

Business Mathematics

Prepared by Dr. Alfaisal Hasan

MTH 112
Business Mathematics

Lecture NO 9



Business Mathematics

Lecture NO 9

MTH 112

Business Mathematics

Prepared by
Dr. Alfaisal Hasan



Chapter

Topics

- Chapter 1: Mathematical preliminaries
- Chapter 2: The straight line and applications
- Chapter 3: Functions and Graphs
- Chapter 4: Limits and Continuity
- Chapter 5: Financial Mathematics
- Chapter 6: Differentiation
- Chapter 7: Methods of Differentiation
- Chapter 8: Function of several variables
- **Chapter 9: Integration and applications**
- Chapter 10: Linear algebra and applications

Differentiation

$f'(x)$ “ f prime of x ”

Remember that

y' “ y prime”

$\frac{dy}{dx}$ “the derivative of y with respect to x ”

$\frac{d}{dx}$ is a verb. “Take the derivative with respect to x ...”

Differentiation

Remember that

- Differentiation is the process of computing the derivative of a function.

You may be asked to:

- Differentiate.
- Derive.
- Find the derivative of...

Remember that

$$\frac{d}{dx}[c] = 0, \quad c \text{ is a constant}$$

The derivative of a constant function is zero.

Differentiation

Remember that

$$\frac{d}{dx}[x^N] = Nx^{N-1}, \quad N \text{ is any real number}$$

$$\frac{d}{dx}[x] = 1$$

Remember that

$$\frac{d}{dx}[c(f(x))] = c(f'(x)), \quad c \text{ is a constant}$$

- The derivative of a constant times a function is equal to the constant times the derivative of the function.

Differentiation

Remember that

$$\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$$

The derivative of a sum is the sum of the derivatives.

$$\frac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x)$$

The derivative of a difference is the difference of the derivatives.

Remember that

The derivative of the product of two differentiable functions is given by

$$y = f(x).g(x)$$

$$\frac{dy}{dx} = y' = f'(x).g(x) + f(x).g'(x)$$

$$\frac{dy}{dx} = y' = f'.g + f.g'$$

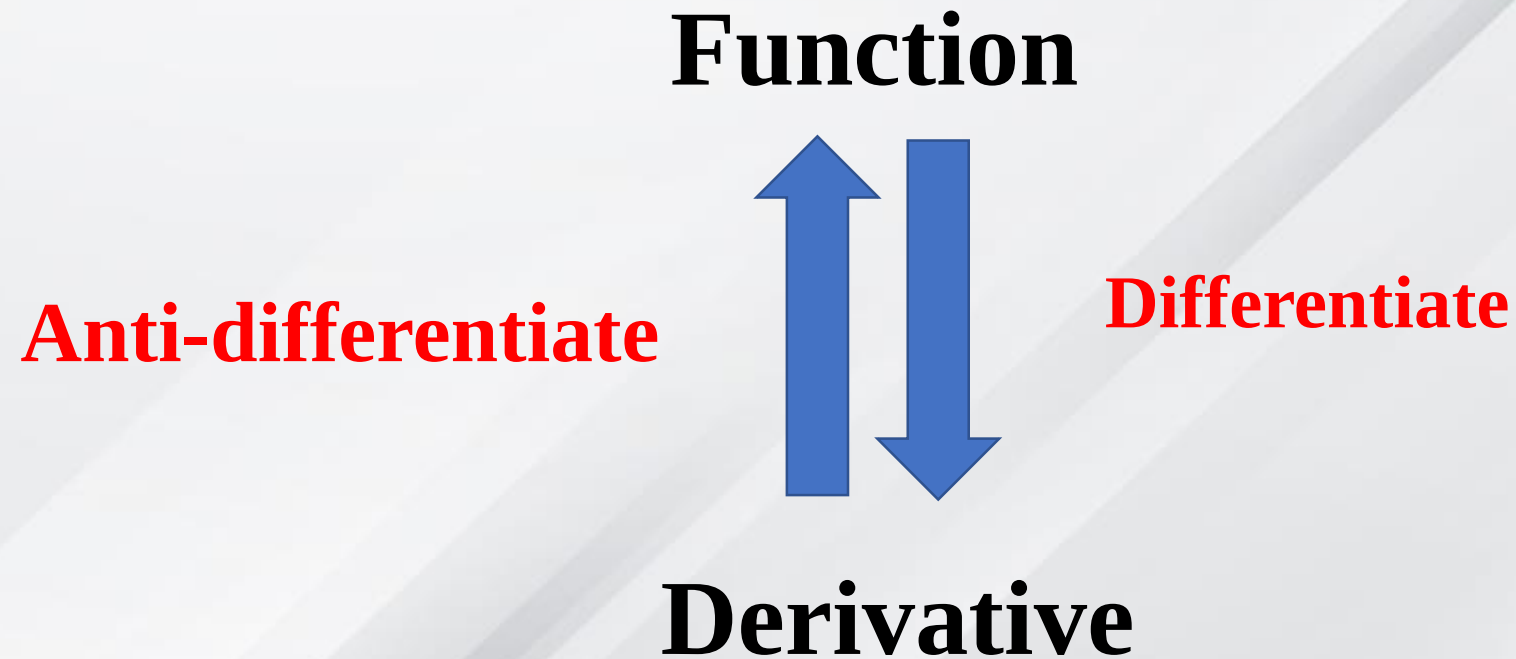
Remember that

- The derivative of the quotient of two differentiable functions is given by

$$\frac{d}{dx} \left[\frac{f(x)}{g(x)} \right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2} \quad (g(x) \neq 0)$$

Integration and Applications

- Integration or Anti-derivative of the Function



Integration and Applications

$$f(x) = x^3$$

$$f(x) = x^3 + 4$$

$$f(x) = x^3 - 7$$

Differentiate

$$f'(x) = 3x^2$$

Anti-differentiate ???

$$f'(x) = 3x^2$$

Anti-differentiate will be

$$f(x) = x^3 + C$$

C Is the constant of integration

Integration and Applications

Chapter

We can write $\int (3x^2) dx = x^3 + C$

$\int$ Is the integration sign

dx Means that integration with respect to x

C Is the constant of integration

$(3x^2)$ Is the derivative of x^3

Integration and Applications

In general

$$\int f'(x) dx = f(x) + C$$

$\int$ Is the integration sign

dx Means that integration with respect to x

C Is the constant of integration

$f'(x)$ Is the derivative of $f(x)$

Integration and Applications

Find $\int (4x^3) dx$

$$\int (4x^3) dx = x^4 + C$$

Integration and Applications

Chapter

In general

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

Integration and Applications

Basic Integration Rules

Differentiation Formula

$$\frac{d}{dx}[C] = 0$$

$$\frac{d}{dx}[kx] = k$$

$$\frac{d}{dx}[kf(x)] = kf'(x)$$

$$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$$

$$\frac{d}{dx}[x^n] = nx^{n-1}$$

Integration Formula

$$\int 0 \, dx = C$$

$$\int k \, dx = kx + C$$

$$\int kf(x) \, dx = k \int f(x) \, dx$$

$$\int [f(x) \pm g(x)] \, dx = \int f(x) \, dx \pm \int g(x) \, dx$$

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1 \quad \text{Power Rule}$$

Integration and Applications

$$\int kx^n dx = \frac{kx^{n+1}}{n+1} + C$$

Integration and Applications

Find

$$\int 3x \, dx$$

Integration and Applications

$$\int 3x \, dx = 3 \int x \, dx$$

$$= 3 \int x^1 \, dx$$

$$= 3 \left(\frac{x^2}{2} \right) + C$$

$$= \frac{3}{2} x^2 + C$$

Integration and Applications

Find

$$\int \frac{1}{x^2} dx$$

Integration and Applications

Chapter

$$\begin{aligned} F(x) &= \int \frac{1}{x^2} dx \\ &= \int x^{-2} dx \\ &= \frac{x^{-1}}{-1} + C = \frac{-1}{x} + C \end{aligned}$$

Integration and Applications

Find

$$\int \sqrt{x} \, dx$$

Integration and Applications

$$\begin{aligned}\int \sqrt{x} \, dx \\&= \int x^{\frac{1}{2}} \, dx \\&= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + C \\&= \frac{2}{3} x^{\frac{3}{2}} + C\end{aligned}$$

Integration and Applications

Find

$$\int \frac{1}{\sqrt{x}} dx$$

Integration and Applications

Chapter

$$\begin{aligned}\int \frac{1}{\sqrt{x}} dx &= \int x^{-\frac{1}{2}} dx \\&= \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C \\&= 2x^{\frac{1}{2}} + C \\&= 2\sqrt{x} + C\end{aligned}$$

Integration and Applications

Find

$$\int \frac{1}{\sqrt[3]{x}} dx$$

Integration and Applications

Chapter

$$\int \frac{1}{\sqrt[3]{x}} dx = \int x^{-\frac{1}{3}} dx$$

$$= \frac{x^{\frac{2}{3}}}{\frac{2}{3}} + C$$

$$= \frac{3}{2} x^{\frac{2}{3}} + C$$

Integration and Applications

Find

$$\int (x^6 + 5x - 4) dx$$

Integration and Applications

$$\int (x^6 + 5x - 4) dx$$

$$= \int (x^6) dx + \int (5x) dx + \int (-4) dx$$

$$= \frac{x^7}{7} + \frac{5x^2}{2} - 4x + C$$

Integration and Applications

Find

$$\int \left(10x^4 + \frac{1}{x^2} + 6\sqrt{x} - 3 \right) dx$$

Integration and Applications

Chapter

$$\begin{aligned}\int \left(10x^4 + \frac{1}{x^2} + 6\sqrt{x} - 3 \right) dx &= \\ \int 10x^4 dx + \int x^{-2} dx + \int 6x^{\frac{1}{2}} dx - \int 3 dx &= \\ = \frac{10x^5}{5} + \frac{x^{-1}}{-1} + \frac{6x^{\frac{3}{2}}}{\frac{3}{2}} - 3x + C &= \\ = 2x^5 - \frac{1}{x} + 4\sqrt{x^3} - 3x + C\end{aligned}$$

Integration and Applications

Chapter

$$\int f'(x) dx = f(x) + C$$

$$\int_a^b f'(x) dx = [f(x)]_a^b + \cancel{C}$$

$$= f(b) - f(a)$$

Integration and Applications

Find

$$\int_0^2 3x \, dx$$

Integration and Applications

Chapter

$$\begin{aligned}\int_0^2 3x \, dx &= 3 \int_0^2 x \, dx \\&= \left[3 \frac{x^2}{2} \right]_0^2 \\&= \frac{3}{2} [x^2]_0^2 \\&= \frac{3}{2} ((2^2) - (0^2)) = 6\end{aligned}$$

Integration and Applications

Chapter

Find

$$\int_1^3 \frac{1}{x^2} dx$$

Integration and Applications

Chapter

$$\begin{aligned}\int_1^3 \frac{1}{x^2} dx &= \int_1^3 x^{-2} dx \\&= \left[\frac{x^{-1}}{-1} \right]_1^3 \\&= \left[\frac{-1}{x} \right]_1^3 = \left(-\frac{1}{3} \right) - \left(-\frac{1}{1} \right) \\&= \frac{2}{3}\end{aligned}$$

Integration and Applications

Find

$$\int_0^4 \sqrt{x} \, dx$$

Integration and Applications

Chapter

$$\begin{aligned} & \int_0^4 \sqrt{x} \, dx \\ &= \int_0^4 x^{\frac{1}{2}} \, dx \\ &= \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^4 \\ &= \frac{2}{3} \left[x^{\frac{3}{2}} \right]_0^4 = \frac{2}{3} \left(\left(4^{\frac{3}{2}} \right) - \left(0^{\frac{3}{2}} \right) \right) = \frac{16}{3} \end{aligned}$$

Integration and Applications

Find

$$\int_4^9 \frac{1}{\sqrt{x}} dx$$

Integration and Applications

$$\begin{aligned}\int_4^9 \frac{1}{\sqrt{x}} dx &= \int_4^9 x^{-\frac{1}{2}} dx \\&= \left[\frac{x^{\frac{1}{2}}}{\frac{1}{2}} \right]_4^9 = \left[2x^{\frac{1}{2}} \right]_4^9 \\&= 2 \left[\sqrt{x} \right]_4^9 = 2 \left[(\sqrt{9}) - (\sqrt{4}) \right] \\&= 2[3 - 2] = 2\end{aligned}$$

Integration and Applications

Find

$$\int_{-2}^3 (x^6 + 5x - 4) dx$$

Integration and Applications

Chapter

$$\begin{aligned} & \int_{-2}^3 (x^6 + 5x - 4) dx \\ &= \left[\frac{x^7}{7} + \frac{5x^2}{2} - 4x \right]_{-2}^3 \\ &= \left[\frac{(3)^7}{7} + \frac{5(3)^2}{2} - 4(3) \right] - \left[\frac{(-2)^7}{7} + \frac{5(-2)^2}{2} - 4(-2) \right] \\ &= \frac{4525}{14} \end{aligned}$$

Integration and Applications

Chapter

$$\int_a^a f'(x) dx = [f(x)]_a^a$$

$$= f(a) - f(a) = 0$$

$$\int_a^a f'(x) dx = 0$$

Integration and Applications

Chapter

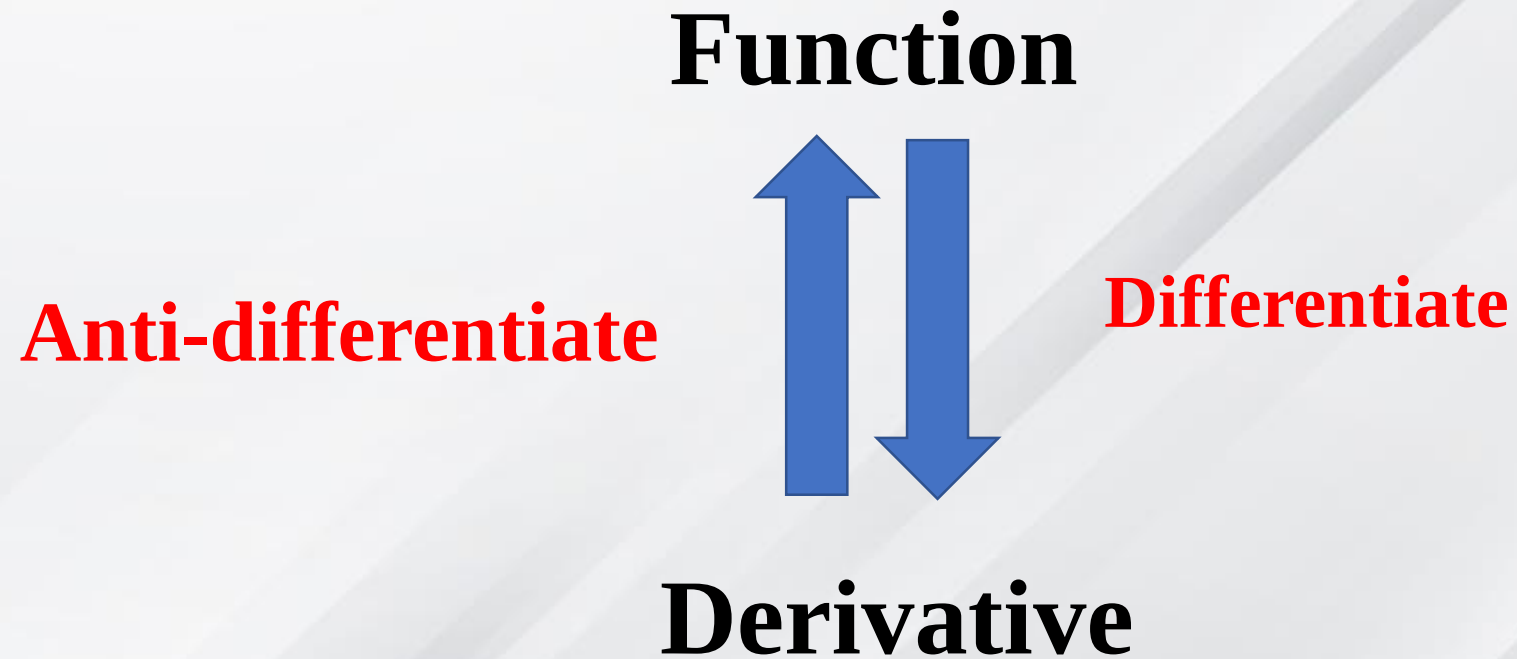
Find

$$\int_3^3 (x^6 + 5x - 4) dx$$

Without solution

$$\int_3^3 (x^6 + 5x - 4) dx = 0$$

- Integration or Anti-derivative of the Function



Integration and Applications

$$f(x) = x^3$$

$$f(x) = x^3 + 4$$

$$f(x) = x^3 - 7$$

Differentiate

$$f'(x) = 3x^2$$

Anti-differentiate ???

Integration and Applications

$$f'(x) = 3x^2$$

Anti-differentiate will be

$$f(x) = x^3 + C$$

C Is the constant of integration

Integration and Applications

Chapter

We can write $\int (3x^2) dx = x^3 + C$

$\int$ Is the integration sign

dx Means that integration with respect to x

C Is the constant of integration

$(3x^2)$ Is the derivative of x^3

Integration and Applications

Chapter

In general

$$\int f'(x) dx = f(x) + C$$

$\int$ Is the integration sign

dx Means that integration with respect to x

C Is the constant of integration

$f'(x)$ Is the derivative of $f(x)$

Integration and Applications

Chapter

Find $\int (4x^3) dx$

$$\int (4x^3) dx = x^4 + C$$

Integration and Applications

In general

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

Integration and Applications

Basic Integration Rules

Differentiation Formula

$$\frac{d}{dx}[C] = 0$$

$$\frac{d}{dx}[kx] = k$$

$$\frac{d}{dx}[kf(x)] = kf'(x)$$

$$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$$

$$\frac{d}{dx}[x^n] = nx^{n-1}$$

Integration Formula

$$\int 0 \, dx = C$$

$$\int k \, dx = kx + C$$

$$\int kf(x) \, dx = k \int f(x) \, dx$$

$$\int [f(x) \pm g(x)] \, dx = \int f(x) \, dx \pm \int g(x) \, dx$$

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1 \quad \text{Power Rule}$$

Integration and Applications

$$\int kx^n dx = \frac{kx^{n+1}}{n+1} + C$$

Un Solved Examples

Chapter

Find

$$\int \sqrt{x^3} \, dx$$

Un Solved Examples

Chapter

Find

$$\int \frac{1}{\sqrt{x^5}} dx$$

Un Solved Examples

Chapter

Find

$$\int (x^9 + 5x^2 - 14) dx$$

Un Solved Examples

Chapter

Find

$$\int_1^3 \frac{1}{x^3} dx$$

Un Solved Examples

Chapter

Find

$$\int_1^2 (x^3 + 5x - 4) dx$$

Un Solved Examples

Chapter

Find

$$\int_1^1 (x^3 + 5x - 4) dx$$



Thank You