

Mathematics

Lecture 1

MAT 122
Introduction to Linear Algebra

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- Lecture 1: Introduction to Linear Algebra
- Lecture 2: Scalar and Vector
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- **Linear Algebra Definition**
- **Linear Algebra Goals**
- **Linear Algebra Application**
- **System of linear equations definition**
- **How to solve system of linear equations Algebraically**

What is Linear Algebra?

Linear algebra is the branch of Mathematics concerning linear equations. Linear Algebra is the branch of mathematics aimed at solving systems of linear equations with a finite number of unknowns.

Linear Algebra is one of the most widely used mathematical theories around. Linear Algebra finds applications in virtually every area of mathematics, including Multivariate Calculus, Differential Equations, and Probability Theory.

Where is Linear Algebra used?

It is also widely applied in fields like physics, chemistry, economics, psychology, and engineering. You are even relying on methods from Linear Algebra every time you use an Internet search like Google, the Global Positioning System (GPS), or a cellphone.

Linear algebra is also used in most sciences and fields of engineering, because it allows modeling many natural phenomena, and computing efficiently with such models.

For nonlinear systems, which cannot be modeled with linear algebra, it is often used for dealing with first-order approximations, using the fact that the differential of a multivariate function at a point is the linear map that best approximates the function near that point.

What is the Linear Algebra Goals?

Linear Algebra would like to obtain answers to the following questions:

- **Characterization of solutions:** Are there solutions to a given system of linear equations? How many solutions are there?
- **Finding solutions:** How does the solution set look? What are the solutions?

Applications of Linear Algebra

- Matrices in Engineering, such as a line of springs.
- Graphs and Networks, such as analyzing networks.
- Markov Matrices, Population, and Economics, such as population growth.
- Linear Programming, the simplex optimization method.

- Fourier Series: Linear Algebra for functions, used widely in signal processing.
- Linear Algebra for statistics and probability, such as least squares for regression.
- Computer Graphics, such as the various translation, rescaling and rotation of images.

Definition:

A system of linear equations is a collection of linear equations in two or more variables.

There are three methods to solve a system of linear equations:

- 1) By elimination.
- 2) By substitution.
- 3) By matrices.

Let's Practice

Example: (1) Find x and y

$$x + y = 9 \quad (1)$$

$$x - y = 3 \quad (2)$$

Solution:

$$x + y = 9 \quad (1)$$

$\oplus$

$$\underline{x - y = 3 \quad (2)}$$

we need to eliminate (y)

After adding the two equations: -

$$2x = 12 \quad \text{Then } x = \frac{12}{2} = 6$$

Then will substitute in any equation to get (y), we will substitute in equation (1).

$$6 + y = 9 \quad \Rightarrow \therefore y = 9 - 6 = 3$$

Then $x = 6$ and $y = 3$

Find x & y

$$5x - y = 13 \quad (1)$$

$$2x + 3y = 12 \quad (2)$$

Solution

We will eliminate (y) because it has different signs.

To do that we should multiply the first equation by 3 and the second one by 1

$$\begin{array}{r} 15x - 3\cancel{y} = 39 \\ + \quad 12x + 3\cancel{y} = 12 \\ \hline 17x = 51 \quad \therefore x = 3 \end{array}$$

Then we will substitute in any equation (the originals) to get (y) By sub. In eq. 1

$$5(3) - y = 13$$

$$\therefore 5(3) - 13 = y$$

$$\therefore y = 2$$

$$\text{Then } x = \underline{3} \text{ \& } y = 2$$

Find x & y

$$2x + 4y = 16 \quad (1)$$

$$\underline{3x - 5y = -9} \quad (2)$$

Solution

here we will try to eliminate (x) so, we will multiply equation (1) by 3 and equation (2) by (-2)

$$\begin{array}{r} \cancel{6}x + 12y = 48 \\ -\cancel{6}x + 10y = 18 \\ \hline 22y = 66 \quad \boxed{\therefore y = 3} \end{array}$$

Then substitute in equation (1)

$$2x + 4(3) = 16$$

$$2x = 16 - 12$$

$$\therefore x = 2$$

Find x & y

$$2x + 3y = 5 \quad (1)$$

$$4x + 6y = 15 \quad (2)$$

To eliminate y, we will multiply equation (1) by (6) and equation (2) by (3)

$$\cancel{12}x + \cancel{18}y = \cancel{30}$$

$$\cancel{-12}x - \cancel{18}y = \cancel{-30}$$

Zero

The system has no solution, so it is inconsistent.

Find x & y

$$x + y = 9 \quad (1)$$

$$x - y = 3 \quad (2)$$

Solution :

From equation (1) we can get

$$\boxed{X = 9 - y}$$

Then we will substitute in equation (2) by $x = 9 - y$

$$(9 - y) - y = 3$$

$$9 - 2y = 3$$

$$\boxed{\therefore y = 3}$$

Then to get the value of x we will substitute in the equation $X = 9 - y$

$$X = 9 - 3 = 6$$

Then $X = 6$ and $y = 3$

Find x & y

$$5x - y = 13 \quad (1)$$

$$2x + 3y = 12 \quad (2)$$

Solution:

From equation (1)

$$Y = 5x - 13$$

Then substitute in equation (2)

$$2x + 3(5x - 13) = 12$$

$$2x + 15x - 39 = 12$$

$$17x = 51$$

$$\therefore x = 3$$

By substitute in $y = 5x - 13$

$$\therefore Y = \underline{\underline{5}}(3) - 13 = 15 - 13 = 2$$

Then $x = 3$ and Y = 2

Find x & y

$$2x + 4y = 16 \quad (1)$$

$$3x - 5y = -9 \quad (2)$$

From equation (1)

$$2x = 16 - 4Y$$

$$\boxed{X = 8 - 2Y}$$

By substitute in equation (2)

$$3(\underline{8} - 2Y) - 5Y = -9$$

$$24 - 6y - 5y = -9$$

$$24 - 11y = -9$$

$$-11y = -33$$

$$\boxed{Y = 3}$$

By substitute in $X = 8 - 2y$

$$X = 8 - 2(3) = 2$$

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