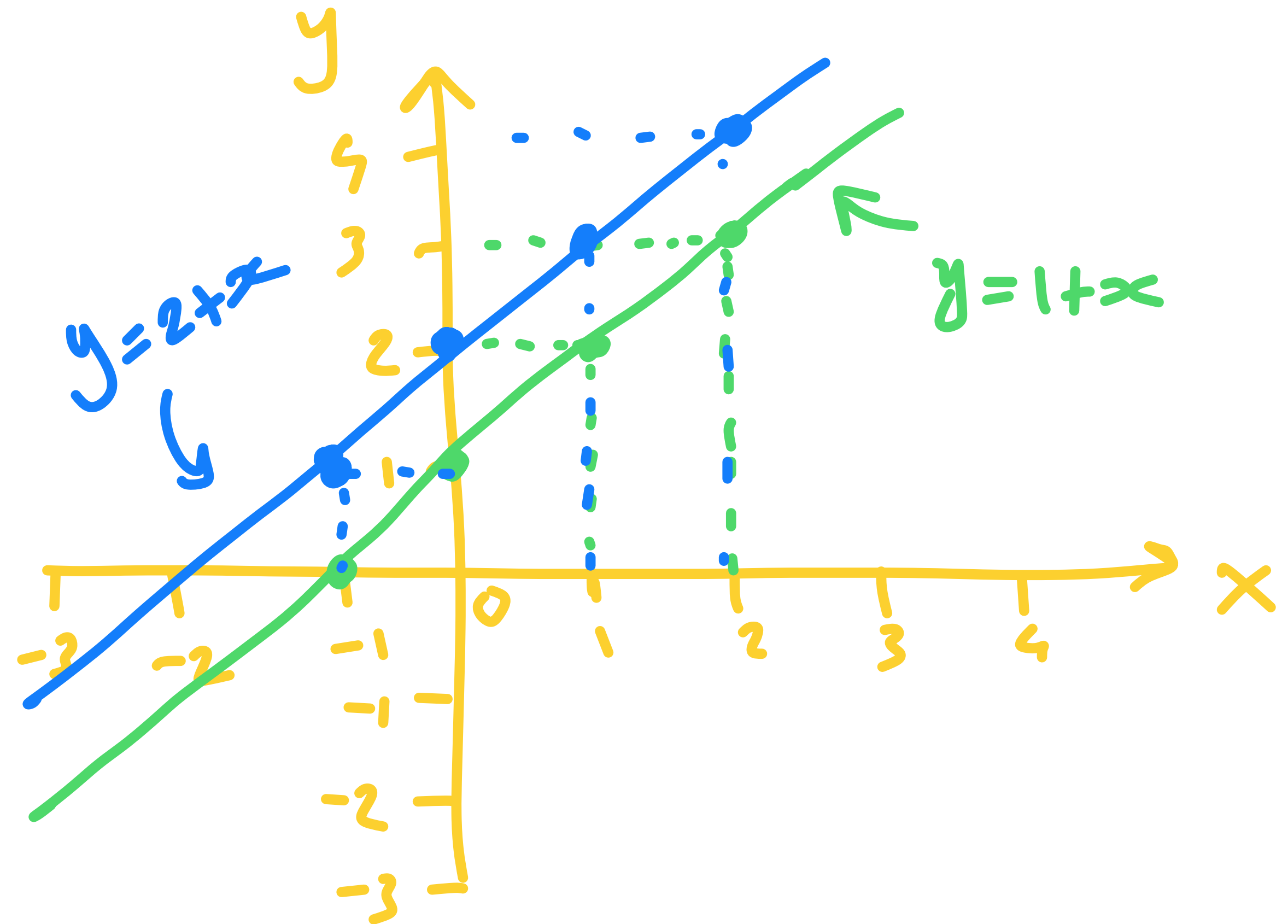
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Solution: ⑥

$$y = 1 + x$$
$$y = 2 + x$$

The lines will never meet
since they are parallel and
thus will never have a point
(solution) in common.



~~Solution:~~
Example

Given the simultaneous equations

$$y = 2 - x$$

$$2y = 4 - 2x$$

- a) solve for x & y algebraically
b) " " " " graphically.

Solution:

$$y = 2 - x \rightarrow \textcircled{1}$$
$$\frac{2y}{7} = \frac{4}{7} - \frac{x}{7} \rightarrow \textcircled{2}$$

$$y = 2 - x$$
$$y = 2 - x$$

eqn. $\textcircled{1}$ & $\textcircled{2}$ are the same.

There is only one eqn in two unknowns.

Solution:

$$y = 2 - x$$

$$x = 2 \rightarrow y = 0$$

$$x = 3 \rightarrow y = -1$$

$$x = 5 \rightarrow y = -3$$

There is an infinite number of (x, y) pairs :
which satisfy eqns ① & ②.

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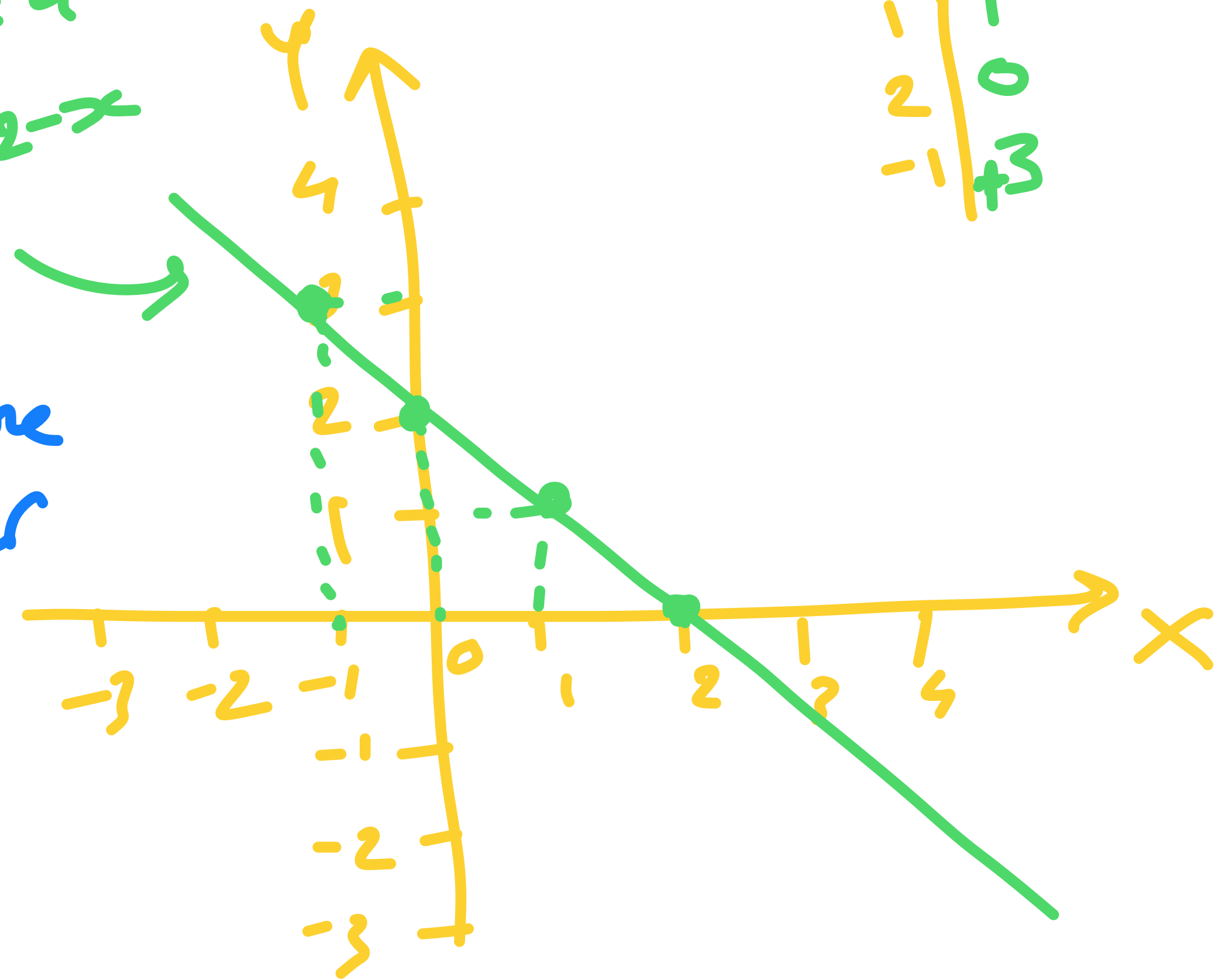
Simultaneous equations -No solution Example

Solution:

$$y = 2 - x$$

$$2y = 4 - 2x$$
$$y = 2 - x$$

These eqns represent coincident lines, therefore, every point on one line is also a point on the other line.



x	y
0	2
1	1
2	0
-1	3

Solution:

Since a line has infinitely many points,
there are infinitely many solutions in common.

Three simultaneous eqns. in three unknowns

Ex solve the eqn

$$\begin{array}{rcl} 2x + y - z & = & 4 \quad \textcircled{1} \\ x + y - z & = & 3 \quad \textcircled{2} \\ 2x + 2y + z & = & 12 \quad \textcircled{3} \end{array}$$

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Simultaneous equations -Three equations

Solution

$$2x + y - \cancel{z} = 4 \quad (1)$$

$$2x + 2y + \cancel{z} = 12 \quad (3)$$

$$\rightarrow 4x + 4y + 0 = 16 \quad (4) = (1) + (3)$$

$$x + y - \cancel{z} = 3 \quad (2)$$

$$2x + 2y + \cancel{z} = 12 \quad (3)$$

$$\rightarrow 3x + 3y + 0 = 15 \quad (5) = (2) + (3)$$

$$2x + y - z = 4 \quad (1)$$

$$x + y - z = 3 \quad (2)$$

$$2x + 2y + z = 12 \quad (3)$$



Solution

$$\begin{array}{rcl} 4x + 3y & = & 16 \quad (4) \\ -3x + 3y & = & 15 \quad (5) \\ \hline x & = & 1 \end{array}$$

Solution

$x=1$, substitute into equations (4) or (5) to solve for y .

$$(4)(1) + 3y = 16$$

(4)

$$4 + 3y = 16$$

$$\begin{array}{r} 3y = 12 \\ \hline y = 4 \end{array}$$

$$\Rightarrow y = 4$$



Solution

To find z , substitute $x=1$, $y=4$ into any of the eqn. ①, ② or ③

$$x + y - z = 3$$

$$1 + 4 - z = 3$$

$$5 - z = 3$$

$$\cancel{-z} = \cancel{-2}$$

→

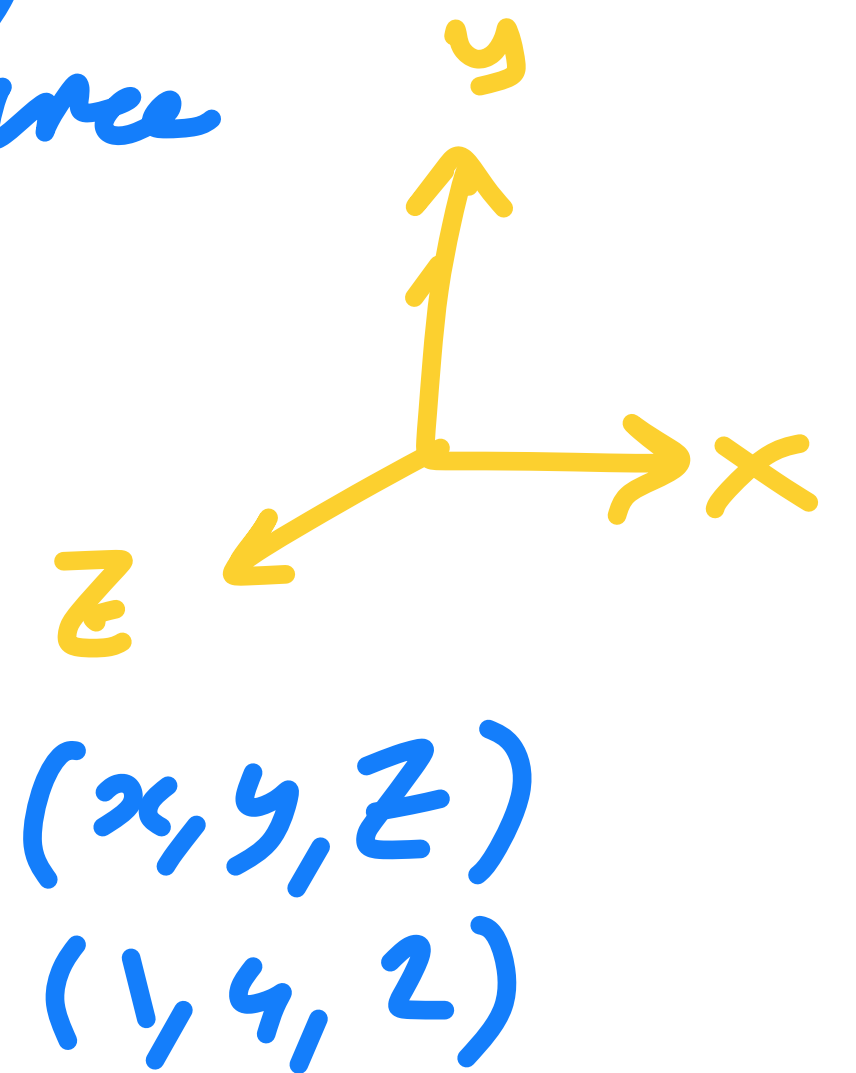
$$z = 2$$



Solution

Therefore, the values which satisfy all three
eqns ①, ② and ③ are

$$\boxed{x=1}, \quad \boxed{y=4} \quad \text{and} \quad \boxed{z=2}.$$



Good market equilibrium occurs when the quantity demanded (Q_d) by consumers and the quantity supplied (Q_s) by producers of a good or service are equal

market equilibrium occurs when the price that a consumer is willing to pay (P_d) is equal to the price that a producer is willing to accept (P_s).

$$Q_d = Q_s ,$$

$$P_d = P_s$$



Example

The demand and supply functions for a good are given as

Demand function

$$P_d = 100 - 0.5Q_d$$

Supply function

$$P_s = 10 + 0.5Q_s$$

Calculate the equilibrium price and quantify algebraically and graphically.

Solution

Market equilibrium occurs when $Q_d = Q_s$ and $P_d = P_s$

$$P_d = 100 - 0.5 Q_d$$

$$P_s = 10 + 0.5 Q_s$$

$$P_d = P_s$$

$$100 - 0.5 Q_d = 10 + 0.5 Q_s$$



Solution

$$100 - 0.5Q = 10 + 0.5Q$$

$$100 - 10 = 0.5Q + 0.5Q$$

$$90 = Q \quad \text{equilibrium quantity}$$

Solution

By substituting $Q=90$ into either equation
to solve for the equilibrium price

$$P = 100 - 0.5Q$$

$$P = 100 - (0.5)(90)$$

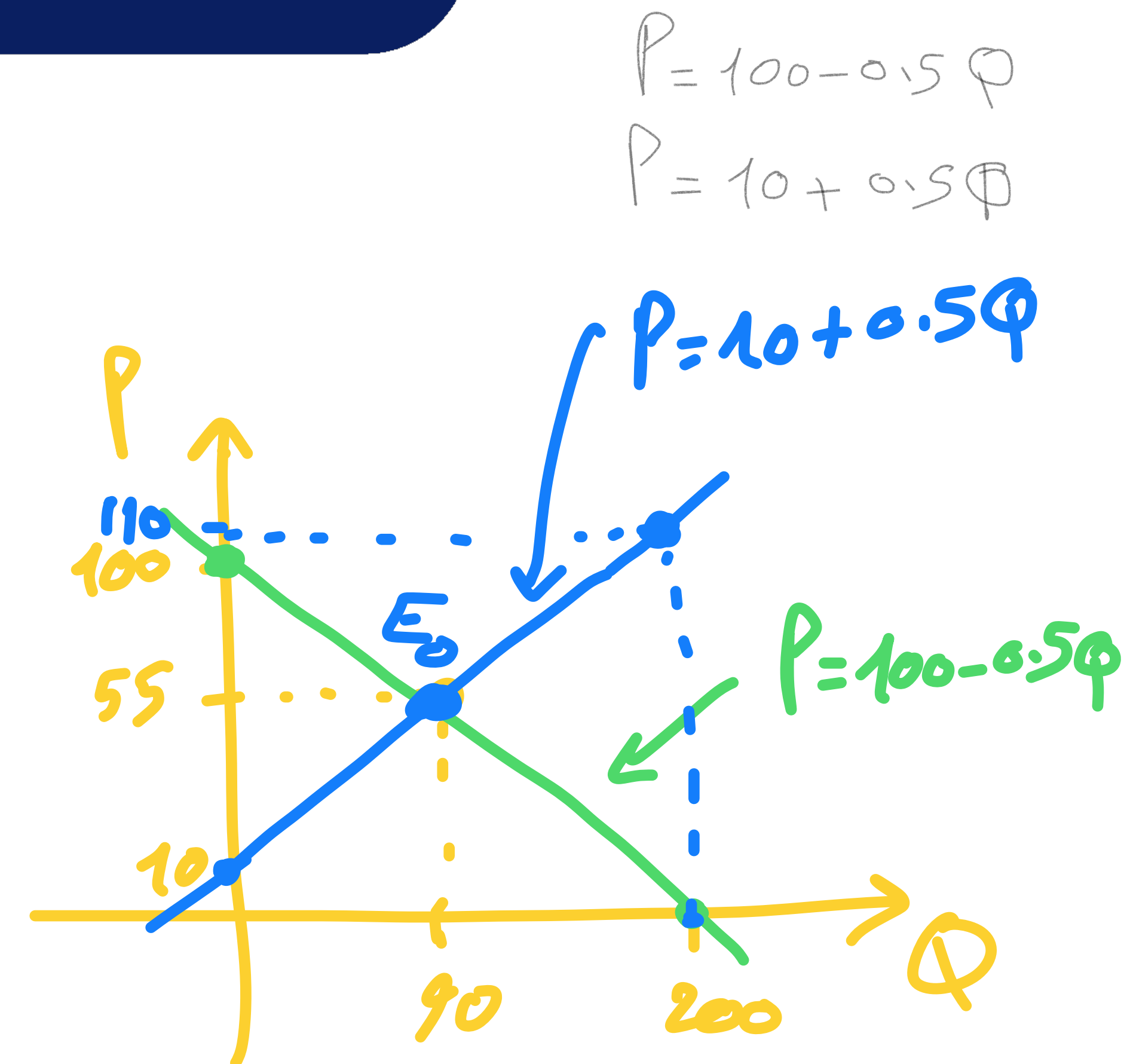
$$P = 100 - 45$$

$$P = 55$$

Solution

The market equilibrium at point E_0 with equilibrium quantity, 90 and equilibrium price £55.

The consumer pays £55 for the good which also the price that the producer receives for the good.



Price ceiling

. Price ceiling is used by governments in cases where they believe that the equilibrium price is too high for the consumer to pay. Thus, price ceiling operates below market equilibrium and are aimed to protecting consumers.

Maximum Price Control

Ex

The demand and supply functions for a good are given by

Demand function

$$P_d = 100 - 0.5Q$$

' supply function

$$P_s = 10 + 0.5Q$$

$$P = £55$$
$$Q = 90$$

A) analyse the effect of the introduction of a price ceiling of £40 in this market.

B) Calculate the profit made by blackmarketters if a black market operated in this market



Solution The equilibrium price and quantity are £55, 90

The price ceiling of £40 is below the equilibrium price of £55

Its effect is analysed by comparing the levels of quantity demanded and supplied at $\bar{P} = £40$



Solution

The quantity demanded at $P=40$

$$P_d = 100 - 0.5 Q_d$$

$$40 = 100 - 0.5 Q_d$$

$$\frac{-60}{-0.5} = \frac{-0.5 Q_d}{-0.5}$$

$$120 = Q_d$$

Solution

The quantity supplied at $P=40$ is

$$P_s = 10 + 0.5Q_s$$

$$\leftarrow 40 = 10 + 0.5Q_s$$

$$\frac{30}{0.5} = \frac{0.5Q_s}{0.5}$$

$$\boxed{60 = Q_s}$$



Solution

$$Q_d = 120$$

$$Q_s = 60$$

There is an excess demand (XD)

$$\begin{aligned} XD &= Q_d - Q_s \\ &= 120 - 60 = 60 \end{aligned}$$

This is referred to as a shortage in the market.



Solution

(b)

$$Q = 60$$

$$P_d = 100 - 0.5 Q_d$$

$$P_d = 100 - (0.5 \times 60) \\ = £70$$

$P_d = 70$ is the price consumers are willing to pay

Solution
(b)

$$Q = 60$$

$$\begin{aligned} TR &= (60)(£70) \\ &= 4200 \end{aligned}$$

$$\begin{aligned} \pi &= TR - TC \\ &= 4200 - 2400 \\ &= 1800 \end{aligned}$$

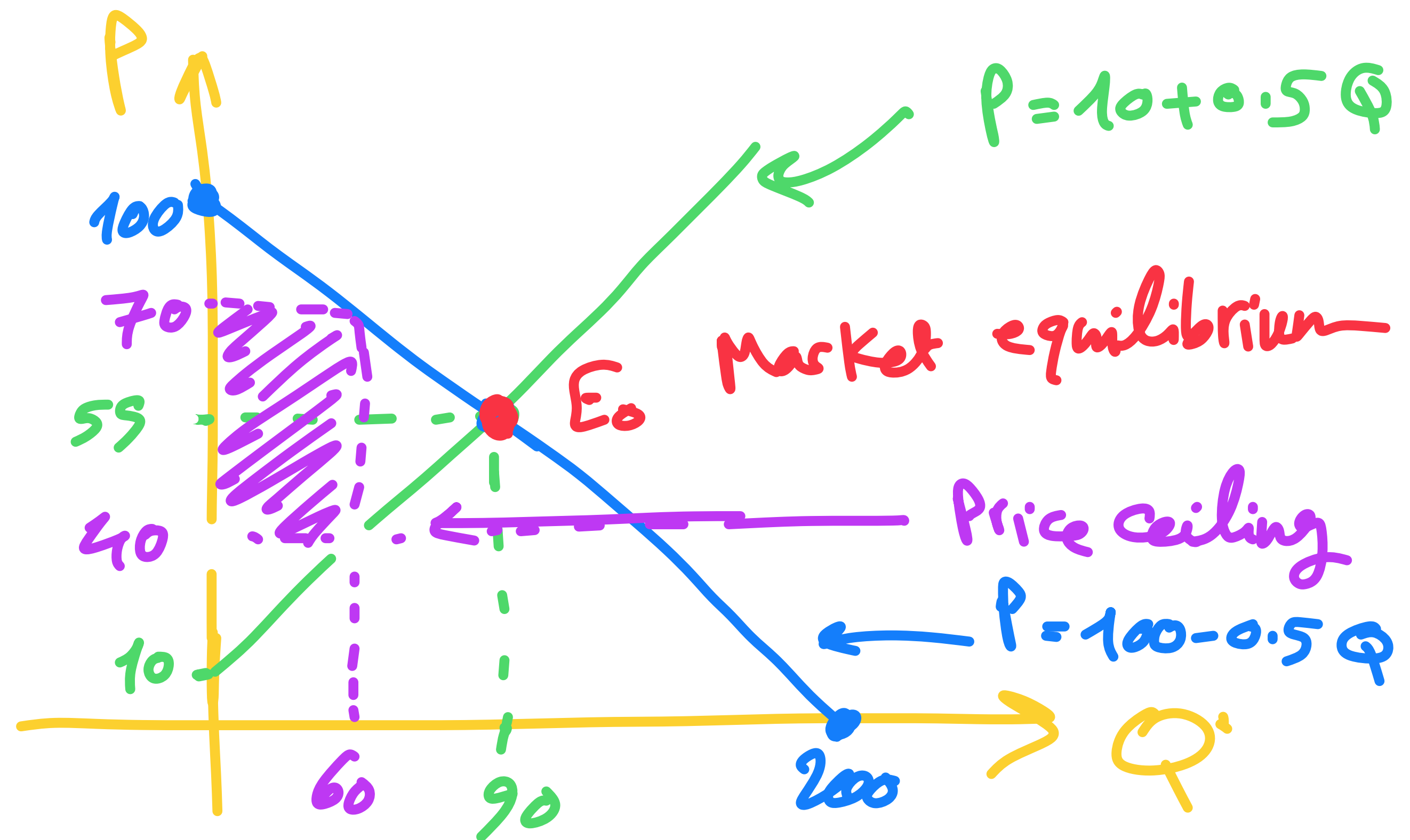
$$\begin{aligned} TC &= (60)(£40) \\ &= 2400 \end{aligned}$$

The profit (π) for the black marketers.

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Equilibrium in the goods and labour markets

Solution
(b)



Thank you

