

Master of Financial Technology

FNT 615
Finance Concepts





Time Value of Money

Chapter 5



Chapter 04

Intended Learning Outcomes

After reading this chapter, you should be able to:

1. Explain the mechanics of compounding and bringing the value of money back to the present.
2. Understand Annuities
3. Determine the future or present value of a sum when there are nonannual compounding periods.
4. Determine the present value of an uneven stream of payments and understand perpetuities.

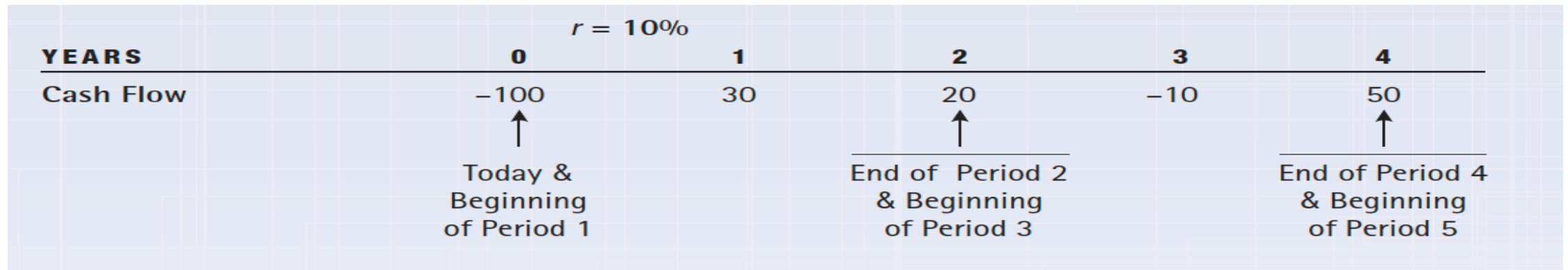
1. Compound Interest, Future, and Present Value

Using Timelines to Visualize Cash Flows

- We begin our study of the time value of money with some basic tools for visualizing the time pattern of cash flows.
- While timelines seem simple at first glance, they can be a tremendous help for more complicated problems.
- As a first step in visualizing cash flows, we can construct a **timeline**, a linear representation of the timing of cash flows.
- **A timeline identifies the timing and amount of a stream of cash flows—both cash received, and cash spent—along with the interest rate it earns.**
- Timelines are a **critical first step** used by financial analysts to solve financial problems.

Using Timelines to Visualize Cash Flows

- To illustrate how to construct a timeline, consider the following example, where we receive annual cash flows over the course of 4 years.
- The following timeline shows our cash inflows and outflows from time period 0 (the present) until the end of year 4:



Using Timelines to Visualize Cash Flows

- For our purposes, **time periods are identified on the top of the timeline**, and in this example, the time periods are measured in years, which are indicated on the far left of the timeline.
- Thus, **time period 0** is both today and the beginning of the first year.
- The dollar amount of the cash flow received or spent at each time period is shown below the timeline.
- **Positive values** represent cash inflows.
- **Negative values** represent cash outflows.
- For example, in the timeline shown, a \$100 cash outflow occurs today, or at time 0, followed by cash inflows of \$30 and \$20 at the end of years 1 and 2, a negative cash flow (a cash outflow) of \$10 at the end of year 3, and finally a cash inflow of \$50 at the end of year 4.

Using Timelines to Visualize Cash Flows



The **units of measurement on the timeline are time periods** and are typically expressed in years but could be expressed as months, days, or any unit of time.



However, for now, let's assume we're looking at cash flows that occur annually.



Thus, the **distance between 0 and 1** represents the period between today and the end of the first year.



Consequently, **time period 0 indicates today**, while time period 1 represents the end of the first year, which is also the beginning of the second year.



(You can think of it as being both the last second of the first year and the first second of the second year.)



The interest rate, which is 10 percent in this example, is listed above the timeline.

Using Timelines to Visualize Cash Flows

- In this chapter and throughout the text, we will often refer to the **idea of moving money through time**.
- Most business investments involve investing money today. Then, in subsequent years, the investment produces cash that comes back to the business.

Using Timelines to Visualize Cash Flows

- To evaluate an investment, you need to **compare the amount of money the investment requires today with the amount of money the investment will return to you in the future and adjust these numbers for the fact that a dollar today is worth more than a dollar in the future.**
- Timelines simplify solving time value of money problems, and they are not just for beginners; experienced financial analysts use them as well.
- In fact, the more complex the problem you're trying to solve, the more valuable a timeline will be in terms of helping you visualize exactly what needs to be done.

The Time Value of Money

- The time value of money refers to the observation that it is better to receive money sooner than later.
- Money that you have in hand today can be invested to earn a positive rate of return, producing more money tomorrow.
- For that reason , a dollar today is worth more than a dollar in the future.



Compound Interest

- Most of us encounter the concept of compound interest at an early age.
- Compound interest occurs when interest paid on the investment during the first period is added to the principal; then, during the second period, interest is earned on this new sum.
- For example, suppose we place \$100 in a savings account that pays 6 percent interest, compounded annually. How will our savings grow?
- At the end of the first year we have earned 6 percent, or \$6 on our initial deposit of \$100, giving us a total of \$106 in our savings account, thus,
 - **Value at the end of year 1 = present value * (1 + interest rate)**
= \$100(1 + 0.06)
= \$100(1.06)
= \$106

Compound Interest

- Carrying these calculations one period further, we find that we now earn the 6 percent interest on a principal of \$106, which means we earn \$6.36 in interest during the second year.
- Why do we earn more interest during the second year more than we did during the first?
- Simply because we now earn interest on the sum of the original principal and the interest we earned in the first year.
- In effect we are now earning interest on interest; this is the concept of compound interest.

Value at the end of year 2 = $\$106 * (1.06) = \112.36



Compound Interest

- Looking back, we can see that the future value at the end of year 1, or \$106, is actually equal to the present value times $(1 + r)$, or $\$100(1 + 0.06)$.
- Moving forward to year 2 we find,
- Value at the end of year 2 = present value * $(1 + r)$ * $(1 + r)$ = present value * $(1 + r)^2$
- Carrying this forward into the third year, we find that we enter the year with \$112.36 and we earn 6 percent, or \$6.74 in interest, giving us a total of \$119.10 in our savings account.
- This can be expressed as Value at the end of year 3 = present value * $(1 + r)$ * $(1 + r)$ * $(1 + r)$ = present value * $(1 + r)^3$
- By now a pattern is becoming apparent. We can generalize this formula to illustrate the future value of our investment if it is compounded annually at a rate of r for n years to be

Future value = present value * $(1 + r)^n$



Compound Interest

- Letting FV_n stand for the future value at the end of n periods and PV stand for the present value, we can rewrite this equation as
- $FV_n = PV(1 + r)^n$
- We also refer to $(1 + r)^n$ as the future value factor.
- Thus, to find the future value of a dollar amount, all you need to do is multiply that dollar amount times the appropriate future value factor,

Future value = present value * (future value factor)

- where future value factor = $(1 + r)^n$.

Compound Interest

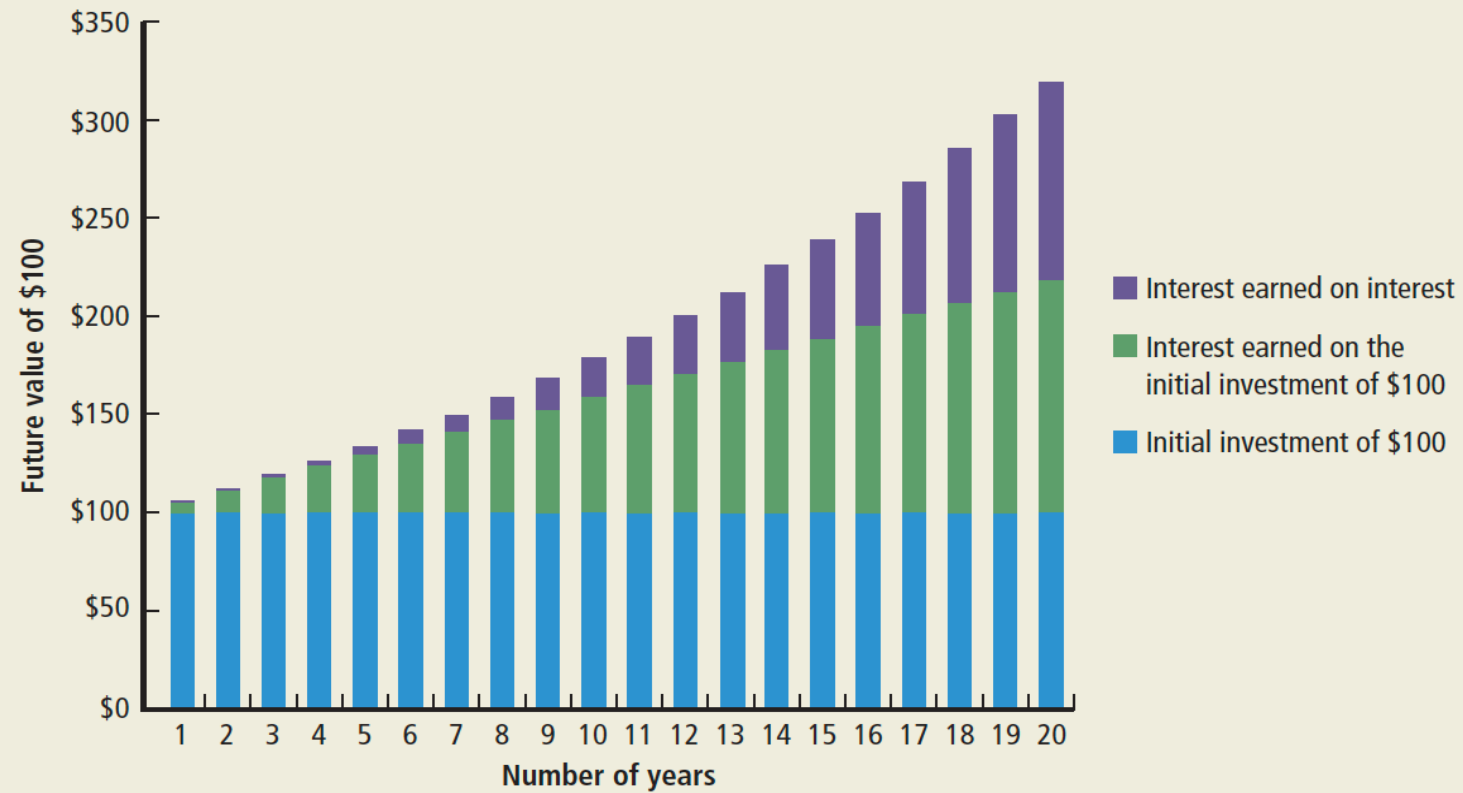


Future value is the amount to which your investment will grow, or a future dollar amount.



Future value factor is the value of $(1 + r)^n$ used as a multiple to calculate an amount's future value.

FIGURE 5-1 \$100 Compounded at 6 Percent Over 20 Years



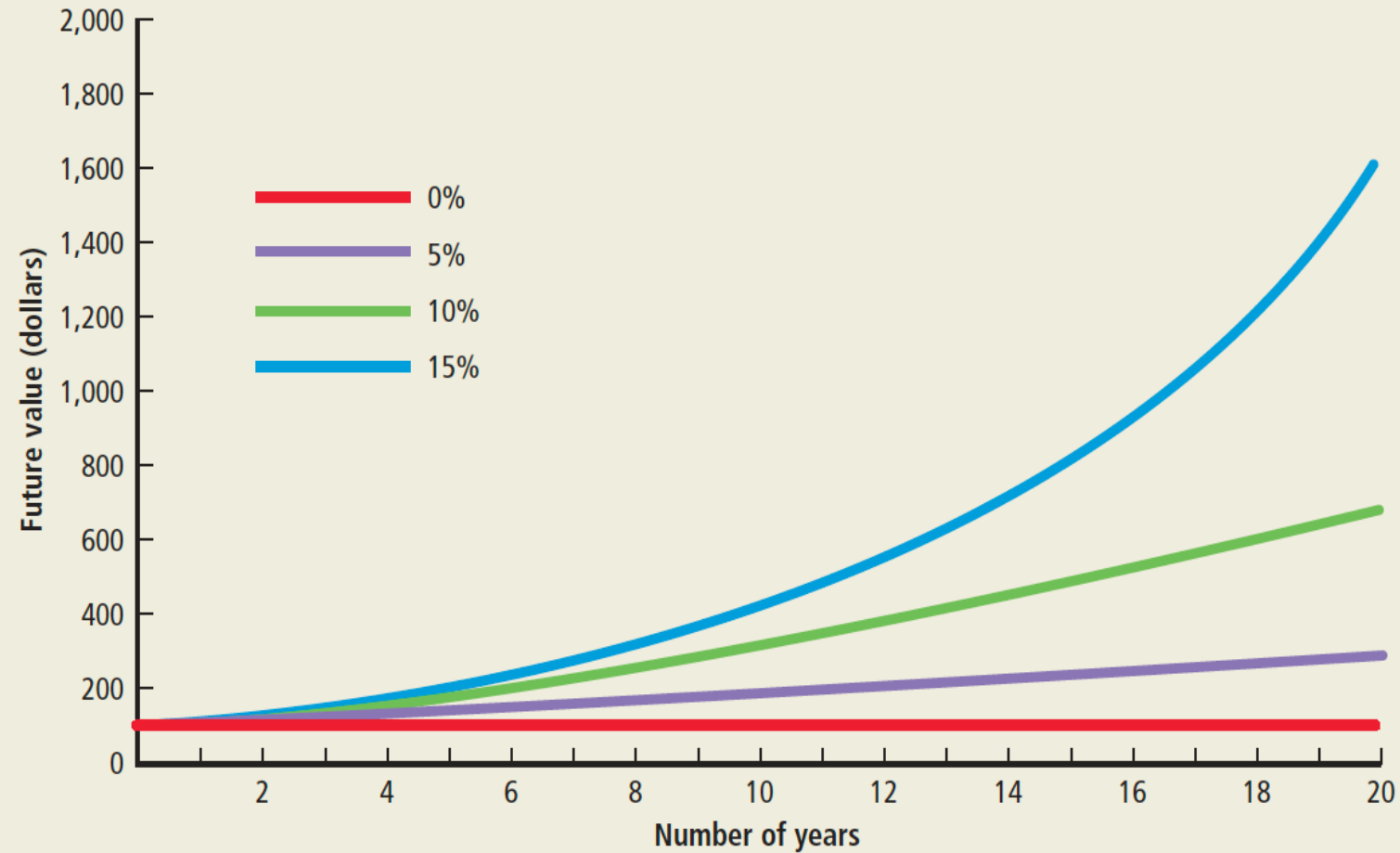
Compound Interest

- Figure 5-1 illustrates how this investment of \$100 would continue to grow for the first 20 years at a compound interest rate of 6 percent.
- Notice how the amount of interest earned annually increases each year.

Compound Interest

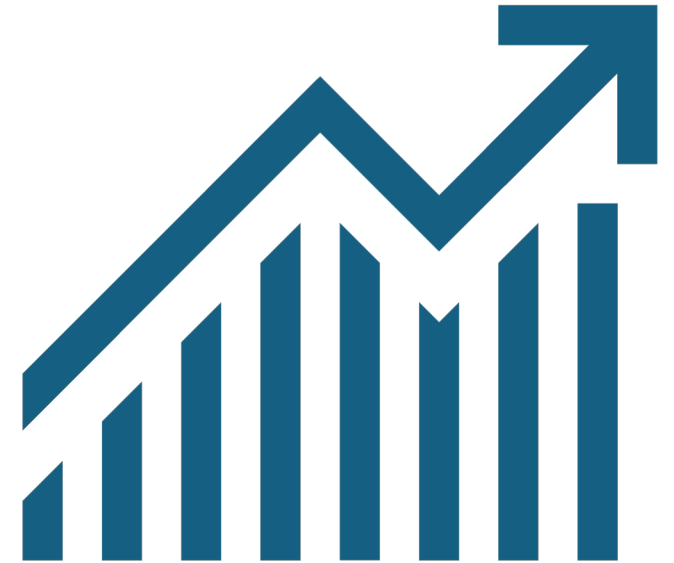
- Again, the reason is that each year interest is received on the sum of the original investment plus any interest earned in the past.
- The situation in which **interest is earned on interest that was earned in the past is referred to as compound interest.**
- If you only earned interest on your initial investment, it would be referred to **as simple interest.**

FIGURE 5-2 The Future Value of \$100 Initially Deposited and Compounded at 0, 5, 10, and 15 Percent



Future Value

- When we examine the relationship between the number of years an initial investment is compounded for and its future value, as shown graphically in Figure 5-2, we see that we can **increase the future value of an investment by either**
 - Increasing the number of years for which we let it compound or
 - Compounding it at a higher interest rate.
- We can also see this from equation (5-1) because an increase in either r or n while the present value is held constant results in an increase in the future value.



EXAMPLE 5.1**Calculating the future value of an investment**

If we place \$1,000 in a savings account paying 5 percent interest compounded annually, how much will our account accrue to in 10 years?

STEP 1: FORMULATE A SOLUTION STRATEGY

The future value of the savings account takes into account the present value of the account and multiplies it by the annual interest rate each year it is applied, that is,

$$\text{Future value} = \text{present value} \times (1 + r)^n \quad (5-1)$$

where r = annual interest rate and n = the number of years.

STEP 2: CRUNCH THE NUMBERS

Substituting present value, percent, and 10 years into equation (5-1), we get the following:

$$\begin{aligned} FV_n &= \$1,000(1 + 0.05)^{10} \\ &= \$1,000(1.62889) \\ &= \$1,628.89 \end{aligned} \quad (5-1)$$

STEP 3: ANALYZE YOUR RESULTS

At the end of the 10 years, we will have \$1,628.89 in our savings account.

Questions

1. The time value of money is the opportunity cost of passing up the earning potential of a dollar today.
2. A timeline identifies the timing and amount of a stream of cash flows, along with the interest rate it earns.
3. If you only earned interest on your initial investment, and not on previously earned interest, it would be called simple interest.

Questions

4. An investment earning simple interest is preferred over an investment earning compound interest because the simplicity adds value.
5. \$10,000 invested at 10% per year for 5 years earns interest equal to \$6,105.10; therefore, \$10,000 invested at 10% per year for 10 years will earn interest equal to \$12,210.20 (2 times \$6,105.10).
6. Tim has \$100 in a bank account paying 2% interest per year. At the end of 5 years, Tim's bank account balance will be \$110 if interest is not compounded but will be greater than \$110 if interest is compounded.

Answer

1. True
2. True
3. True
4. False
5. False
6. true

Present Value

- We are now going to look at the reverse question: **What is the value in today's dollars of a sum of money to be received in the future?**
- In this case we are moving future money back to the present.
- We will determine the present value of a lump sum, which in simple terms is the **current value of a future payment**.

Present Value

- In fact, we will be doing nothing other than **inverse compounding (discounting)**.
- In compounding, we talked about the compound interest rate and the initial investment.
- In determining the present value, we will talk about the discount rate and present value of future cash flows.

Present Value

$$\text{Present value} = \text{future value at the end of year } n \times \left[\frac{1}{(1 + r)^n} \right]$$

or

$$PV = FV_n \left[\frac{1}{(1 + r)^n} \right]$$

$$\text{Present value} = \text{future value (present value factor)}$$

where

$$\text{Present value factor} = \left[\frac{1}{(1 + r)^n} \right]$$

EXAMPLE 5.4**Calculating the discounted value to be received in 10 years**

What is the present value of \$500 to be received 10 years from today if our discount rate is 6 percent?

STEP 1: FORMULATE A SOLUTION STRATEGY

The present value to be received can be calculated using equation (5-2) as follows:

$$\text{Present value} = FV_n \left[\frac{1}{(1 + r)^n} \right] \quad (5-2)$$

STEP 2: CRUNCH THE NUMBERS

Substituting $FV = \$500$, $n = 10$, and $r = 6$ percent into equation (5-2), we find:

$$\begin{aligned} \text{Present value} &= \$500 \left[\frac{1}{(1 + 0.06)^{10}} \right] \\ &= \$500(0.5584) \\ &= \$279.20 \end{aligned}$$

STEP 3: ANALYZE YOUR RESULTS

Thus, the present value of the \$500 to be received in 10 years is \$279.20.

Example

- You're on vacation in a rather remote part of Florida and see an advertisement stating that if you take a sales tour "you will be given \$100 just for taking the tour." However, the \$100 that you get is in the form of a savings bond that will not pay you the \$100 for 10 years. What is the present value of \$100 to be received 10 years from today if your discount rate is 6 percent?

STEP 1: FORMULATE A SOLUTION STRATEGY

The present value of our savings bond can be computed using equation (5-2) as follows:

$$\text{Present value} = FV_n \left[\frac{1}{(1 + r)^n} \right] \quad (5-2)$$

STEP 2: CRUNCH THE NUMBERS

Substituting $FV = \$100$, $n = 10$, and $r = 6$ percent into equation (5-2), we compute the present value as follows:

$$\begin{aligned} \text{Present value} &= \$100 \left[\frac{1}{(1 + 0.06)^{10}} \right] \\ &= \$100(0.5584) \\ &= \$55.84 \end{aligned}$$

STEP 3: ANALYZE YOUR RESULTS

Thus, the value in today's dollars of that \$100 savings bond is only \$55.84.

Present Value



Again, we have only one present-value–future-value equation; that is, equations (5-1) and (5-2) are different formats of the same equation.



We have introduced them as separate equations to simplify our calculations; in one case we are determining the value in future dollars, and in the other case the value in today's dollars.



In either case, the reason is the same: to compare values on alternative investments and to recognize that the value of a dollar received today is not the same as that of a dollar received at some future date.



In other words, **we must measure the dollar values in dollars of the same time period.**

Present Value



For example, if we looked at these projects—one that promised \$1,000 in 1 year, one that promised \$1,500 in 5 years, and one that promised \$2,500 in 10 years—the **concept of present value allows us to bring their flows back to the present and make those projects comparable.**



Moreover, because all present values are comparable (they are all measured in dollars of the same time period), we can add and subtract the present value of inflows and outflows to determine the present value of an investment.

EXAMPLE 5.6**Calculating the present value of an investment**

What is the present value of an investment that yields \$1,000 to be received in 7 years and \$1,000 to be received in 10 years if the discount rate is 6 percent?

STEP 1: FORMULATE A SOLUTION STRATEGY

The present value of our investment can be computed using equation (5-2) for each yield, then adding these values together as follows:

$$\text{Present value} = FV_n \left[\frac{1}{(1 + r)^n} \right] + FV_n \left[\frac{1}{(1 + r)^n} \right] \quad (5-2)$$

STEP 2: CRUNCH THE NUMBERS

Substituting into equation (5-2), we compute the future value as follows:

$$\begin{aligned} \text{Present value} &= \$1,000 \left[\frac{1}{(1 + 0.06)^7} \right] + \$1,000 \left[\frac{1}{(1 + 0.06)^{10}} \right] \\ &= \$665.06 + \$558.39 = \$1,223.45 \end{aligned}$$

STEP 3: ANALYZE YOUR RESULTS

Again, present values are comparable because they are measured in the same time period's dollars.

With a financial calculator, this becomes a three-step solution, as shown in the margin. First, you'll solve for the present value of the \$1,000 received at the end of 7 years, then you'll solve for the present value of the \$1,000 received at the end of 10 years, and finally, you'll add the two present values together. Remember, once you've found the present value of those future cash flows you can add them together because they're measured in the same period's dollars.

Questions

7. You have a savings bond that will be worth \$750 when it matures in 3 years, but you need cash today. If the current going rate of interest is 5%, what is your bond worth if you sell it today (rounded to the nearest dollar)?

- A) \$675
- B) \$648
- C) \$625
- D) \$612

8. How much money do I need to place into a bank account that pays a 1.08% rate in order to have \$500 at the end of 7 years?

- A) \$332.54
- B) \$751.81
- C) \$463.78
- D) \$629.51

Questions

9. A car manufacturer offers either \$2,000 cash back or zero percent financing for 5 years. A rational consumer will always take the cash back because money received today is worth more than money received in the future.
10. The present value of a single future sum of money is inversely related to both the number of years until payment is received and the discount rate.

Questions

11. The present value of a single future sum

- A) increases as the number of discount periods increases.
- B) is generally larger than the future sum.
- C) depends upon the number of discount periods.
- D) increases as the discount rate increases.

12. The present value of \$1,000 to be received in 5 years is _____ if the discount rate is 12.78%.

- A) \$368
- B) \$494
- C) \$548
- D) \$687

Questions

13. What is the present value of \$11,463 to be received 7 years from today? Assume a discount rate of 3.5% compounded annually and round to the nearest \$1.

- A) \$5,790
- B) \$6,508
- C) \$7,210
- D) \$9,010

14. How much money must be put into a bank account yielding 6.42% (compounded annually) in order to have \$1,671 at the end of 11 years (round to nearest \$1)?

- A) \$921
- B) \$886
- C) \$843
- D) \$798

Questions

- 15. How much money do I need to place into a bank account that pays a 1.08% rate in order to have \$500 at the end of 7 years?
 - A) \$332.54
 - B) \$751.81
 - C) \$463.78
 - D) \$629.51
- 16. Assuming two investments have equal lives, a high discount rate tends to favor
 - A) the investment with large cash flow early.
 - B) the investment with large cash flow late.
 - C) the investment with even cash flow.
 - D) neither investment since they have equal lives.

Answers

- 7. B
- 8. C
- 9. False
- 10. True
- 11. C
- 12. C
- 13. D
- 14. C
- 15. C
- 16. A

Thank You

