

Probability and Statistics

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Lecture 3

جامعة الجميع الذكية

STA 201
Probability



جامعة الجميع الذكية
EVERYONE'S SMART UNIVERSITY

3.1 Basic Concepts of Probability and Counting

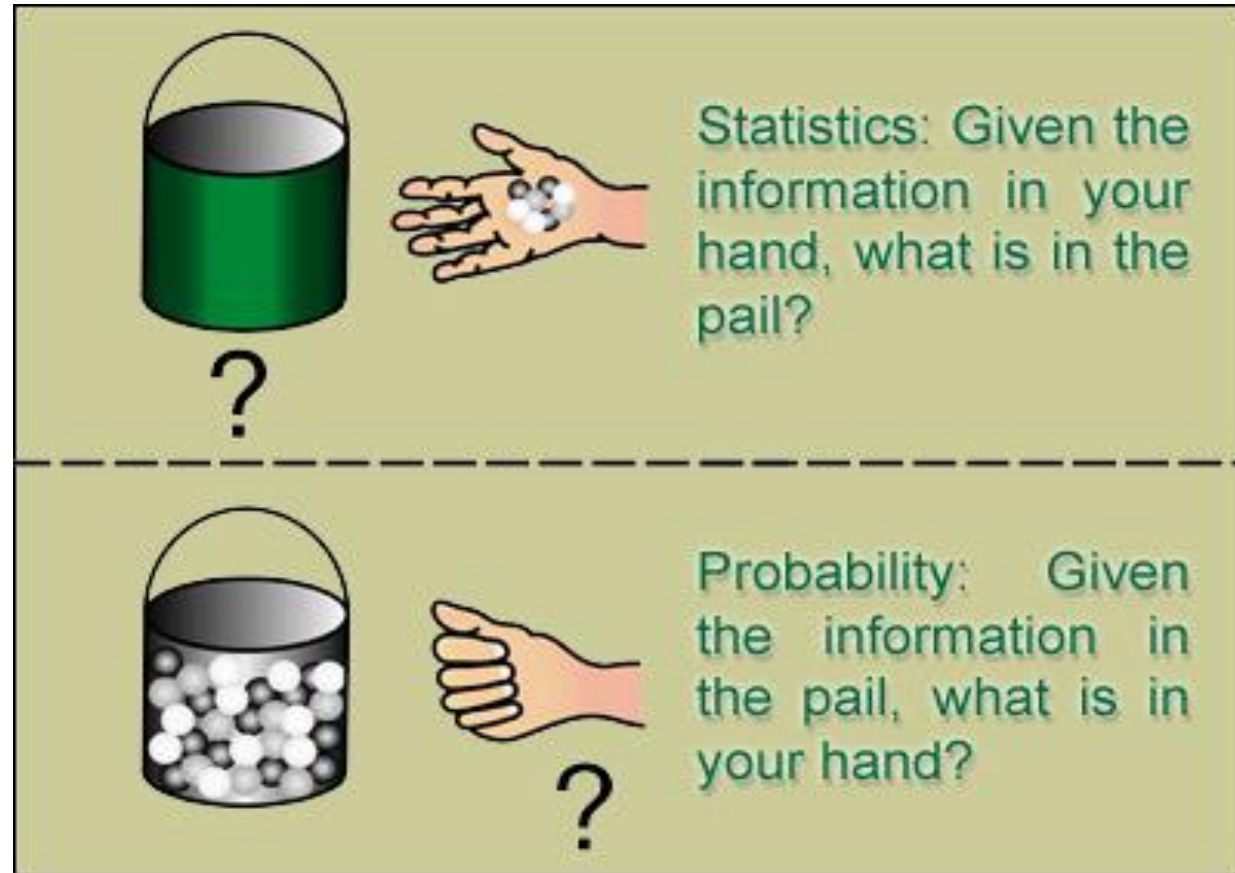
3.2 Conditional Probability and the Multiplication Rule

3.3 The Addition Rule

3.4 Additional Topics in Probability and Counting

3.1 Basic concepts of Probability and Counting

- Identify the sample space of a probability experiment
- Identify simple events
- Distinguish among classical probability, empirical probability, and subjective probability
- Determine the probability of the complement of an event
- Use a tree diagram and the Fundamental Counting Principle to find probabilities



Probability experiment

An action, or trial, through which specific results (counts, measurements, or responses) are obtained.

Outcome

The result of a single trial in a probability experiment.

Sample Space

The set of all possible outcomes of a probability experiment.

Event

Consists of one or more outcomes and is a subset of the sample space.

Probability experiment: Roll a die

Outcome: $\{3\}$

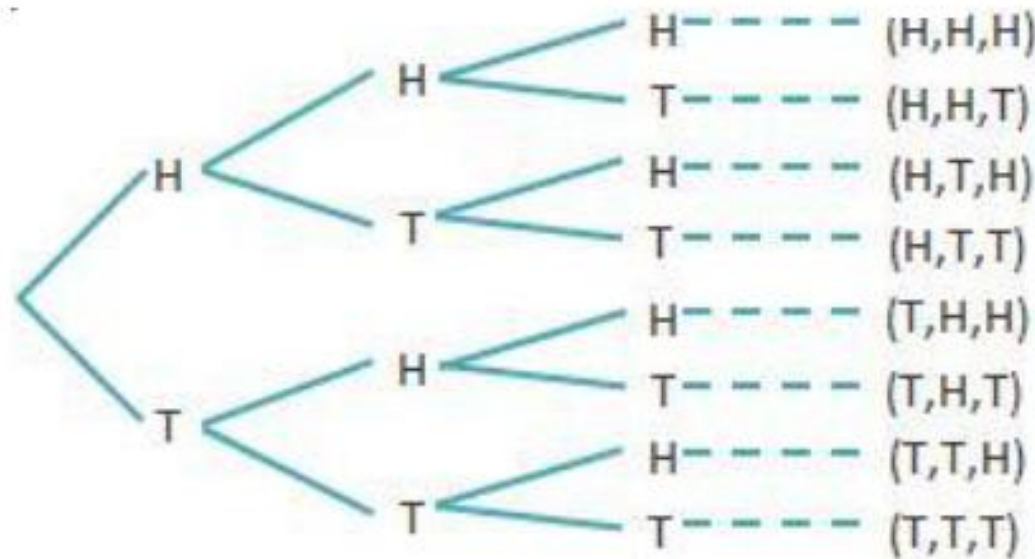
Sample space: $\{1, 2, 3, 4, 5, 6\}$

Event: $\{\text{Die is odd}\} = \{1, 3, 5\}$



Example: Identifying the Sample Space

A probability experiment consists of tossing a three coins. Describe the sample space.



Solution: {HHH, HHT, HTT, HTH, THH, THT, TTH, TTT}

Classical (equally probable outcomes)

$$P(E) = \frac{\text{Number of outcomes in event E}}{\text{Number of outcomes in sample space}}$$

Empirical

$$P(E) = \frac{\text{Frequency of event E}}{\text{Total Frequency}}$$

Subjective

Intuition, educated guesses, and estimates.



1. Event A: rolling a 3 Event A = {3}

$$P(\text{rolling a 3}) = \frac{1}{6} \approx 0.167$$

2. Event B: rolling a 7 Event B = { } (7 is not in the sample space)

$$P(\text{rolling a 7}) = \frac{0}{6} = 0$$

3. Event C: rolling a number less than 5

Event C = {1, 2, 3, 4}

$$P(\text{rolling a number less than 5}) = \frac{4}{6} \approx 0.667$$

You survey a sample of 1000 employees at a company and record the age of each. Find the probability of randomly choosing an employee who is between 25 and 34 years old.

Event A = Age is between 25 and 34

$$P(\text{age 25 to 34}) = \frac{f}{n} = \frac{366}{1000} = 0.366$$

Employee ages	Frequency, f
15 to 24	54
25 to 34	366
35 to 44	233
45 to 54	180
55 to 64	125
65 and over	42
	$\Sigma f = 1000$

Subjective Probability

Intuition, educated guesses, and estimates.

e.g. A doctor may feel a patient has a 90% chance of a full recovery.

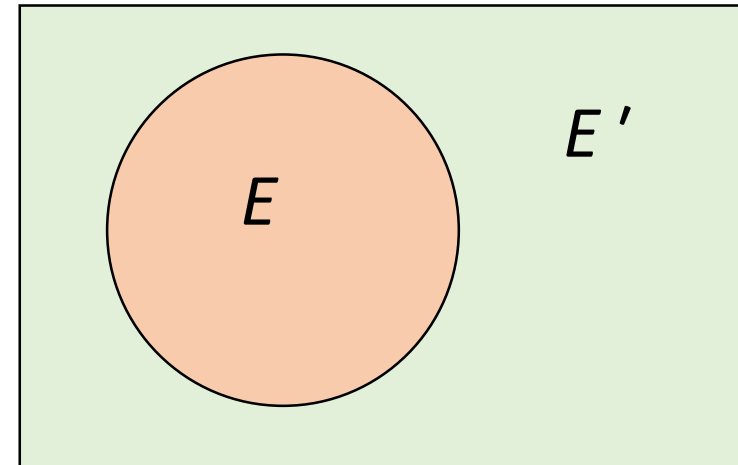
Probability = 0.9



Complement of event E

The set of all outcomes in a sample space that are **not** included in event E .

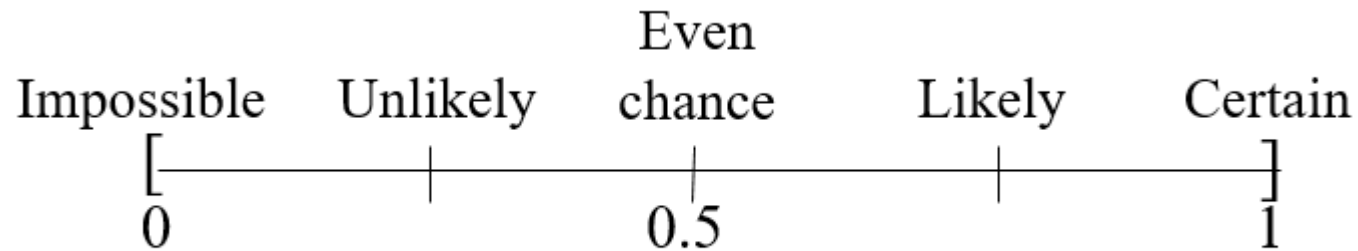
- $P(E') + P(E) = 1$
- $P(E) = 1 - P(E')$
- $P(E') = 1 - P(E)$



Range of probabilities rule

The probability of an event E is between 0 and 1, inclusive.

$$0 \leq P(E) \leq 1$$



Section 3.2 Conditional Probability and the Multiplication Rule

Conditional Probability

The probability of an event occurring, given that another event has already occurred

Denoted $P(B | A)$ (read “probability of B , given A ”)

Conditional Probability Rule:

$$P(B|A) = \frac{P(B \text{ and } A)}{P(A)}$$

Example: Finding Conditional Probabilities

The table shows the results of a study in which researchers examined a child's IQ and the presence of a specific gene in the child. Find the probability that a child has a high IQ, given that the child has the gene.

	Gene Present	Gene not present	Total
High IQ	33	19	52
Normal IQ	39	11	50
Total	72	30	102

There are 72 children who have the gene. So, the sample space consists of these 72 children.

	Gene Present	Gene not present	Total
High IQ	33	19	52
Normal IQ	39	11	50
Total	72	30	102

Of these, 33 have a high IQ

$$P(\text{High IQ} | \text{gene present}) = \frac{P(\text{High IQ and gene present})}{P(\text{gene present})}$$

$$P(B | A) = P(\text{high IQ} | \text{gene present}) = \frac{33}{72} \approx 0.458$$

Example: Probabilities from Contingency Table

The results of responses when a sample of adults in 3 cities was asked if they liked a new juice is:

	Omaha	Seattle	Miami	Total
Yes	100	150	150	400
No	125	130	95	350
Undecided	75	170	5	250
Total	300	450	250	1000

One of the responses is selected at random. Find:

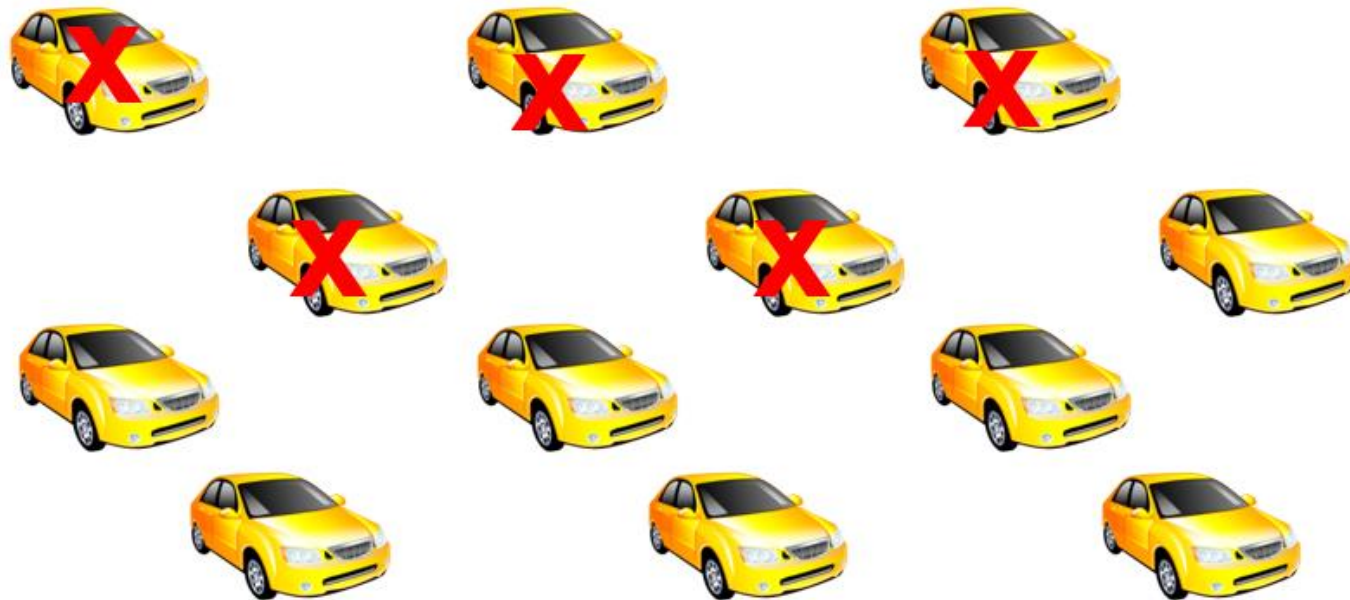
1. $P(\text{Yes})$
2. $P(\text{Seattle})$
3. $P(\text{Miami})$
4. $P(\text{No} \mid \text{Miami})$ = probability the answer is “no” given that adult is a resident of Miami

	Omaha	Seattle	Miami	Total
Yes	100	150	150	400
No	125	130	95	350
Undecided	75	170	5	250
Total	300	450	250	1000

1. $P(\text{Yes}) = 400 / 1000 = 0.4$
2. $P(\text{Seattle}) = 450 / 1000 = 0.45$
3. $P(\text{Miami}) = 250 / 1000 = 0.25$
4. $P(\text{No} | \text{given Miami}) = 95 / 250 = 0.38$

Imagine this

Two cars are selected from a production line of 12 where 5 are defective. Find the probability **both cars are defective**.



To find the probability that two events, A and B will occur in sequence, multiply the probability A occurs by the conditional probability B occurs, given A has already occurred.

A = first car is defective B = second car is defective.

$$\mathbf{P(A \text{ and } B) = P(A) \times P(B|A)}$$

$$P(A) = 5/12$$

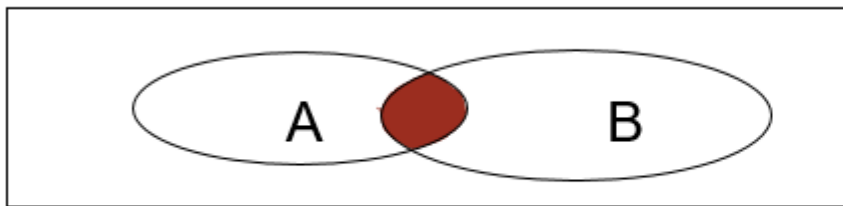
$$P(B|A) = 4/11$$

$$P(A \text{ and } B) = 5/12 \times 4/11 = 5/33 = 0.1515$$

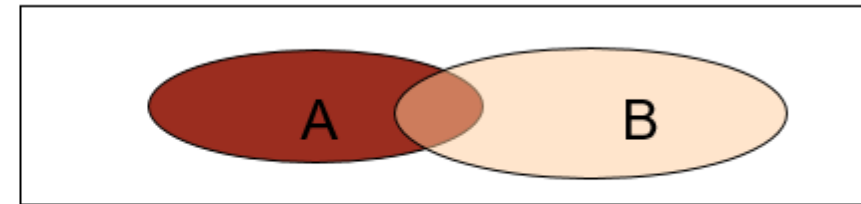
Compare “A and B” to “A or B”

The compound event “A and B” means that A and B *both* occur in the same trial. Use the multiplication rule to find $P(A \text{ and } B)$.

The compound event “A or B” means either A can occur without B, B can occur without A or both A and B can occur.



A and B



A or B

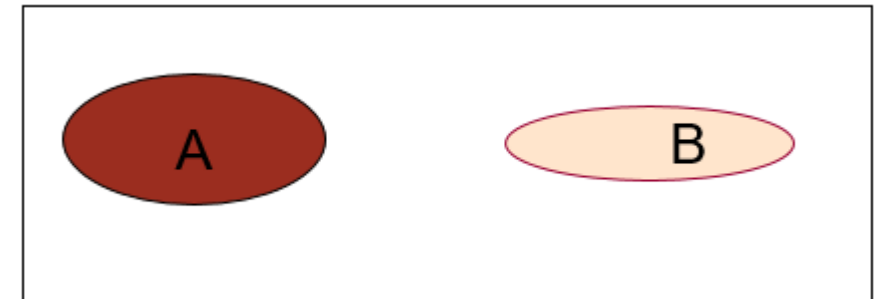
Two events, A and B, are mutually exclusive if they cannot occur in the same trial.

A = A person is under 21 years old

B = A person is running for the U.S. Senate

A = A person was born in Philadelphia

B = A person was born in Houston



Mutually exclusive if $P(A \text{ and } B) = 0$

When event A occurs it excludes event B in the same trial.

If two events can occur in the same trial, they are NOT mutually exclusive.

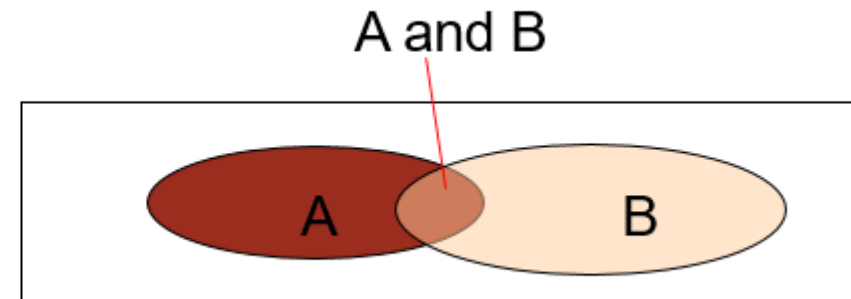
A = A person is under 25 years old

B = A person is a lawyer

A = A person was born in Philadelphia

B = A person watches “Jeopardy” on TV

Non-mutually exclusive
 $P(A \text{ and } B) \neq 0$



The probability that one or the other of two events will occur is: $P(A) + P(B) - P(A \text{ and } B)$

A card is drawn from a deck. Find the probability it is a king or it is red.

The probability that one or the other of two events will occur is: $P(A) + P(B) - P(A \text{ and } B)$

A card is drawn from a deck. Find the probability it is a king or it is red.

A = the card is a king

B = the card is red.

$$P(A) = 4/52 \text{ and } P(B) = 26/52$$

$$\text{but } P(A \text{ and } B) = 2/52$$

$$\begin{aligned} P(A \text{ or } B) &= 4/52 + 26/52 - 2/52 \\ &= 28/52 = 0.538 \end{aligned}$$

The Additive Rule

A blood bank catalogs the types of blood given by donors during the last five days. A donor is selected at random.

- 1- Find the probability the donor has type O or type A blood.
- 2- Find the probability the donor has type B or is Rh-negative

	Type O	Type A	Type B	Type AB	Total
Rh-Positive	156	139	37	12	344
Rh-Negative	28	25	8	4	65
Total	184	164	45	16	409



1- Find the probability the donor has type O or type A blood.

The events are mutually exclusive (a donor cannot have type O blood and type A blood)

	Type O	Type A	Type B	Type AB	Total
Rh-Positive	156	139	37	12	344
Rh-Negative	28	25	8	4	65
Total	184	164	45	16	409

$$P(\text{type O or type A}) = P(\text{type O}) + P(\text{type A})$$

$$\begin{aligned}
 &= \frac{184}{409} + \frac{164}{409} \\
 &= \frac{348}{409} \approx 0.851
 \end{aligned}$$

2- Find the probability the donor has type B or is Rh-negative

	Type O	Type A	Type B	Type AB	Total
Rh-Positive	156	139	37	12	344
Rh-Negative	28	25	8	4	65
Total	184	164	45	16	409

$$\begin{aligned}
 &P(\text{type B or Rh - neg}) \\
 &= P(\text{type B}) + P(\text{Rh - neg}) - P(\text{type B and Rh - neg}) \\
 &= \frac{45}{409} + \frac{65}{409} - \frac{8}{409} = \frac{102}{409} \approx 0.249
 \end{aligned}$$

3. 4 Additional Topics in Probability and Counting

1- Permutation

2- Combination

Permutation of n objects taken r at a time

The number of different permutations of n distinct objects taken r at a time

$${}_nP_r = \frac{n!}{(n-r)!} \quad \text{where } r \leq n$$

Find the number of ways of forming three-digit codes from 10 numbers in which no digit is repeated.

Solution

$$\begin{aligned} {}_{10}P_3 &= \frac{10!}{(10-3)!} = \frac{10!}{7!} \\ &= \frac{10 \cdot 9 \cdot 8 \cdot \cancel{7} \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{7} \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}} \\ &= 720 \text{ ways} \end{aligned}$$

Combination of n objects taken r at a time

A selection of r objects from a group of n objects without regard to order

$${}_nC_r = \frac{n!}{(n-r)!r!}$$

A state's department of transportation plans to develop a new section of interstate highway and receives 16 bids for the project. The state plans to hire four of the bidding companies. How many different combinations of four companies can be selected from the 16 bidding companies?

Solution:

- $n = 16, r = 4$
- Order is not important

$$\begin{aligned}
 {}_{16}C_4 &= \frac{16!}{(16-4)!4!} \\
 &= \frac{16!}{12!4!} \\
 &= \frac{16 \cdot 15 \cdot 14 \cdot 13 \cdot 12!}{12! \cdot 4 \cdot 3 \cdot 2 \cdot 1} \\
 &= 1820 \text{ different combinations}
 \end{aligned}$$

**Basic Concepts of
Probability and Counting**

Conditional Probability:

$$P(B|A) = \frac{P(B \text{ and } A)}{P(A)}$$

**Additional Topics in
Probability and Counting**

1- Permutation

2- Combination

Multiplication Rule: (and)

$$P(A \text{ and } B) = P(A) \times P(B|A)$$

The Addition Rule: (or)

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$