

Probability and Statistics

Prepared by Dr. Alfaisal Hasan

STA 201
Probability and Statistics

Lecture NO 3



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Chapter

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الفصل الثاني: الإحصاء الوصفي

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Probability

3.1 Basic Concepts of Probability and Counting

3.2 Conditional Probability and the Multiplication Rule

3.3 The Addition Rule

3.4 Additional Topics in Probability and Counting

3.1 المفاهيم الأساسية للاحتتمالات

3.2 الاحتمال الشرطي وقاعدة الضرب

3.3 قاعدة الإضافة

3.4 4 موضوعات إضافية في الاحتمالات

Chap Basic concepts of Probability and Counting

- Identify the sample space of a probability experiment
- Identify simple events
- Distinguish among classical probability, empirical probability, and subjective probability
- Determine the probability of the complement of an event
- Use a tree diagram and the Fundamental Counting Principle to find probabilities

تحديد فضاء العينة للتجربة الاحتمالية

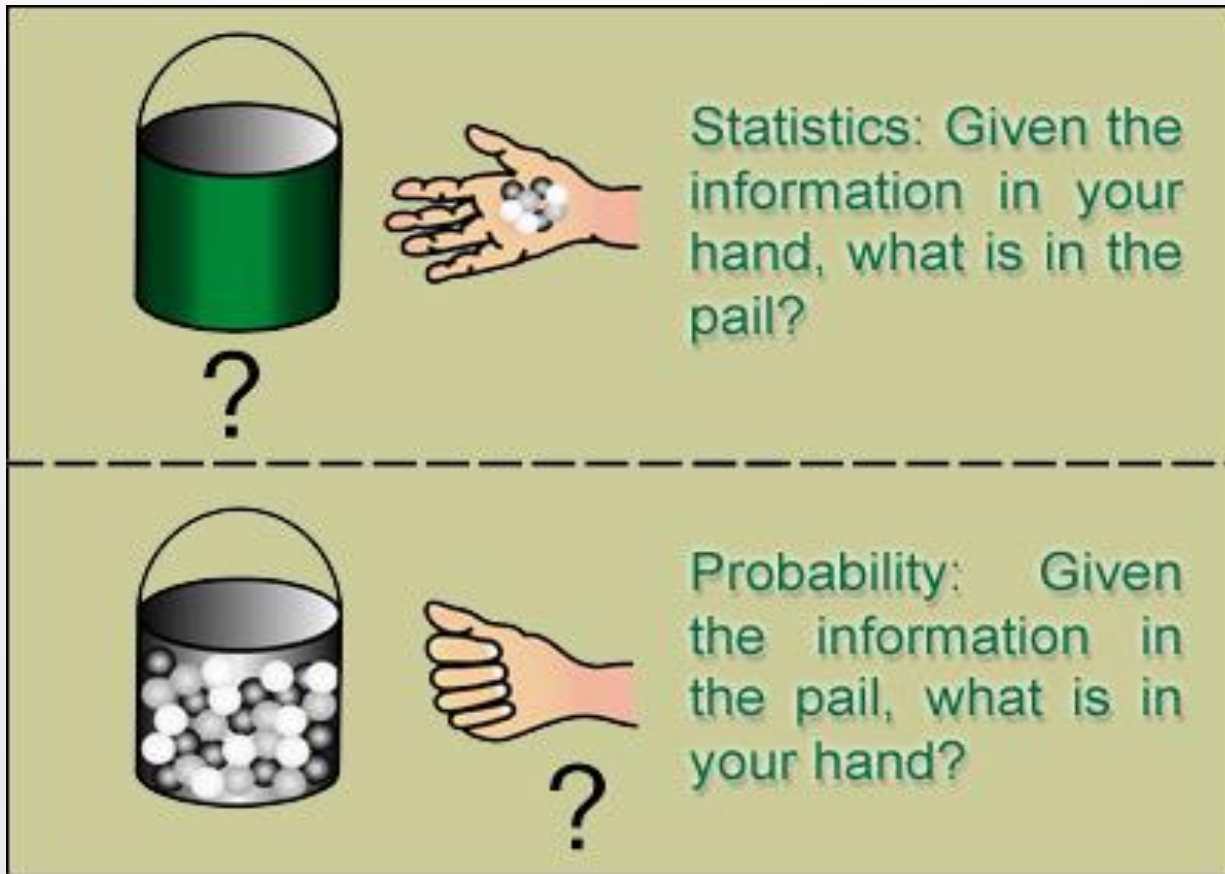
تحديد الأحداث البسيطة

يُميز بين الاحتمال الكلاسيكي والاحتمال التجريبي والاحتمال الذاتي

تحديد احتمال الحدث المكمل

استخدم مخطط الشجرة البيانية ومبدأ العد الأساسي لإيجاد الاحتمالات

Chap Probability and Statistics



الاحصاء

عن طريق المعلومات التي في يديك نستطيع أن نعلم ما بداخل

الصندوق

الاحتمال

عن طريق المعلومات التي في الصندوق نستطيع أن نعلم ما بداخل

يديك

Probability Experiments

Chapter

Probability experiment

An action, or trial, through which specific results (counts, measurements, or responses) are obtained.

Outcome

The result of a single trial in a probability experiment.

Sample Space

The set of all possible outcomes of a probability experiment.

Event

Consists of one or more outcomes and is a subset of the sample space.

تجربة الاحتمالية

إجراء أو تجربة يتم من خلالها الحصول على نتائج محددة (أعداد أو قياسات أو استجابات).

مخرجات

نتيجة تجربة واحدة في تجربة الاحتمالية.

فضاء العينة

مجموعة كل النتائج الممكنة للتجربة الاحتمالية.

حدث

يتكون من نتيجة واحدة أو أكثر، وهو عبارة عن مجموعة فرعية من فضاء العينة.

Probability Experiments

Probability experiment: Roll a die

Outcome: $\{3\}$

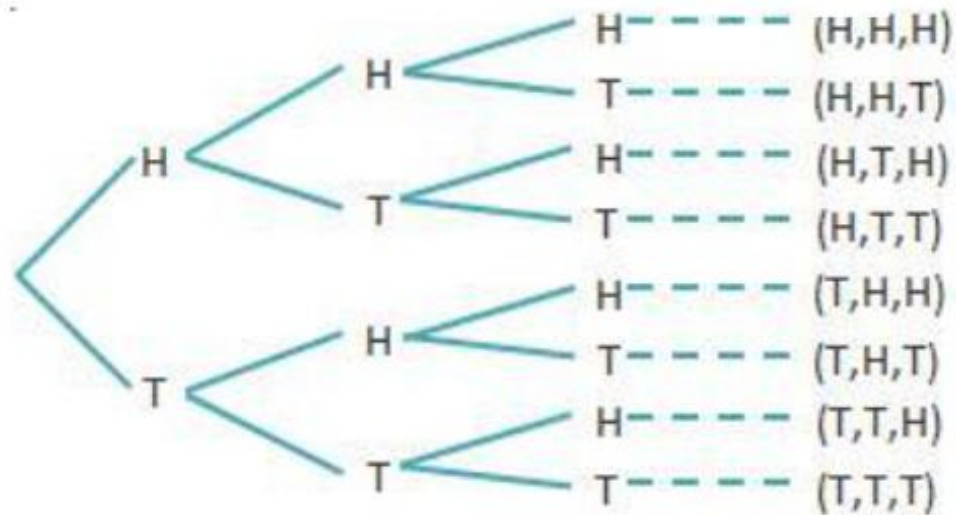
Sample space: $\{1, 2, 3, 4, 5, 6\}$

Event: $\{\text{Die is odd}\} = \{1, 3, 5\}$



Example: Identifying the Sample Space

A probability experiment consists of tossing a three coins.
Describe the sample space.



Solution: {HHH, HHT, HTT, HTH, THH, THT, TTH, TTT}

Types of Probabilities

Classical (equally probable outcomes)

$$P(E) = \frac{\text{Number of outcomes in event E}}{\text{Number of outcomes in sample space}}$$

Empirical

$$P(E) = \frac{\text{Frequency of event E}}{\text{Total Frequency}}$$

Subjective

Intuition, educated guesses, and estimates.

Example of Finding Classical Probability

1. Event A: rolling a 3 Event A = {3}

$$P(\text{rolling a 3}) = \frac{1}{6} \approx 0.167$$



2. Event B: rolling a 7 Event B = { } (7 is not in the sample space)

$$P(\text{rolling a 7}) = \frac{0}{6} = 0$$

3. Event C: rolling a number less than 5

Event C = {1, 2, 3, 4}

$$P(\text{rolling a number less than 5}) = \frac{4}{6} \approx 0.667$$

Example of Finding Empirical Probabilities

Chapter

You survey a sample of 1000 employees at a company and record the age of each. Find the probability of randomly choosing an employee who is between 25 and 34 years old.

Event A = Age is between 25 and 34

$$P(\text{age 25 to 34}) = \frac{f}{n} = \frac{366}{1000} = 0.366$$

Employee ages	Frequency, f
15 to 24	54
25 to 34	366
35 to 44	233
45 to 54	180
55 to 64	125
65 and over	42
	$\Sigma f = 1000$

Example of Subjective Probabilities

Subjective Probability

Intuition, educated guesses, and estimates.

e.g. A doctor may feel a patient has a 90% chance of a full recovery.

Probability = 0.9



الاحتمال الذاتي

الحدس والتخمينات المتعلمة والتقدير.

على سبيل المثال قد يشعر الطبيب أن لدى

المريض فرصة بنسبة 90% للشفاء التام.

الاحتمال = 0.9

Definition: Complementary Events

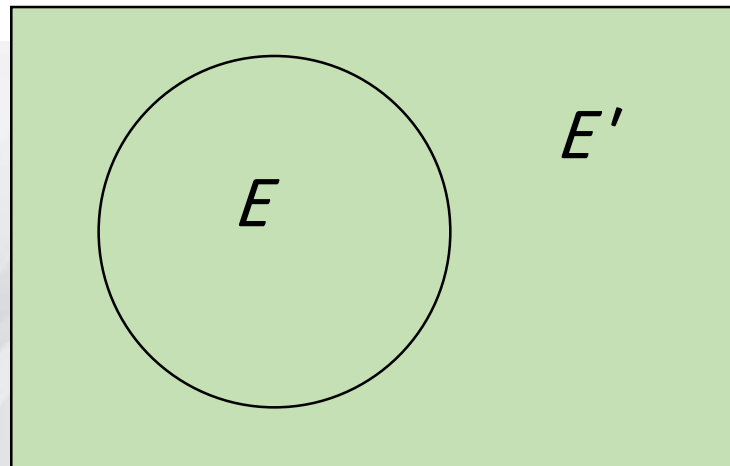
Complement of event E

The set of all outcomes in a sample space that are not included in event E .

مجموعة جميع النتائج في فضاء العينة التي لم يتم تضمينها في الحدث

$$P(E') + P(E) = 1$$

- $P(E) = 1 - P(E')$
- $P(E') = 1 - P(E)$



Example

Chapter

If the probability of an event is equal to 0.4,
Find the probability of its complement event

إذا كان احتمال وقوع حدث يساوي 0.4
أوجد احتمال الحدث المكمل له

Example

Chapter

If the probability of an event is equal to 0.4,
Find the probability of its complement event

إذا كان احتمال وقوع حدث يساوي 0.4
أوجد احتمال الحدث المكمل له

$$P(A) = 0.4$$

$$P(A') = 1 - P(A)$$

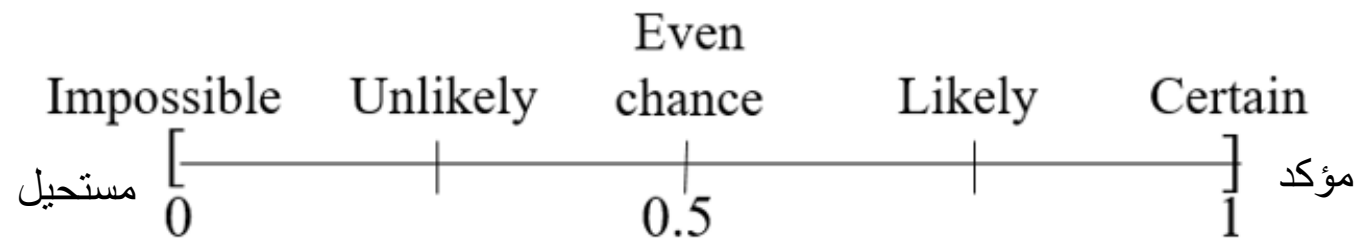
$$P(A') = 1 - 0.4 = 0.6$$

Range of Probabilities Rule

Range of probabilities rule

The probability of an event E is between 0 and 1, inclusive.

$$0 \leq P(E) \leq 1$$



Example

Chapter

If the probability of an event is equal to 1.0,
Find the probability of its complement event

إذا كان احتمال وقوع حدث يساوي 1.0
أوجد احتمال الحدث المكمل له

Example

Chapter

If the probability of an event is equal to 1.0,
Find the probability of its complement event

إذا كان احتمال وقوع حدث يساوي 1.0
أوجد احتمال الحدث المكمل له

$$P(A) = 1.0$$

Certain Event

حدث مؤكد

$$P(A') = 1 - P(A)$$

$$P(A') = 1 - 1 = 0$$

Impossible Event

حدث مستحيل

Section 3.2 Conditional Probability and the Multiplication Rule

Conditional Probability

The probability of an event occurring, given that another event has already occurred

Denoted $P(B | A)$ (read “probability of B , given A ”) بشرط أن

Conditional Probability Rule: الاحتمال الشرطي

$$P(B|A) = \frac{P(B \text{ and } A)}{P(A)}$$

Example

Chapter

If the probability of an event A, $P(A) = 0.6$, $P(B \text{ and } A) = 0.3$
Find the probability of B given that A

$$P(A) = 0.6, P(B \text{ and } A) = 0.3 \quad \text{إذا كان}$$

أوجد احتمال حدوث B بشرط حدوث A

Example

Chapter

If the probability of an event A, $P(A) = 0.6$, $P(B \text{ and } A) = 0.3$
Find the probability of B given that A

$$P(A) = 0.6, P(B \text{ and } A) = 0.3 \quad \text{إذا كان}$$

أوجد احتمال حدوث B بشرط حدوث A

$$P(B / A) = \frac{P(B \text{ and } A)}{P(A)}$$

$$P(B / A) = \frac{0.3}{0.6} = 0.5$$

Example: Finding Conditional Probabilities

The table shows the results of a study in which researchers examined a child's IQ and the presence of a specific gene in the child. Find the probability that a child has a high IQ, given that the child has the gene.

	Gene Present	Gene not present	Total
High IQ	33	19	52
Normal IQ	39	11	50
Total	72	30	102

الجدول التالي يوضح نتائج دراسة لباحث تم فيها اختبار وجود جين الذكاء في أطفال. أوجد احتمال الطفل الذي تم اختياره عشوائياً من بين العينة أن يكون له معدل ذكاء عالي **بشرط** وجود جين الذكاء به.

Solution: Finding Conditional Probabilities

There are 72 children who have the gene. So, the sample space consists of these 72 children.

	Gene Present	Gene not present	Total
High IQ	33	19	52
Normal IQ	39	11	50
Total	72	30	102

Of these, 33 have a high IQ

$$P(\text{High IQ} | \text{gene present}) = \frac{P(\text{High IQ and gene present})}{P(\text{gene present})}$$

$$P(B | A) = P(\text{high IQ} | \text{gene present}) = \frac{33}{72} \approx 0.458$$

Example: Probabilities from Contingency Table

The results of responses when a sample of adults in 3 cities was asked if they liked a new juice is:

	Omaha	Seattle	Miami	Total
Yes	100	150	150	400
No	125	130	95	350
Undecided	75	170	5	250
Total	300	450	250	1000

One of the responses is selected at random. Find:

1. $P(\text{Yes})$
2. $P(\text{Seattle})$
3. $P(\text{Miami})$
4. $P(\text{No} \mid \text{Miami})$ = probability the answer is “no” given that adult is a resident of Miami

Solution

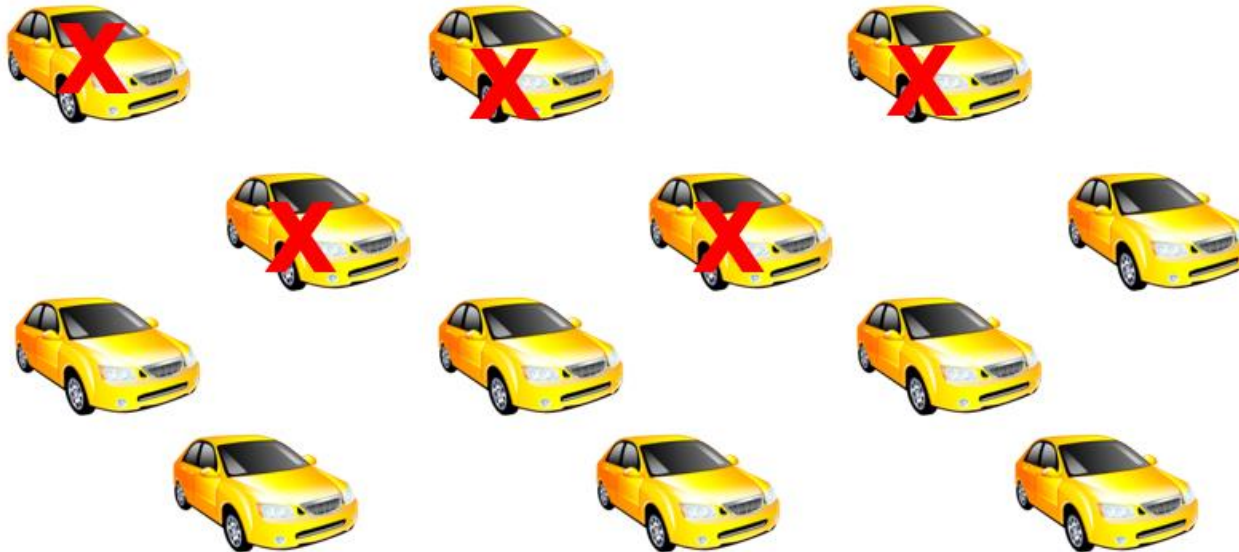
	Omaha	Seattle	Miami	Total
Yes	100	150	150	400
No	125	130	95	350
Undecided	75	170	5	250
Total	300	450	250	1000

1. $P(\text{Yes}) = 400 / 1000 = 0.4$
2. $P(\text{Seattle}) = 450 / 1000 = 0.45$
3. $P(\text{Miami}) = 250 / 1000 = 0.25$
4. $P(\text{No} | \text{given Miami}) = 95 / 250 = 0.38$

Imagine this

Chapter

Two cars are selected from a production line of 12 where 5 are defective. Find the probability **both cars are defective**.



تم اختيار سيارتين من 12 سيارة منتجة مع وجود
5 بهم عيوب في الصناعة. أوجد احتمال أن
السيارتين التي تم اختيارهما بصورة عشوائية
بهما عيوب صناعة

Multiplication Rule

To find the probability that two events, A and B will occur in sequence, multiply the probability A occurs by the conditional probability B occurs, given A has already occurred.

A = first car is defective B = second car is defective.

$$\mathbf{P(A \text{ and } B) = P(A) \times P(B|A)}$$

$$P(A) = 5/12$$

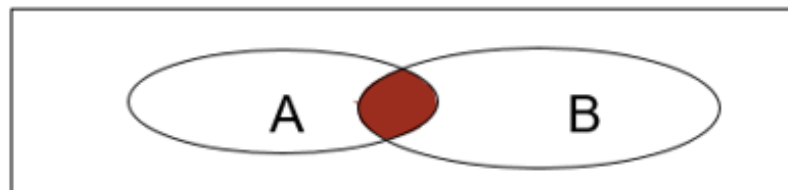
$$P(B|A) = 4/11$$

$$P(A \text{ and } B) = 5/12 \times 4/11 = 5/33 = 0.1515$$

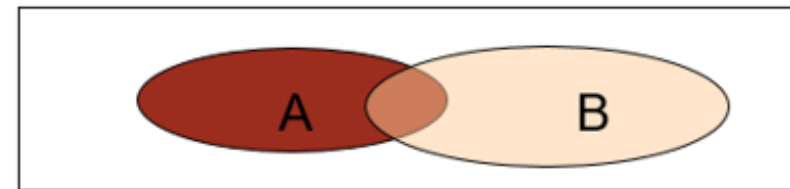
Compare “A and B” to “A or B”

The compound event “A and B” means that A and B *both* occur in the same trial. Use the multiplication rule to find $P(A \text{ and } B)$.

The compound event “A or B” means either A can occur without B, B can occur without A or both A and B can occur.



A and B



A or B

Mutually Exclusive Events

الأحداث المتنافية

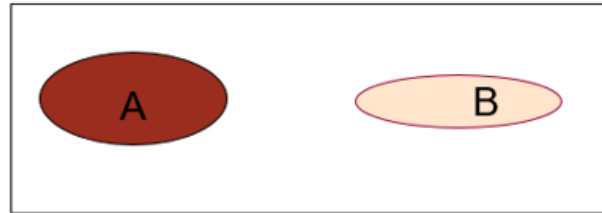
Two events, A and B, are mutually exclusive if they cannot occur in the same trial.

A = A person is under 21 years old

B = A person is running for the U.S. Senate

A = A person was born in Philadelphia

B = A person was born in Houston



Mutually exclusive if $P(A \text{ and } B) = 0$

When event A occurs it excludes event B in the same trial.

Non-Mutually Exclusive Events

If two events can occur in the same trial, they are NOT mutually exclusive.

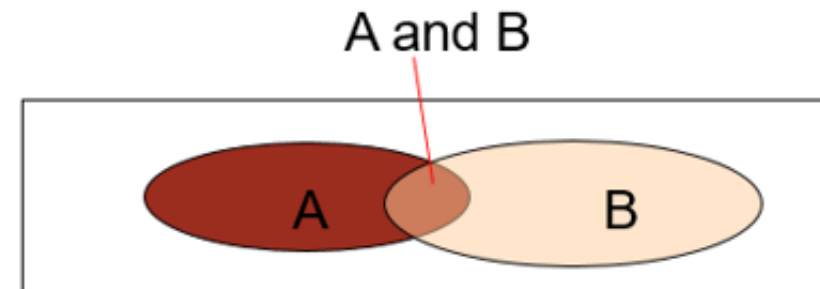
A = A person is under 25 years old

B = A person is a lawyer

A = A person was born in Philadelphia

B = A person watches “Jeopardy” on TV

Non-mutually exclusive
 $P(A \text{ and } B) \neq 0$



Section 3.3: The Additive Rule

The probability that one or the other of two events will occur is: $P(A) + P(B) - P(A \text{ and } B)$

A card is drawn from a deck. Find the probability it is a king or it is red.

Example

Chapter

If the probability, $P(A) = 0.6$, $P(B) = 0.5$, $P(A \text{ and } B) = 0.2$
Find the probability of A or B occur

إذا كان $P(A) = 0.6$, $P(B) = 0.5$, $P(A \text{ and } B) = 0.2$

أوجد احتمال حدوث A أو B

Example

Chapter

If the probability, $P(A) = 0.6$, $P(B) = 0.5$, $P(A \text{ and } B) = 0.2$
Find the probability of A or B occur

$$P(A) = 0.6, P(B) = 0.5, P(A \text{ and } B) = 0.2 \quad \text{إذا كان}$$

أوجد احتمال حدوث B أو A

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(A \text{ or } B) = 0.6 + 0.5 - 0.2 = 0.9$$

The Additive Rule

The probability that one or the other of two events will occur is: $P(A) + P(B) - P(A \text{ and } B)$

A card is drawn from a deck. Find the probability it is a king or it is red.

A = the card is a king

B = the card is red.

$$P(A) = 4/52 \text{ and } P(B) = 26/52$$

$$\text{but } P(A \text{ and } B) = 2/52$$

$$\begin{aligned} P(A \text{ or } B) &= 4/52 + 26/52 - 2/52 \\ &= 28/52 = 0.538 \end{aligned}$$

The Additive Rule

A blood bank catalogs the types of blood given by donors during the last five days. A donor is selected at random.

- 1- Find the probability the donor has type O or type A blood.
- 2- Find the probability the donor has type B or is Rh-negative

	Type O	Type A	Type B	Type AB	Total
Rh-Positive	156	139	37	12	344
Rh-Negative	28	25	8	4	65
Total	184	164	45	16	409



يقوم بنك الدم بفهرسة أنواع الدم التي قدمها المتبرعون خلال الأيام الخمسة الماضية. يتم اختيار المتبرع بشكل عشوائي.

1- أوجد احتمال أن يكون المتبرع من فصيلة الدم O أو فصيلة الدم A

2- أوجد احتمال أن يكون المتبرع من فصيلة الدم B أو سالب عامل Rh

The Additive Rule

1- Find the probability the donor has type O or type A blood.

The events are mutually exclusive (a donor cannot have type O blood and type A blood)

	Type O	Type A	Type B	Type AB	Total
Rh-Positive	156	139	37	12	344
Rh-Negative	28	25	8	4	65
Total	184	164	45	16	409

$$P(\text{type O or type A}) = P(\text{type O}) + P(\text{type A})$$

$$\begin{aligned}
 &= \frac{184}{409} + \frac{164}{409} \\
 &= \frac{348}{409} \approx 0.851
 \end{aligned}$$

The Additive Rule

2- Find the probability the donor has type B or is Rh-negative

	Type O	Type A	Type B	Type AB	Total
Rh-Positive	156	139	37	12	344
Rh-Negative	28	25	8	4	65
Total	184	164	45	16	409

$$\begin{aligned}
 &P(\text{type B or Rh-neg}) \\
 &= P(\text{type B}) + P(\text{Rh-neg}) - P(\text{type B and Rh-neg}) \\
 &= \frac{45}{409} + \frac{65}{409} - \frac{8}{409} = \frac{102}{409} \approx 0.249
 \end{aligned}$$

3. 4 Additional Topics in Probability and Counting

1- Permutation

التباديل

2- Combination

التوافيق

Permutation

Permutation of n objects taken r at a time

The number of different permutations of n distinct objects taken r at a time

$${}_n P_r = \frac{n!}{(n - r)!} \quad \text{where } r \leq n$$

عدد التباديل المختلفة لعدد n من العناصر
مأخوذة من r في المرة الواحدة

Permutation

Find the number of ways of forming three-digit codes from 10 numbers in which no digit is repeated.

Solution

$$\begin{aligned}
 {}_{10}P_3 &= \frac{10!}{(10-3)!} = \frac{10!}{7!} \\
 &= \frac{10 \cdot 9 \cdot 8 \cdot \cancel{7} \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{7} \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}} \\
 &= 720 \text{ ways}
 \end{aligned}$$

أوجد عدد طرق تكوين رموز مكونة من ثلاثة أرقام من 10 أرقام لا يتكرر فيها أي رقم.

Chapter 10 Combination

Combination of n objects taken r at a time

A selection of r objects from a group of n objects without regard to order

$${}_n C_r = \frac{n!}{(n-r)!r!}$$

عدد التوافيق المختلفة لاختيار r من العناصر
مأخوذة من n في العناصر دون النظر في
الترتيب

Combination

A state's department of transportation plans to develop a new section of interstate highway and receives 16 bids for the project. The state plans to hire four of the bidding companies. How many different combinations of four companies can be selected from the 16 bidding companies?

Solution:

- $n = 16, r = 4$
- Order is not important

$$\begin{aligned} {}_{16}C_4 &= \frac{16!}{(16-4)!4!} \\ &= \frac{16!}{12!4!} \\ &= \frac{16 \cdot 15 \cdot 14 \cdot 13 \cdot 12!}{12! \cdot 4 \cdot 3 \cdot 2 \cdot 1} \\ &= 1820 \text{ different combinations} \end{aligned}$$

تخطط وزارة النقل بالولاية لتطوير قسم جديد من الطريق السريع بين الولايات وتتلقى 16 عرضاً للمشروع. وتخطط الدولة لتوظيف أربع من الشركات المتقدمة بالعطاءات. ما هو عدد المجموعات المختلفة المكونة من أربع شركات التي يمكن اختيارها من بين الشركات الـ 16 المتنافسة؟

Un Solved Examples

Chapter

The number of ways of forming 4-digit code from 9 numbers in which no digits is repeated

عدد طرق تكوين كود مكون من 4 أرقام من 9 أرقام لا يتكرر فيها أي رقم

Un Solved Examples

Chapter

The probability of an event for obtaining an odd number from a die = 1.0

احتمال حصول حدث ما على عدد فردي من حجر النرد = 1.0

Un Solved Examples

Chapter

The probability of an event for obtaining 9 from a die = 1.0

احتمال وقوع حدث للحصول على 9 من حجر النرد = 1.0

Un Solved Examples

Chapter

If the probability of an event is equal to 0.3,
Find the probability of its complement event

إذا كان احتمال وقوع حدث يساوي 0.3
أوجد احتمال الحدث المكمل له

Chapter Summary

Chapter

**Basic Concepts of
Probability and Counting**

Conditional Probability:

$$P(B|A) = \frac{P(B \text{ and } A)}{P(A)}$$

**Additional Topics in
Probability and Counting**

1- Permutation

2- Combination

Multiplication Rule: (and)

$$P(A \text{ and } B) = P(A) \times P(B|A)$$

The Addition Rule: (or)

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$



Thank You